

# Non-Similarity Transformations of MHD flow of Third Grade Fluid over a Horizontal Circular Cylinder in the Presence of Radiation and Suction

K. Madhavi<sup>1</sup>, N. Nagendra<sup>1\*</sup>, G.S.S. Raju<sup>2</sup>, V. Ramachandra Prasad<sup>1</sup>

Department of Mathematics, Madanapalle Institute of Technology and Science, Madanapalle, Andhra Pradesh, India.<sup>1</sup>

Department of Mathematics, JNTUA College of Engineering Pulivendula, Pulivendula, Andhra Pradesh, India.<sup>2</sup>

**ABSTRACT:** In the present paper, we analysed MHD boundary layer flow and heat transfer in third grade fluid over a horizontal circular cylinder in the presence of radiation and suction effects. We also considered thermal slip effect. The related nonlinear coupled ordinary partial differential equations subjected to their boundary conditions by the use of a non-similarity transformation. These equations are solved numerically by applying an efficient and unconditionally stable Keller Box finite difference scheme. The dependence of velocity and temperature profiles on the various pertinent parameters like third grade fluid parameter, material fluid parameters, magneto hydrodynamic parameter, radiation parameter, suction parameter and slip parameters are shown and discussed by sketching the graphs.

**KEYWORDS:** MHD; Suction; Thermal radiation; Heat transfer; Third grade fluid; Horizontal circular cylinder.

## I. INTRODUCTION

Many modern engineering applications involve the study of non-Newtonian fluids. Analysis of non-Newtonian fluids has been the effort of several investigations during the past few decades because of its extensive engineering and industrial applications. Especially, flow and heat transfer of non-Newtonian fluids play important role in food engineering, petroleum production, power engineering and in polymer solutions and in polymer melt in the plastic processing industries. Thus researchers are attracted to investigate its thermo physical properties.

Magneto hydrodynamic flows also arise in rheological wire coating processes [1]. An interesting non-Newtonian model developed for chemical engineering system. This rheological model has certain benefits over the other non-Newtonian formulations, including simplicity, ease of computation and physical robustness. Sumalatha C and Shankar B [2] have discussed Casson Fluid Flow over Nonlinear Stretching Sheet in presence of Radiations and Heat Source/Sink. Subba Rao et al. [3] analysed Heat and Mass Transfer of Non-Newtonian Fluid Past a Vertical Plate with Suction and Injection. For instance, Hayat et al [4] have examined MHD on the boundary layer of axisymmetric flow of third-grade fluid over stretching cylinder. Hayat and Mustafa [5] studied the analysis of a third grade fluid over a stretching surface with heat and mass transfer. Yuru soy and Pakdermirli [6] analyzed the heat transfer flow of third- grade fluid in a pipe. Massoudi and Christie [7] studied the fully developed flow of an incompressible, thermodynamically compatible fluid of grade three in a pipe. Anwar Beg et al. [8] explained the numerical solution presented for the natural convective dissipate heat transfer an incompressible third-grade non-Newtonian fluid flowing past an infinite porous plate embedded in Darcy – Forcheimer porous medium. Oluwole Daniel Makinde [9,10] explained to investigate the thermal criticality for a reactive third-grade liquid flowing steadily between two parallel isothermal plates. Bikash Sahoo [11] analysis the two-dimensional stagnation-point flow and heat transfer analysis of the thermodynamically compatible third grade.

The objective of the present study is to examine ‘Thermal slip effect on MHD flow of Third Grade Fluid over a Horizontal Circular Cylinder in the Presence of Radiation and Suction’. Numerical solutions for the velocity and the temperature distributions are obtained using a Keller-Box finite difference method.

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Vol. 6, Special Issue 13, July 2017

## II. MATHEMATICAL MODEL

The regime under investigation is illustrated in Fig. 1. Represents steady, incompressible hydromagnetic Third grade non-Newtonian boundary layer flow and heat transfer from a cylindrical body under radial magnetic field is considered.

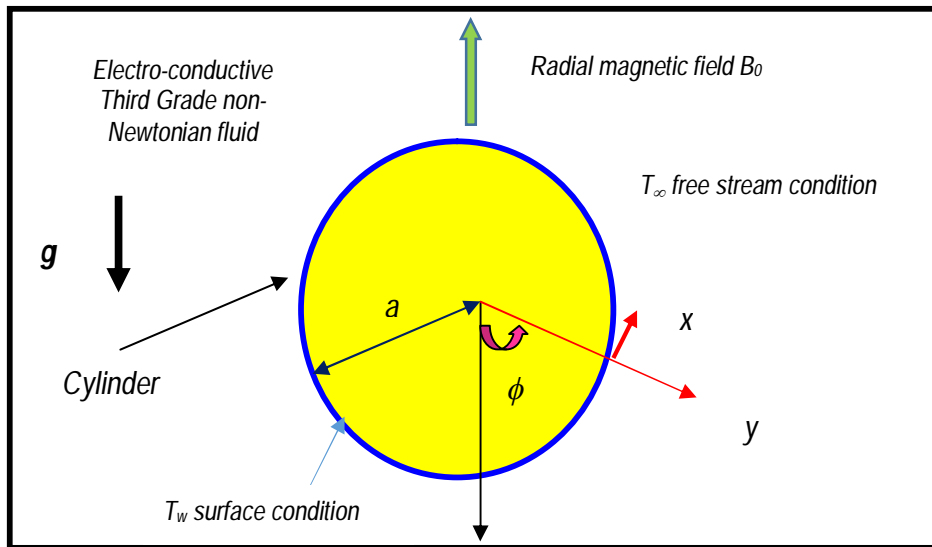


Fig 1: Magnetohydrodynamic non-Newtonian heat transfer from a cylinder

Here x-coordinate is measured along the circumference of the horizontal cylinder from the lowest point and the y-coordinate is measured normal to the surface, with 'a' denoting the radius of the horizontal cylinder.  $\Phi = x/a$  is the angle of orientation of the y-axis with respect to the vertical  $0 \leq \Phi \leq \pi$ . The gravitational acceleration,  $g$  acts downwards. Both the horizontal cylinder and the fluid are maintained initially at the same temperature. Instantaneously they are raised to a temperature  $T_w > T_\infty$  i.e. the ambient temperature of the fluid which remains unchanged.

The corresponding velocities in the x and y directions are u and v respectively. The governing conservation equations can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho} \left[ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + 3 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right] + 2 \frac{\alpha_2}{\rho} \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y \partial x} + \frac{6\beta}{\rho} \left( \frac{\partial u}{\partial y} \right)^3 \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} + g \beta (T - T_\infty) \sin\left(\frac{x}{a}\right) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \tag{3}$$

Where u and v are the velocity components in the x- and y- directions respectively. The boundary conditions are prescribed at the surface and the edge of the boundary layer regime, respectively as follows:

$$\text{At } y=0, \quad u=0, \quad v=-V_w, \quad -K \frac{\partial T}{\partial y} = h_w (T_w - T) \tag{4}$$

$$\text{As } y \rightarrow \infty, \quad u \rightarrow 0, \quad T \rightarrow T_\infty$$

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Here  $T_\infty$  is the free stream temperature,  $k$  is the thermal conductivity,  $h_w$  is the Convective heat transfer coefficient and  $T_w$  is the convective fluid temperature.

The stream function  $\psi$  is defined by  $u = \frac{\partial \psi}{\partial y}$  and  $v = -\frac{\partial \psi}{\partial x}$ , and therefore, the continuity equation (1) is automatically satisfied. In order to render the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\xi = \frac{x}{a}, \eta = \frac{y}{a} Gr^{\frac{1}{4}}, f = \frac{\psi}{v \xi} Gr^{-\frac{1}{4}}, \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty}, Gr = \frac{g \beta (T_w - T_\infty) a^3}{\nu^3} \quad (5)$$

In view of the transformation defined in eqn. (8) the boundary layer equations (5) - (6) are reduced to the following coupled, nonlinear, dimensionless partial differential equations for momentum and energy for the regime:

$$f''' + 2E_1 f f'' - E_1 f' f'' - 3E_1 (f'')^2 + 2E_2 (f'')^2 + 6A (f'')^2 f''' + \frac{\sin \xi}{\xi} \theta - M f'' - f'^2 + f f'' = \xi \left( f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} - E_1 f' \frac{\partial f''}{\partial \xi} - E_1 f'' \frac{\partial f'}{\partial \xi} + E_1 f' \nu \frac{\partial f}{\partial \xi} + (3E_1 - 2E_2) f'' \frac{\partial f''}{\partial \xi} \right) \quad (6)$$

$$\frac{1}{Pr} \left( 1 + \frac{4F}{3} \right) \theta'' + (1 + \xi \cot \xi) f \theta' = \xi \left( f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) \quad (7)$$

Where  $E_1, E_2$  and  $A$  are fluid parameters,  $M$  is the magnetic parameter,  $Pr$  is the Prandtl number and  $F$  is the radiation parameter. These quantities are defined as follows.

$$E_1 = \frac{\alpha_1}{\rho a^2} Gr^{\frac{1}{2}}, E_2 = \frac{\alpha_2}{\rho a^2} Gr^{\frac{1}{2}}, A = \frac{\beta \nu \xi^2}{\rho a^4} Gr^{\frac{3}{2}}, M = \frac{\sigma B_0^2 a^2}{\rho \nu} Gr^{-\frac{1}{2}}, Pr = \frac{\nu}{\alpha} \text{ and } F = \frac{k k^*}{4 \sigma^* T_\infty^3}$$

The transformed dimensionless boundary conditions are:

$$\text{At } \eta = 0, \quad f = 0, \quad f' = 1, \quad \theta = 1 + S_T \theta'(0)$$

$$\text{As } \eta \rightarrow \infty, \quad f' \rightarrow 0, \quad \theta \rightarrow 0$$

(8)

Here primes denote the differentiation with respect to  $\eta$  and  $S_T = \frac{a h_w}{k} Gr^{\frac{1}{4}}$

The skin friction coefficient (shear stress at the cylinder surface) and Nusselt number (heat transfer rate) can be defined using the transformation described above with the following expressions

$$\frac{1}{2} Gr^{\frac{-3}{4}} C_f = \xi f''(\xi, 0) \quad (9)$$

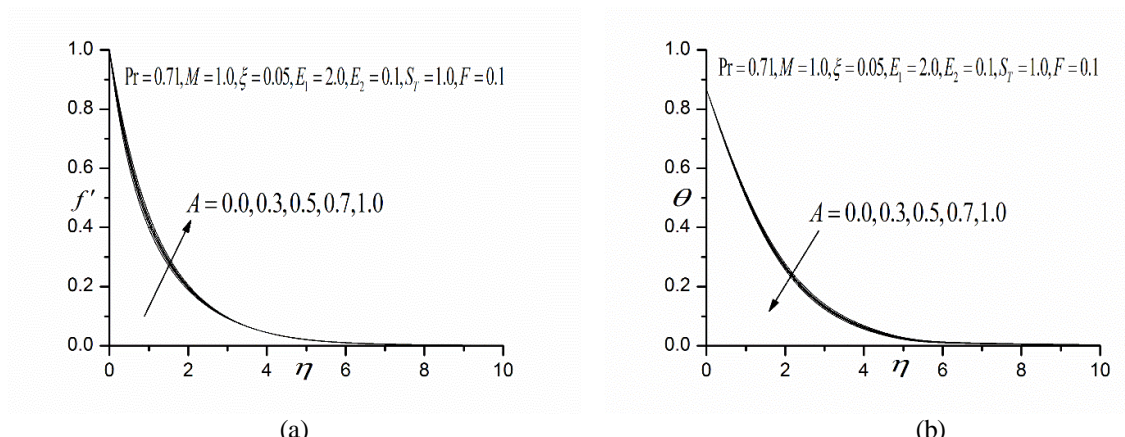
$$Gr^{\frac{-1}{4}} Nu = -\theta'(\xi, 0) \quad (10)$$

### III. NUMERICAL SOLUTION

In this study, the efficient Keller-Box implicit difference method has been employed to solve the general flow model defined by equations (6) – (7) with boundary conditions (8). This method was originally developed for low speed aerodynamic boundary layers and this system is developed by Keller [12]. and This technique has been described briefly in Cebeci and Bradshaw [13]. This method has been employed in a diverse range of industrial multi-physical fluid dynamics problems. These include boundary layer flows [14]-[15]. This method remains among the most powerful, versatile and accurate computational finite difference schemes employed in modern viscous fluid dynamics simulations.

**IV. RESULTS AND DISCUSSIONS**

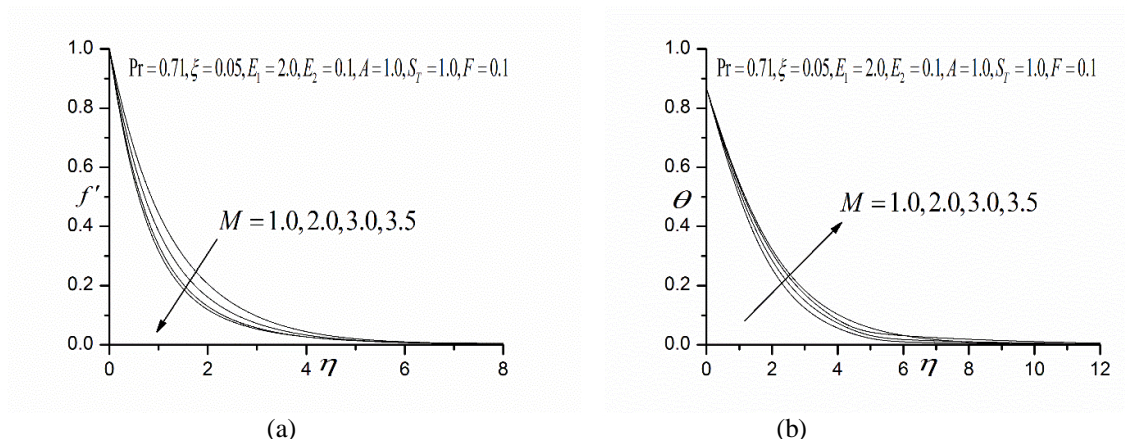
Comprehensive solutions have been obtain and are presented in Figs.2-6. The numerical problem comprises two independent variables ( $\xi, \eta$ ), two dependent fluid dynamic variables ( $f, \theta$ ) and 7 thermo-physical and body force control parameters, namely  $A, E_1, E_2, M, F, f_w$  and  $S_T$  are prescribed. The following default parameter values i.e.  $Pr = 0.71, M = 1.0, \xi = 0.05, E_1 = 2.0, E_2 = 0.1, A = 1.0, S_T = 1.0, F = 0.1$  are prescribed.



**Fig. 2** Effect of A on (a) Velocity Profiles, (b) Temperature Profiles

In Figs. 2(a)-2(b) illustrate the influence of Third grade fluid parameter ( $A$ ) on velocity and temperature profiles. Third grade fluid parameter of zero corresponds to a purely Newtonian fluid and infinite Third grade fluid parameter corresponds to a purely elastic solid. Intermediate values correlate quite well with actual polymeric viscoelastic properties, it is observed that, the boundary layer flow is accelerated with increasing Third grade fluid parameter  $A$ , a weak enhancement in the velocity throughout the boundary layer and opposite behaviour is observed in temperature profiles.

Figs. 3(a)-3(b) demonstrates the effect of the Magneto hydrodynamic parameter  $M$  on the velocity ( $f'$ ) and the temperature ( $\theta$ ). When increases the Magneto hydrodynamic parameter  $M$  with decreases the velocity and increases the temperature profiles.



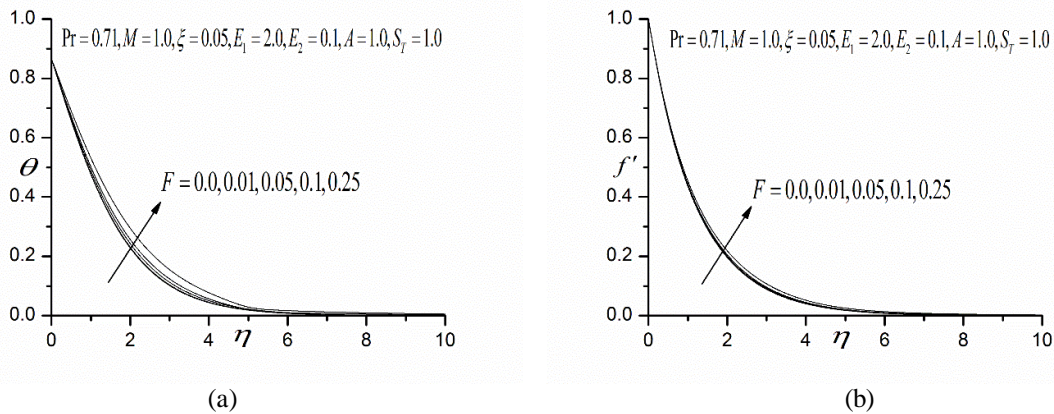
**Fig. 3** Effect of M on (a) Velocity Profiles, (b) Temperature Profiles

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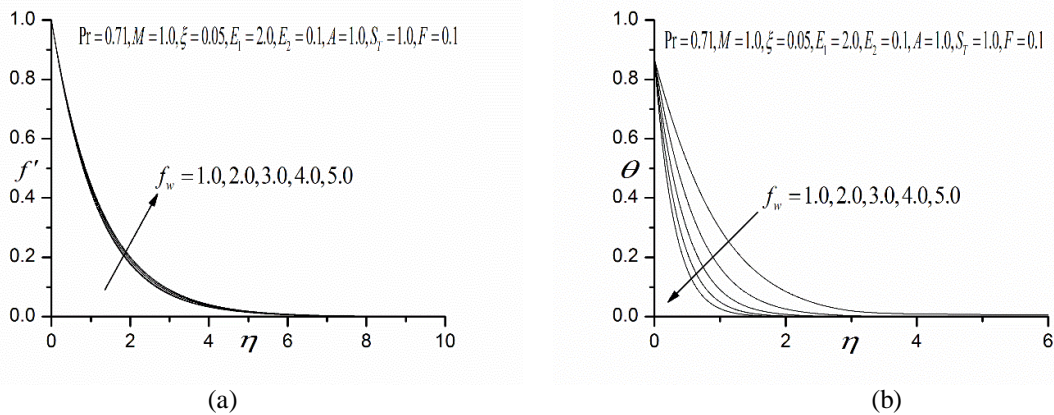
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Vol. 6, Special Issue 13, July 2017

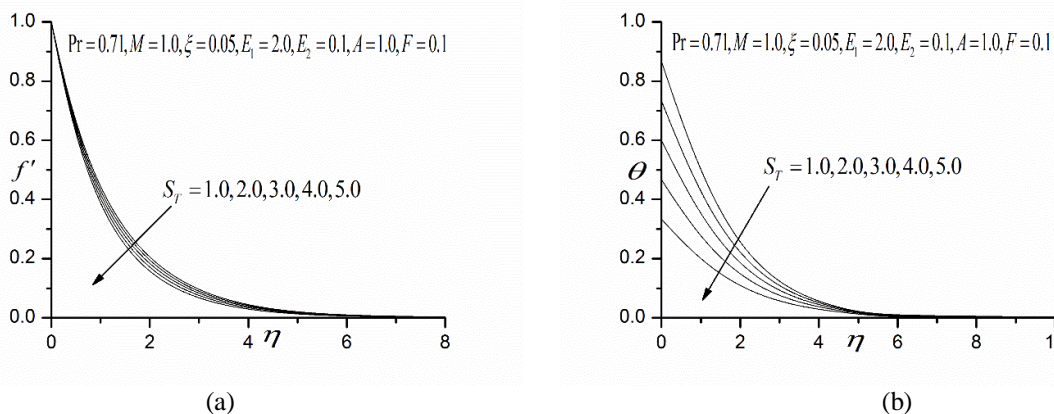
Figs. 4(a) and 4(b) shows the effects of radiation parameter  $F$ , It is observe that the velocity and temperature profiles are converges and close to the boundary layer when increasing the radiation parameter. The flow is accelerated and velocity is increased. The temperature profiles are also increases when increasing the radiation parameter.



**Fig. 4** Effect of  $F$  on (a) Velocity Profiles, (b) Temperature Profiles



**Fig. 5** Effect of  $f_w$  on (a) Velocity Profiles, (b) Temperature Profiles



**Fig. 6** Effect of  $S_T$  on (a) Velocity Profiles, (b) Temperature Profiles

# International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 6, Special Issue 13, July 2017

Figs. 5(a) and 5(b) illustrate the influence of the suction parameter on the transient velocity and temperature. As the suction parameter  $f_w$  increases a week increment observed in the velocity and temperature decreases throughout the boundary layer.

Figs. 6(a) and 6(b) illustrate the influence of the thermal slip parameter  $S_T$  on the transient velocity and temperature. As  $S_T$  increases the velocity and temperature decreases. The reason is the temperature buoyancy effects to decreases yielding a reduction in the fluid velocity. The increasing in the velocity and temperature profiles is accompanied by simultaneous reductions in velocity and temperature on the boundary layer.

## V. CONCLUSION

A theoretical study has been conducted to simulate the MHD flow of Third grade fluid over a horizontal circular cylinder in the presence of radiation and Suction effects also discussed thermal Slip. The transformed momentum and heat boundary layer equations have been solved computationally with Keller box finite difference method. The present study has shown

- Increasing viscoelastic Third grade fluid parameter (A) increases the velocity flow and decreases thermal boundary layer thickness throughout the boundary layer.
- Increasing Magnetohydrodynamic parameter (M) on velocity profiles decelerates the flow whereas it enhances temperature.
- Increasing Thermal radiation effect (F) enhances the velocity and temperature profiles throughout the boundary layer.
- Increasing Suction parameter ( $f_w$ ) on velocity profiles accelerates the flow whereas it reduces the temperature.
- Increasing thermal slip parameter ( $S_T$ ) reduces velocities and strongly reduces the temperatures.

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**Vol. 6, Special Issue 13, July 2017**

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