

Influence of Heat and Mass Transfer on MHD Flow Due To Stretching Surface with Velocity Slip

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ABSTRACT: The aim of this article is to analyze the velocity slip effects on a steady two-dimensional MHD flow of viscous, incompressible and electrically conducting fluid over a stretching sheet through porous medium in the presence of suction/injection. The governing equations are solved using the Runge-Kutta fourth order method along with shooting technique. The effects of the various parameters on the velocity, shear stress, temperature, temperature gradient, concentration and concentration gradient are illustrated graphically and discussed in detail.

KEYWORDS: Stretching sheet, Suction/injection, velocity slip, shooting technique and porous medium

I. INTRODUCTION

In fact, nearly two hundred years ago, Navier proposed a general boundary condition that incorporates the possibility of fluid slip at a solid boundary. Navier's proposed condition assumes that the velocity, v_x , at a solid surface is proportional to the shear stress at the surface [1, 2]. The fluid slippage phenomenon at the solid boundaries appear in many applications such as in microchannels and in applications, where a thin film of light oil is attached to the moving plates or when the surface is coated with special coatings such as thick monolayer of hydrophobic octadecyltrichlorosilane [3]. Also, wall slip can occur in the working fluid contains concentrated suspensions [4]. However, the literature lacks studies that take into account the possibility of fluid slippage at the wall under vibrating conditions. Effect of chemical reaction on slip flow of MHD Casson fluid over a stretching sheet with heat and mass transfer was studied by Gangadhar [5].

The stretching problems for steady flow have been used in various engineering and an industrial process, such as non-Newtonian fluid flows through porous medium. The boundary layer flow over a stretching sheet was first studied by Sakiadis [6, 7]. Later, Crane [8] extended this idea for the two dimensional flow over a stretching sheet problem. Gupta and Gupta [9], Carragher and Crane [10], and Dutta et al. [11] studied the heat transfer in the flow over a stretching surface considering different aspects of the problem. Problem of flow and heat transfer through a porous medium over a stretching surface is considered by Cortell [12], Chauhan and Agrawal [13], and Chauhan and Rastogi [14]. Abel et al. [15] studied heat transfer in MHD slip flow of a second grade fluid through a porous medium past a stretching sheet with non-uniform heat source or sink. Also, the MHD stretching problem has been studied by several authors, namely Rashidi et al. [16], Erfani et al. [17], Takhar et al. [18], Jafar et al. [19], Ishak et al. [20], etc.

The aim of the present work is to investigate the numerical solution of the steady boundary layer flow of a MHD stretching sheet with suction/injection and porous medium involving boundary conditions of velocity slip effect. The governing partial differential equations are first transformed into ordinary differential equations, before being solved numerically using the shooting technique for some values of the governing parameters. We have extended the work of Yasin et al. [21] to study the effect of MHD heat and mass transfer stretching sheet with radiation effect. The results are analyzed for various physical parameters such as magnetic field, Thermal Grashof number, Solutal Grashof number, Prandtl number, Schmidt number, Stretching sheet, Velocity slip parameter and suction/injection parameter on the flow, heat and mass transfer characteristics.

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II. MATHEMATICAL FORMULATION

Consider a steady two dimensional, electrically conducting viscous and incompressible fluid over a permeable stretching sheet with porous medium and buoyancy effects are taken into consideration. The velocity of the stretching sheet is assumed in the form $\lambda u_w(x)$, with $\lambda > 0$ for a stretching surface, where x - and y - axes are measured along the stretching surface and normal to it, respectively, and the flow being confined to $\lambda > 0$. It is assumed that the surface is permeable and the mass flux velocity is v_0 with $v_0 < 0$ for suction and $v_0 > 0$ for injection. It is also assumed that the constant temperature and concentration at the surface of the sheet are T_w and C_w , while those of the ambient (inviscid) fluid are T_∞ and C_∞ . The system of equations, which models the flow is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u + g \beta_T (T - T_\infty) + g \beta_C (C - C_\infty) - \frac{\nu}{k'} u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho C_p} (T - T_\infty) \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_\infty) \tag{4}$$

We assume that Eqs. (1) - (4) are subjected to the boundary conditions

$$\begin{aligned} v = v_0, u = \lambda u_w(x) + L \frac{\partial u}{\partial y}, T = T_w, C = C_w \text{ at } y = 0 \\ u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \end{aligned} \tag{5}$$

Where we assume that $u_w(x) = ax$, with $a > 0$

By using the Rosseland approximation (Brewster [22]) and following the papers by Ishak [23] and Babu et al. [24], the energy Eq. (3) can be written as

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(1 + \frac{16 \sigma^* T_\infty^3}{3 k k^*} \right) \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) \tag{6}$$

Where k is the thermal conductivity, σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient.

We look for a similarity solution of Eqs. (1), (2), (4) and (6) of the following form:

$$\begin{aligned} \psi = \sqrt{av} x f(\eta), \theta(\eta) = (T - T_\infty)/(T_w - T_\infty) \\ \phi(\eta) = (C - C_\infty)/(C_w - C_\infty), \eta = y \sqrt{a/v} \end{aligned} \tag{7}$$

Where $\psi(x, y)$ is the stream function, which is defined in the usual way as $u = \partial \psi / \partial y$ and $v = -\partial \psi / \partial x$. Thus, we have

$$u = a x f'(\eta), v = -\sqrt{av} f(\eta) \tag{8}$$

Where primes denote differentiation with respect to η . Thus, we take the dimensionless parameter S and δ are defined as

$$\begin{aligned} S = -v_0 / \sqrt{av} \\ \delta = \sqrt{\frac{a}{\nu}} L \end{aligned} \tag{9}$$

Where L is the velocity slip length, $S > 0$ corresponds to suction, $S < 0$ for injection and δ is the velocity slip parameter.

Substituting (7) into Eqs. (2), (6) and (4), we obtain the following ordinary differential equations

$$f''' + ff'' - f'^2 + Gr\theta + Gc\phi - (M + K)f' = 0 \tag{10}$$

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$$\frac{1}{Pr} \left(1 + \frac{4}{3} R \right) \theta'' + f \theta' + Q \theta = 0 \quad (11)$$

$$\phi'' + Sc f \phi' - Sc \Gamma \phi = 0 \quad (12)$$

Subject to the boundary conditions

$$f(0) = S, f'(0) = \lambda + \delta f''(0), \theta(0) = 1, \phi(0) = 1$$

$$f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (13)$$

The dimensionless constants $Pr, Sc, M, R, Gr, Gc, Q, K$ and Γ denote the Prandtl number, the Schmidt number, the magnetic parameter, the radiation parameter, thermal Grashof number, solutal Grashof number, the heat source/sink parameter, Permeability parameter and Chemical reaction parameter, which are defined as

$$Pr = \frac{\nu}{\alpha}, Sc = \frac{\nu}{D_B}, M = \frac{\sigma B_0^2}{\rho}, R = \frac{4\sigma^* T_\infty^3}{kk^*}, Gr = \frac{g \beta_T (T_w - T_\infty)}{a^2 x},$$

$$Gc = \frac{g \beta_c (C_w - C_\infty)}{a^2 x}, Q = \frac{Q_0}{a \rho C_p}, K = \frac{\nu}{k'a}, \Gamma = \frac{k_0}{a} \quad (14)$$

III. RESULTS AND DISCUSSION

In order to obtain the physical significance of the problem, we have plotted velocity, temperature, concentration, the rate of heat transfer and the rate of mass transfer for different values of the physical parameters like Magnetic parameter M , suction/injection parameter S , prandtl number Pr , stretching parameter λ , radiation parameter R , heat source/sink parameter Q , porous parameter K , thermal Grashof number Gr , modified Grashof number Gc , chemical reaction parameter Γ and velocity slip parameter δ .

Fig. 1(a)-(c) depicts the effects of the magnetic parameter M on velocity, temperature and concentration fields. It can be seen that (fig. 1(a)) as M increases, the velocity decreases. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the flow. Figs. 1(b) and 1(c) illustrates the temperature and concentration gradient for various values of the magnetic parameter M . It is clear that the temperature and concentration increases as the magnetic parameter increases for both cases of suction and injection significantly after the certain distance η normal to the sheet. From fig. 2, it is observed that the velocity increases initially with an increase in the stretching parameter λ but it decreases significantly after a certain distance η normal to the sheet. Now we see the effects of porous parameter K on the velocity, temperature and concentration profiles. The velocity profiles for several values of porous parameter K are depicted in figure 3(a). From the figure it is noticed that for a fixed value of η , velocity profiles decrease as porous parameter increases for both cases of suction/injection parameter. The dimensionless temperature profiles for various values of porous parameter are demonstrated in figure 3(b). It is seen that the porous parameter affects the temperature distribution in addition to the velocity field with increasing porous the temperature for fixed η increases. Fig. 3(c) illustrates that the fluid concentration along the sheet increases with an increasing values of porous parameter for both cases of suction/injection parameter respectively. From fig. 4, it is noticed that the rate of transport increases with the increasing distance η normal to the sheet. In all cases the velocity vanishes at some large distance from the sheet. With the increasing δ , the stream wise component of the velocity is found to increase.

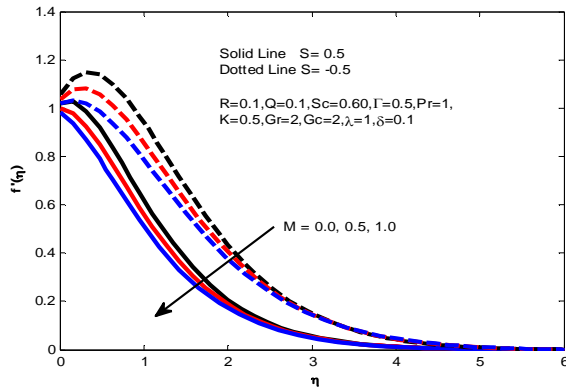


Fig. 1 (a) Velocity profiles for various values of M

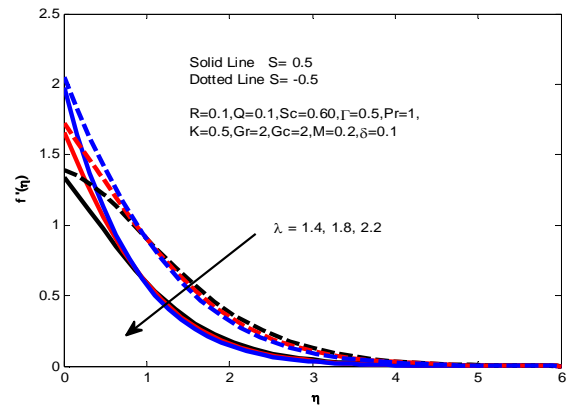


Fig. 2 Velocity profiles for various values of λ

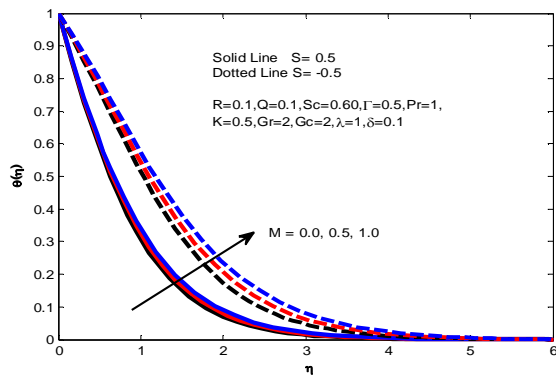


Fig. 1 (b) Temperature profiles for various values of M

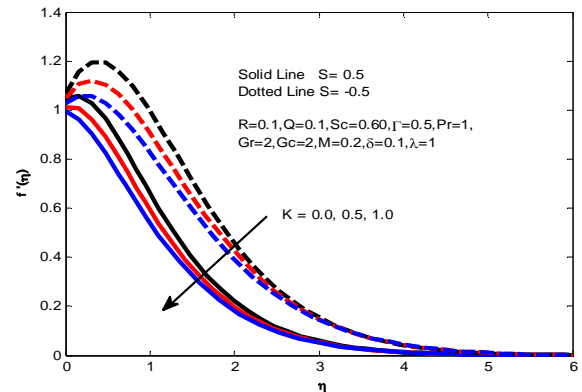


Fig. 3 (a) Velocity profiles for various values of K

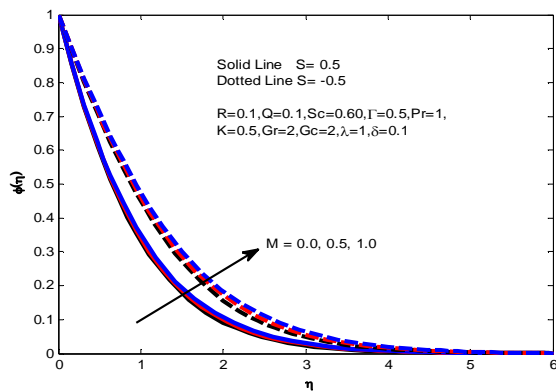


Fig. 1 (c) Concentration profiles for various values of M

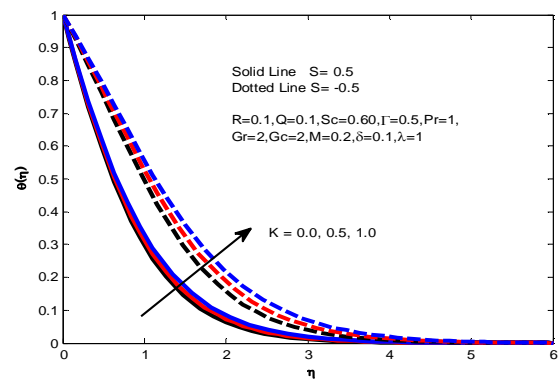


Fig. 3 (b) Temperature profiles for various values of K

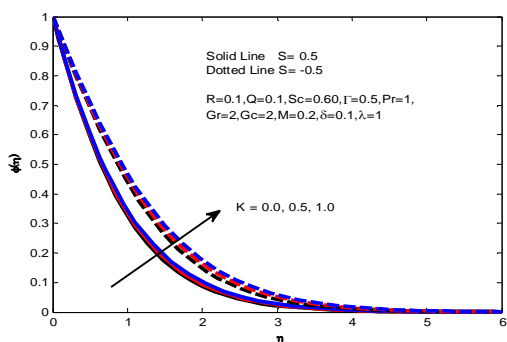


Fig. 3 (c) Concentration profiles for various vales of K

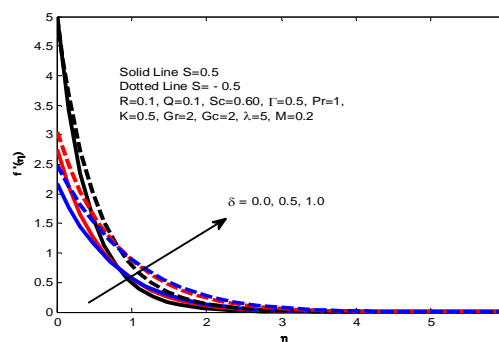


Fig. 4 Velocity profiles for various values of velocity slip parameter

IV. CONCLUSION

The important conclusions of the study are summarized below:

1. The velocity boundary layer decrease with increasing values of magnetic parameter.
2. The rate of transport increases with the increasing distance η normal to the sheet. In all cases the velocity vanishes at some large distance from the sheet. With the increasing δ the stream wise component of the velocity is found to increase. When slip occurs, the flow velocity near the sheet is no longer equal to the stretching velocity of the sheet.
3. The velocity increases initially with an increase in the stretching parameter λ but it decreases significantly after a certain distance η normal to the sheet.
4. The skin friction at the sheet increases with increasing values of chemical reaction parameter, porous parameter, suction/injection parameter, stretching parameter and velocity slip parameter.
5. Except for the porous parameter K , the local Sherwood number increases for increasing values of the other parameters.

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