

# MHD Three Dimensional Flow of a Williamson Fluid over a Stretching Sheet with Velocity Slip

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**ABSTRACT:** An analysis has been carried out for the three dimensional flow of a Williamson fluid in the presence of porous medium and velocity slip. Suitable similarity transformations are employed to reduce the governing partial differential equations into coupled nonlinear ordinary differential equations. These non linear ordinary differential are then solved numerically by using the Runge-Kutta-Fehlberg method. Obtained solutions are compared with previous results in a limiting sense from the literature, demonstrating an excellent agreement. The physical features of non-dimensional Williamson parameter, magnetic field parameter, stretching ratio parameter, velocity slip parameter and porosity parameter have been discussed by plotting the graphs of velocities in both directions.

**KEYWORDS:** MHD, three-dimensional flow, Williamson fluid, porous medium, velocity slip.

## I. INTRODUCTION

Fluids used in engineering and industrial processes are modeled as non-Newtonian fluids. For example, mercury amalgams, lubrications with heavy oils and greases, paints molten plastics and slurries. The rheological properties of such fluids cannot be explained by the Navier Stokes equations alone. It is also pertinent that one single model is not sufficient to describe these non-Newtonian fluids. To overcome this deficiency rheological models have been proposed. Several rheological models have been introduced such as Ellis model Carreau model, Cross model, power law model and Williamson fluid model. Williamson fluid is non-Newtonian fluid model with shear thinning property. Williamson [1] while studying the flow of pseudoplastic materials, proposed a model equation to describe the flow of pseudoplastic fluids and validated with the experimental results. Dapra and Scarpi [2] examined approximate analytical expression for the velocity field for a pulsatile flow of a non-Newtonian Williamson fluid in a rock fracture. Nadeem and Akram [3] analyzed the peristaltic flow of a Williamson fluid in an asymmetric channel. Nadeem et al. [4] investigated the two dimensional flow of a Williamson fluid over stretching sheet. In later study, Nadeem and Hussain [5] examined the effects of nano-sized particle of Williamson fluid over a stretching sheet. Krishnamurthy et al. [6] obtained the effects of radiation and chemical reaction on the steady boundary layer flow of MHD Williamson fluid through porous medium toward a horizontal linearly stretching in the presence of nanoparticles. Hayat et al. [7] discussed the two-dimensional boundary layer flow of an incompressible Williamson fluid over an unsteady permeable stretching surface with thermal radiation. Sinivas Reddy et al. [8] investigated the MHD flow and heat transfer characteristics of Williamson nanofluid over a stretching sheet with variable thickness and variable thermal conductivity. Ellahi et al. [9] discussed the three dimensional peristaltic flow of a Williamson fluid in a rectangular duct. Very recently, Malik et al. [10] explored the three dimensional flow of a Williamson fluid over a linearly stretching surface under the influence of a magnetic field.

In all the foregoing studies the flow field is assumed to obey the no slip boundary condition. To the best of author's knowledge, there are no studies on the three dimensional flow of Williamson fluid over a stretching sheet in the presence of porous medium and velocity slip.

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## II. MATHEMATICAL FORMULATION

We consider the steady incompressible three dimensional Williamson fluid through a porous medium which is continuously stretched in the x and y- directions. The geometrical configuration of the present flow is shown in Fig. 1. A constant magnetic field  $B_0$  is applied in a direction normal to the fluid flow. The magnetic Reynolds number is assumed to be small and thus the induced magnetic field is neglected. The fluid velocities on the surface are given by  $U_w = ax$ ,  $V_w = by$  and  $w = 0$  along the  $xy$ -plane, where a and b are stretching rate constants. The fluid has no lateral motions at  $z \rightarrow \infty$ .

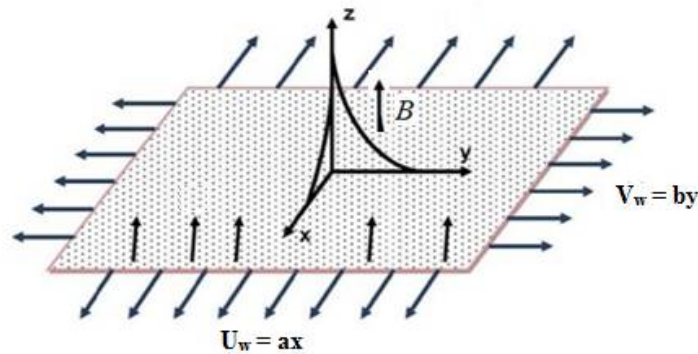


Fig. 1 Physical model and coordinate system

The constitutive equations in Williamson fluid flow for continuity and momentum after using boundary layer approximation is given by

$$\text{div } \mathbf{V} = 0 \tag{1}$$

$$\rho \frac{d\mathbf{V}}{dt} = \text{div } \mathbf{S} + \rho \mathbf{e} \tag{2}$$

where  $\rho$  is the density,  $\mathbf{V}$  is the velocity vector,  $\mathbf{S}$  is the Cauchy stress tensor,  $\mathbf{e}$  represents body force vector and  $d/dt$  represents the material time derivative.

The constitutive equations in Williamson fluid model are given as:

$$\mathbf{S} = p\mathbf{I} + \tau, \tag{3}$$

$$\tau = \left[ \mu_\infty + \frac{\mu_0 - \mu_\infty}{1 - \Gamma \dot{\gamma}} \right] \mathbf{A}_1, \tag{4}$$

Where  $p$  is pressure,  $\mathbf{I}$  is the identity vector,  $\tau$  is the extra stress tensor,  $\mu_0$  and  $\mu_\infty$  are the limiting viscosities at zero and at infinite shear rate,  $\Gamma > 0$  is the time constant,  $\mathbf{A}_1$  is the first Rivlin-Ericson tensor and  $\dot{\gamma}$  is defined as follows

$$\dot{\gamma} = \sqrt{\frac{\pi}{2}}, \quad \pi = \text{trace}(\mathbf{A}_1^2) \tag{5}$$

Where  $\pi$  is the second invariant strain tensor. Here we have only considered the case for which  $\mu_\infty = 0$  and  $< 1$ .

Then, we obtain

$$\tau = \left[ \frac{\mu_0}{1 - \Gamma \dot{\gamma}} \right] \mathbf{A}_1 \text{ or } \tau = \mu_0 (1 + \Gamma \dot{\gamma}) \mathbf{A}_1 \tag{6}$$

The steady boundary layer equations governing the flow, heat and diffusion transport can be expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2} + \sqrt{2} \nu \Gamma \frac{\partial u}{\partial z} \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{k_p^*} u \tag{8}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \nu \frac{\partial^2 v}{\partial z^2} + \sqrt{2} \nu \Gamma \frac{\partial v}{\partial z} \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho} v - \frac{\nu}{k_p^*} v \tag{9}$$

The associated boundary conditions of equations (8) and (9) are as follows:

$$u = U_w(x) + \frac{2 - \sigma \nu}{\sigma \nu} \lambda_0 \frac{\partial u}{\partial z}, \quad v = V_w(x) + \frac{2 - \sigma \nu}{\sigma \nu} \lambda_0 \frac{\partial v}{\partial z}, \quad w = 0 \quad \text{at} \quad z = 0 \tag{10}$$

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$$u \rightarrow 0, v \rightarrow 0, \text{ as } z \rightarrow \infty \tag{11}$$

Here  $u, v$  and  $w$  are the fluid velocity components along  $x, y$  and  $z$ -directions respectively,  $\nu$  is the kinematic viscosity,  $k_p^*$  is the permeability of the porous medium,  $\sigma_v$  is the tangential momentum accommodation coefficient and  $\lambda_0$  is the velocity slip factor.

### III. METHOD OF SOLUTION

Introducing the following similarity transformations

$$u = axf'(\eta), v = ayg'(\eta), w = -\sqrt{av}(f(\eta) + g(\eta)), \eta = z\sqrt{a/\nu} \tag{12}$$

We see that  $u, v$  and  $w$  satisfy equation (7), and equations (8) and (9) take the form

$$f'''' - (f')^2 + We f'' f'''' + (f + g)f'' - (M^2 + k_p)f' = 0 \tag{13}$$

$$g'''' - (g')^2 + We g'' g'''' + (f + g)g'' - (M^2 + k_p)g' = 0 \tag{14}$$

with the boundary conditions

$$f = 0, g = 0, f' = 1 + \lambda f'', g' = c + \lambda g'', \text{ at } \eta = 0 \tag{15}$$

$$f' \rightarrow 0, g' \rightarrow 0 \text{ as } \eta \rightarrow \infty \tag{16}$$

where,  $We = \Gamma x \sqrt{2a^3/\nu}$  is the Williamson parameter,  $M^2 = \sigma B_0^2/\rho a$  is the magnetic parameter,  $k_p = \nu/ak_p^*$  is the porosity parameter,  $c = b/a$  stretching ratio parameter and  $\lambda = \frac{2-\sigma_v}{\sigma_v} \sqrt{\frac{a}{\nu}} \lambda_0$  is the velocity slip parameter.

The physical quantities of engineering interest are the skin friction and heat transfer coefficients. The local skin friction coefficients in the  $x$  and  $y$  directions are given by:

$$C_{fx} = \frac{\tau_{wx}}{\rho U_w^2}, C_{fy} = \frac{\tau_{wy}}{\rho U_w^2} \tag{17}$$

Thus, the wall shear stresses along the  $x$  and  $y$  directions are denoted by

$$\tau_{wx} = -\mu_0 \left( \frac{\partial u}{\partial z} + \frac{\Gamma}{\sqrt{2}} \left( \frac{\partial u}{\partial z} \right)^2 \right)_{z=0} \text{ and } \tau_{wy} = -\mu_0 \left( \frac{\partial v}{\partial z} + \frac{\Gamma}{\sqrt{2}} \left( \frac{\partial v}{\partial z} \right)^2 \right)_{z=0}, \text{ respectively} \tag{18}$$

$$C_{fx} Re_x^{1/2} = f''(0) + \frac{We}{2} f''(0)^2, C_{fy} = g''(0) + \frac{We}{2} g''(0)^2$$

where  $Re_x = \frac{U_w x}{\nu}$  and  $Re_y = \frac{V_w y}{\nu}$  are the local Reynolds numbers,  $C_{fx}$  and  $C_{fy}$  are the local skin friction coefficients along  $x$  and  $y$  directions (i.e.,  $C_{fx}$  and  $C_{fy}$  are the local skin friction coefficients for the primary and secondary flows, respectively).

The ordinary differential equations (13)-(14) are coupled and highly non-linear. These equations with the boundary conditions (15) and (16) are solved using Runge-Kutta-Fehlberg method with shooting technique and obtained numerical solutions. The accuracy of this method is established by comparing our results of skin friction coefficient with those obtained by Malik et al. [10] in the absence of magnetic field and porous medium with no slip condition ( $M = k_p = \lambda = 0$  and  $c = 0.5$ ) for different values of Williamson parameter ( $We$ ). **Table 1** shows that our results are in excellent agreement.

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**Table 1:** Comparison values of skin friction coefficient values along x and y-directions for  $M = k_p = \lambda = 0$  and  $c = 0.5$

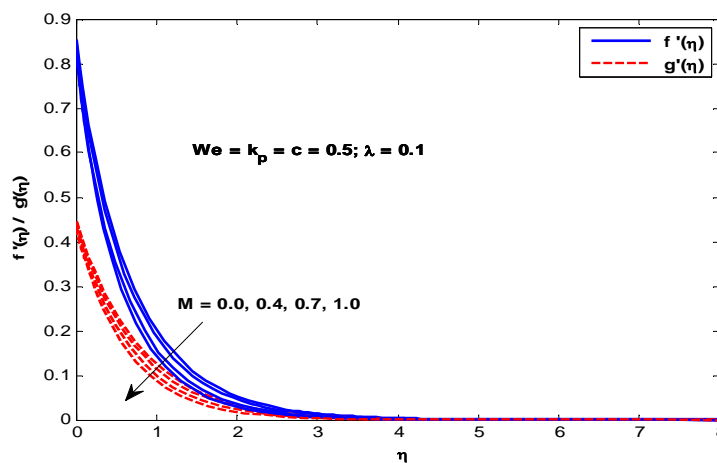
We	$- [f''(0) + we f''^2(0)]$		$- [g''(0) + we g''^2(0)]$	
	Malik et al. [10]	Present results	Malik et al. [10]	Present results
0.1	1.0934	1.0934	0.4661	0.4661
0.2	1.2695	1.2695	0.4841	0.4841
0.3	1.3340	1.3340	0.5025	0.5025
0.4	1.4915	1.4915	0.5220	0.5220

## IV. RESULTS AND DISCUSSION

To understand the effect of the physical parameters that emerged in the analysis on the velocity and temperature distributions the computational results are depicted through their profiles.

**Fig. 2** shows that stronger magnetic field ( $M$ ) strengths reduce the velocities in x and y directions due to the resistive nature of the Lorentz force developed as a result of the application of magnetic field. It is evident from **Fig. 3** that both the velocities are reduced as the Williamson parameter ( $We$ ) increases. As  $We$  in the ratio of relaxation times to the retardation time, increase in  $We$  corresponds to increase in relaxation time. Increase in relaxation time causes increase in viscosity which makes the fluid particle to consume more time to restore their position. **Fig. 4** reveals that increasing values of porous parameter ( $k_p$ ) are found to have a decreasing influence on the x and y components of velocities due to the Darcy resistance offered by the porous medium. **Fig. 5** presents variation of stretching ratio parameter ( $c$ ) on velocities. It is clear that the velocity increases in y-direction for increasing values of stretching ratio parameter and an opposite behavior in the x-direction. The influence of velocity slip parameter ( $\lambda$ ) is found to have a declining effect on velocities in both directions (**Fig. 6**).

From **Fig. 7** it is observed that the skin friction coefficient increases for increasing values of the Williamson parameter and magnetic field parameter in both the directions.



**Fig. 2:** Variation of  $M$  on  $f'(\eta)$  and  $g'(\eta)$

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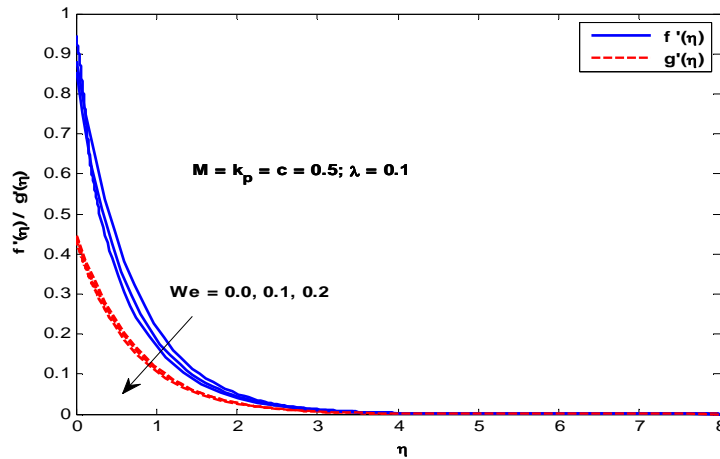


Fig. 3: Variation of  $We$  on  $f'(\eta)$  and  $g'(\eta)$

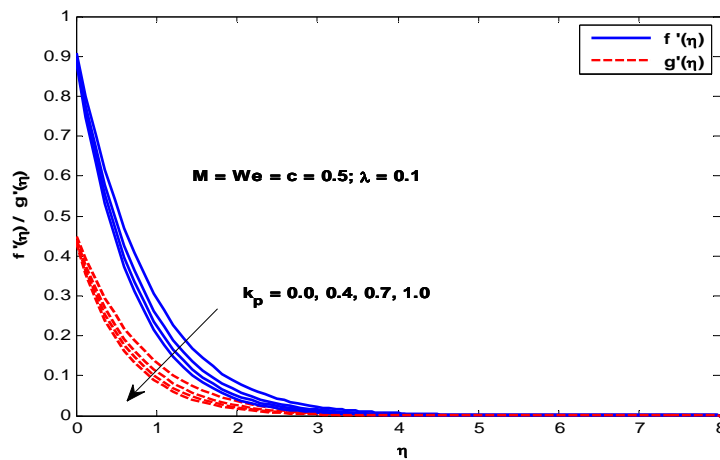


Fig. 4: Variation of  $k_p$  on  $f'(\eta)$  and  $g'(\eta)$

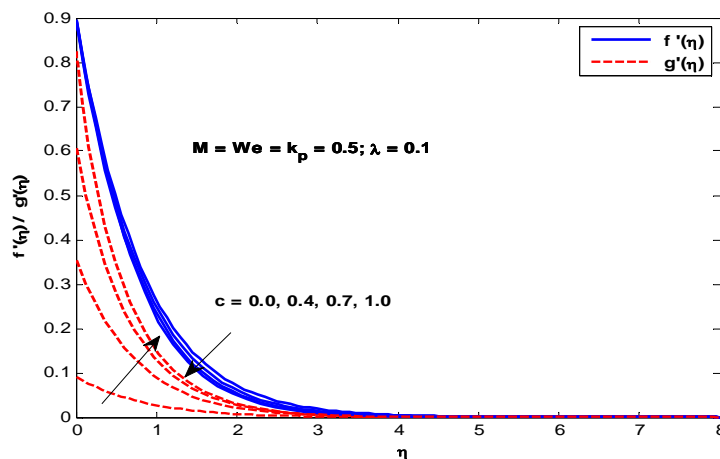
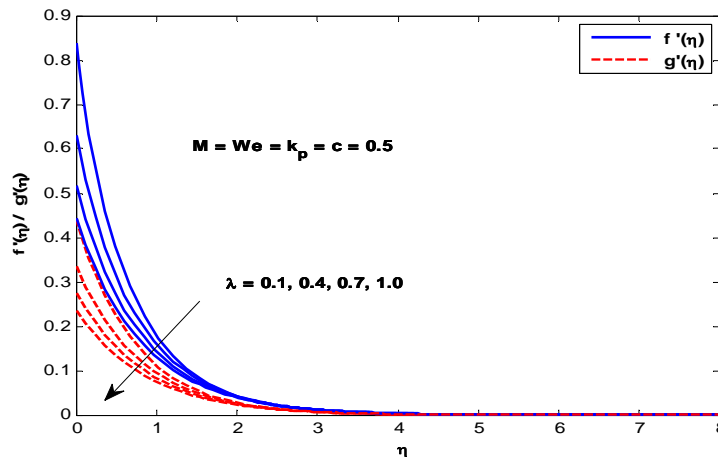
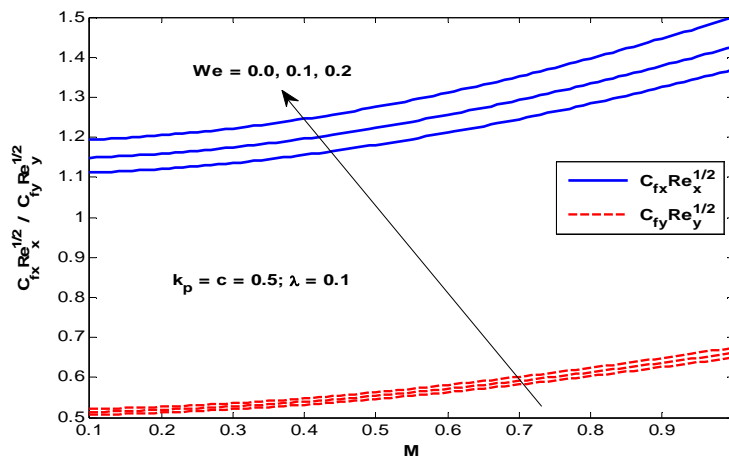


Fig. 5: Variation of  $c$  on  $f'(\eta)$  and  $g'(\eta)$



**Fig. 6:** Variation of  $\lambda$  on  $f'(\eta)$  and  $g'(\eta)$



**Fig. 7:** Variation of  $We$  and  $M$  on  $C_{fx}Re_x^{1/2}$  and  $C_{fy}Re_y^{1/2}$

## V. CONCLUSION

The salient features are summarized as follows:

- ✚ Velocity profiles decrease for increasing values of Williamson parameter.
- ✚ Increasing values of stretching parameter enhance the y-component of the velocity while an opposite effect on the x-component is observed.
- ✚ Velocity slip parameter reduces the velocities in both directions.
- ✚ Skin friction coefficient is enhanced by Williamson parameter and magnetic field parameter in x and y-directions.

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