

Effect of Heat Source on the Flow of a Stretching Wedge in the Presence of Thermophoresis and Viscous Dissipation

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ABSTRACT: In this paper the effect of thermophoresis, viscous heating and internal heat generation on the flow of a viscous fluid on a stretching wedge under the influence of a magnetic field is investigated. The flow heat and mass transfer characteristics for different variations in governing parameters are studied. It is observed that the velocity increases with magnetic field and porous parameter while the temperature and mass concentration are found to decrease for the same set of parameters. Increasing values of wedge angle are seen to increase the velocity.

KEYWORDS: Wedge; Porous medium; MHD, viscous heating, thermophoresis.

I. INTRODUCTION

The fluid flows over wedge shaped bodies find abundant thermal engineering applications in crude oil extraction, geothermal systems, ground water pollution, heat exchangers and storage of nuclear waste etc. Yih[1] investigated the MHD forced convection over a wedge, with viscous dissipation taking a variable wall temperature on the surface of the wedge and the equations governing the motion and temperature are converted into ordinary differential equations and are then solved using implicit finite difference scheme. Chamkha et al.[2] extended the study of Yih[1] by including suction or injection, temperature dependent heat generation or absorption and thermal radiation, and following Yih(1) the governing equations are solved using implicit finite difference scheme of Blottner[3]. Anjali Devi et al.[4] investigated the effects of heat and mass transfer on MHD laminar boundary layer flow over a wedge with suction or injection. Kandaswamy et al.[5] discussed the effect of chemical reaction on the heat and mass transfer in the flow over a porous wedge with thermal radiation in the presence of suction/injection. Chandrasekhar and Baskaran [6] studied the MHD flow over a wedge to analyse the effect of uniform suction and injection. Paramjeeth singh et al.[7] studied the unsteady mixed convection and mass transfer of a viscous incompressible fluid over a vertical wedge with constant suction and injection. Sarojamma et al.[8] studied the unsteady boundary layer flow of a casson fluid past a wedge with wall slip velocity.

In this paper we made an effort to investigate the effect of heat source, thermophoresis, viscous dissipation in the presence of chemical reaction on the heat and mass transfer flow of a viscous fluid past a wedge embedded in a porous medium under the influence of a magnetic field. Employing suitable similarity transformations, the non linear partial differential equations describing the momentum, energy, and concentration are converted into a set of coupled non-linear differential equations. These equations are solved numerically using Runge-Kutta method along with shooting technique. The impact of the all pertinent parameters on the flow variables is discussed through the graphs and tables.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 6, Special Issue 13, July 2017

II. MATHEMATICAL FORMULATION

Consider an unsteady two-dimensional MHD forced convective heat and mass transfer flow of a viscous incompressible electrically conducting fluid along a heated impermeable wedge in the presence of thermophoresis. The angle of the wedge is given by $\Omega = \beta\pi$. The flow is assumed to be in x-direction which is taken along the direction of the wedge and the y-axis is normal to it. The geometry of the problem has been shown in fig.1.

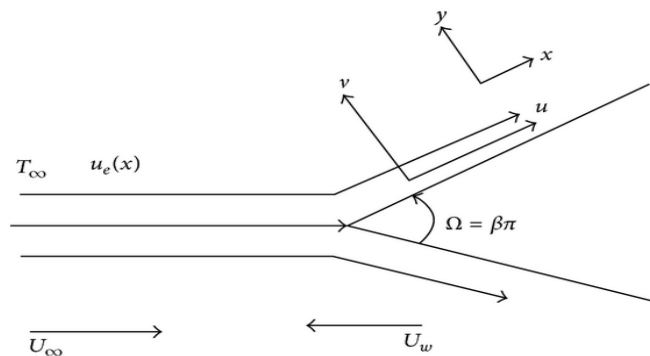


Fig.1. Flow configuration and co-ordinate system

The basic governing equations describing the continuity, momentum, energy and concentration respectively can be written as follows

$$u_x + v_y = 0 \tag{1}$$

$$u_t + uu_x + vv_y = \nu u_{yy} + U_t + \nu U_x - \frac{\sigma B_0^2}{\rho} (u - U) - \frac{\nu}{k_e} (u - U) \tag{2}$$

$$T_t + uT_x + vT_y = \frac{\lambda_g}{\rho c_p} T_{yy} - \frac{1}{\rho c_p} \frac{\partial}{\partial y} (q_r) + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q_0}{\rho c_p} (T - T_\infty) \tag{3}$$

$$c_t + uc_x + vc_y = Dc_{yy} - k_1(c - c_\infty) - \frac{\partial}{\partial y} (V_T(c - c_\infty)) \tag{4}$$

Where u and v are the velocity components in the x and y directions respectively. ν is the kinematic coefficient of viscosity, σ is the electrical conductivity, k_e is the permeability of the porous medium, λ_g is the thermal conductivity of the fluid, T is a fluid temperature, C is the concentration, B_0 is the magnetic induction, ρ is the density of the fluid, c_p is the specific heat at constant pressure and k_1 is the chemical reaction. The third term on the right hand side of the equation (4) represents viscous dissipation effect.

The boundary conditions are

$$u = U_w(x), \quad v = 0, \quad -k \frac{\partial T}{\partial y} = h(T_f - T_w), \quad c = c_w \quad \text{at } y = 0 \tag{5}$$

$$u = U(x), \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{at } y \rightarrow \infty \tag{6}$$

where $U(x,t)$ is the potential flow velocity for the wedge which is taken as follows

$$U(x,t) = \frac{\nu x^m}{\delta^{m+1}} \tag{7}$$

where m is the arbitrary constant and is related to the wedge angle and δ is the time dependent length scale which is taken to be as $\delta = \delta(t)$.

III. METHOD OF SOLUTION

We introduce the following similarity transformation:

$$\eta = y \sqrt{\frac{m+1}{2}} \sqrt{\frac{x^{m-1}}{\delta^{m+1}}} \quad \psi = \sqrt{\frac{2}{m+1}} \frac{\nu x^{\frac{m+1}{2}}}{\delta^{\frac{m+1}{2}}} f(\eta) \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{8}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 6, Special Issue 13, July 2017

Now using equations (7)-(8) into equations (1),(2), and (3) we obtain the following non linear ordinary differential equations:

$$f'''' - \lambda(2 - 2f' - \eta f'') + \beta(1 - f'^2) + ff'' - \frac{2}{m+1} \lambda_k (f' - 1) - M(f' - 1) \left(\frac{2}{m+1}\right) = 0 \quad (9)$$

$$\left(1 + \frac{4}{3}Nr\right)\theta'' + Pr\left(f\theta' + \eta\lambda\theta' + Ec f''^2 + Q\theta\frac{2}{m+1}\right) = 0 \quad (10)$$

$$\varphi'' + Sc(\eta\lambda\varphi' + f\varphi' - \frac{2}{m+1}\gamma\varphi - \tau(\theta''\varphi + \theta'\varphi')) = 0 \quad (11)$$

The transformed boundary conditions:

$$f'(0) = 0, f(0) = f_w, \theta'(0) = -B_i(1 - \theta(0)), \varphi(0) = 1; \quad (12)$$

$$f'(\infty) \rightarrow 1, \theta(\infty) \rightarrow 0, \varphi(\infty) \rightarrow 0 \quad (13)$$

where $\beta = \frac{2m}{m+1}$ is the wedge angle parameter that corresponds to $\Omega = \beta\pi$ for a total angle Ω

of the wedge, $Pr = \frac{\nu\rho c_p}{\lambda_g}$ is the prandtl number, $Sc = \frac{\nu}{D}$ is the Schmidt number, $M = \frac{\sigma B_0^2 x}{\rho U(x,t)}$ is the magnetic field, $\lambda = \frac{\delta^m}{\nu x^{m-1}} \frac{d\delta}{dt}$

is the unsteadiness parameter, $\lambda_k = \frac{x\nu}{U(x,t)k_e}$ is the dimensionless porous medium parameter, $N_r = \frac{4\sigma^* T_\infty^3}{k^* \lambda_g}$ is the thermal

radiation parameter, $Ec = \frac{u^2(x,t)}{(T_f - T_\infty)c_p}$ is the Eckert number, $Q = \frac{Q_0 x}{\rho c_p U(x,t)}$ is the heat generation or absorption parameter, $\tau =$

$-\frac{k}{T_r}(T_f - T_\infty)$ is the thermophoresis parameter.

The parameters of engineering interest for the present problem are the local skin friction coefficient, local nusselt number and local Sherwood number which indicate physically wall shear stress, rate of heat transfer and rate of mass transfer respectively and thermophoretic velocity. These physical quantities obtained from the following expressions:

$$\frac{1}{2}cf\sqrt{2-\beta} = Re^{-1/2}f''(0), \text{ where } \tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (14)$$

$$Nu\sqrt{2-\beta} = Re^{-1/2}\theta'(0), \text{ where } q_w = -\lambda_g\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (15)$$

$$sh\sqrt{2-\beta} = Re^{-1/2}\varphi'(0) \quad (16)$$

The highly non-linear coupled ordinary differential equations (9),(10) and (11) with the boundary conditions (12) and (13) are solved using Runge-kutta Felberg method.

To check the validity of the numerical code, we have calculated the values of $f(0), f'(0)$ and $f''(0)$ for the Falkner-Skan boundary layer equation for the case $\beta = 0, M = 0$ and $\lambda = 0$ for different values of η . Table 1 shows the comparison of the data produced by the present code and those of White [9]. The results are found to be in good agreement.

Table1. Comparison of the present numerical results of Falkner-Skan boundary layer equation for the case of $\beta = M = \lambda = 0$

η	$f(\eta)$		$f'(\eta)$		$f''(\eta)$	
	Present Work	White [9]	Present Work	White [9]	Present Work	White [9]
0.0	0.0000	0.0000	0.0000	0.0000	0.46964	0.46960
0.5	0.05864	0.05864	0.23423	0.23423	0.46503	0.46503
1.0	0.23299	0.23299	0.46063	0.46063	0.43438	0.43438
1.5	0.51503	0.51503	0.66147	0.66147	0.36180	0.36180
2.0	0.88680	0.88680	0.81669	0.81669	0.255669	0.25567
3.0	1.79557	1.79557	0.96905	0.96905	0.067710	0.06771
4.0	2.78389	2.78388	0.99777	0.99777	0.006874	0.00687
5.0	3.78323	3.78323	0.99994	0.99994	0.0002577	0.00026

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 6, Special Issue 13, July 2017

IV. RESULTS AND DISCUSSION

In this paper the unsteady flow of viscous incompressible fluid past a stretching wedge under the influence of a transfer magnetic field in the presence of viscous dissipation, heat generation / absorption, thermophoresis and chemical reaction is investigated.

Fig 2 shows the effect of magnetic field (M) on velocity. It is revealed that the velocity increases rapidly with increase in the strength of the magnetic field near the boundary. However, in this case the applied magnetic field moving with the stream accelerates the fluid resulting in the velocity enhancement and hence the thickness of the boundary layer increases. **Fig 3** illustrates that the wedge angle shows an increase in the velocity and thickness of the associated boundary layers decrease. **Fig 4** shows that the porous parameter (λ_k) has an increasing influence on velocity. From **Fig 5** it is observed that the presence of thermal radiation (Nr increases the temperature significantly throughout the region, and the corresponding boundary layer thickness is increased. Physically we may say that there is a strong augmentation in the rate of heat transfer from the stretching wedge due to radiation parameter.

Fig 6 reveals the temperature profiles for different values of heat generation/absorption parameter (Q). The temperature has higher values in the presence of heat source,

Fig 7 shows the effect of chemical reaction parameter on concentration. It is noticed that there is a pronounced impact on the species concentration as the chemical reaction parameter increases. As the value of the chemical reaction increases we observe that the species concentration decreases.

In **Fig 8** concentration profile for distinct values of thermophoresis parameter (τ) is plotted. It is evident that the concentration decreases as the thermophoresis parameter increases owing to the smaller temperature differences between the hot fluid and the free stream.

Fig 9 presents the drag coefficient on the boundary for a variation in the unsteadiness parameter for different values of the wedge angle. It is noticed that the skin friction coefficient decreases with elapse of time while it increases in magnitude with increasing values of wedge angle.

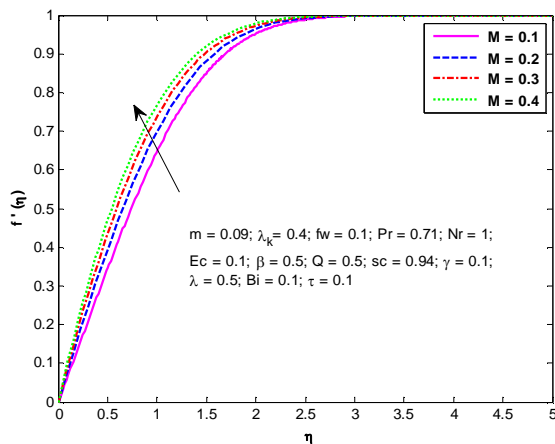


Fig.2 velocity profile for different values of M

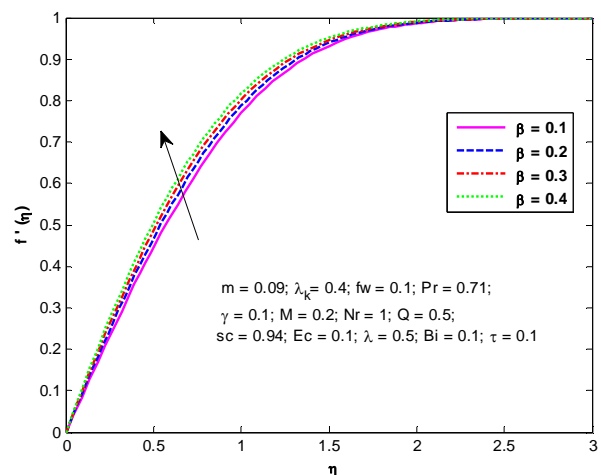


Fig.3 velocity profile for different values of β

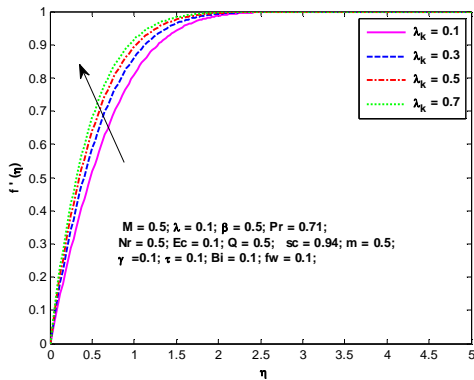


Fig.4 velocity profile for different values of λ_k

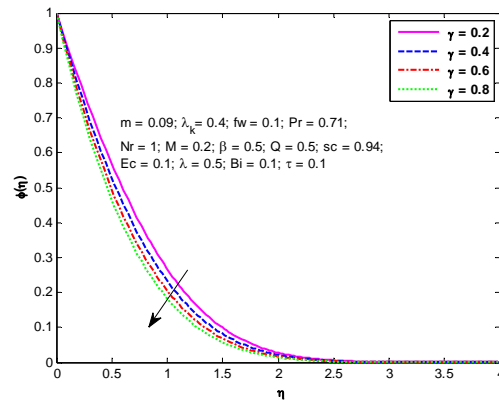


Fig.7 Concentration profile for different values of γ

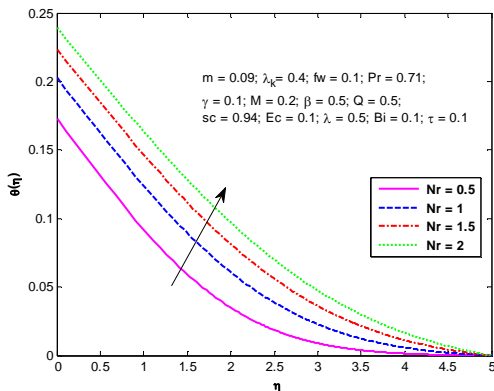


Fig.5 Temperature profile for different values of Nr

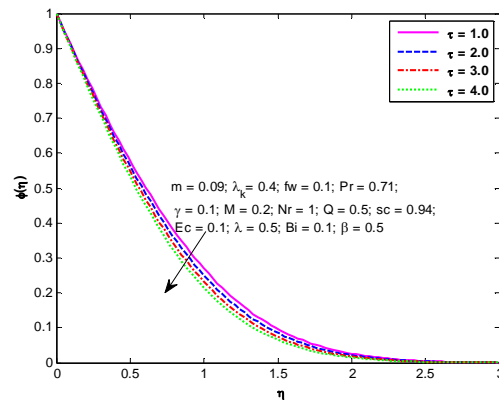


Fig.8 Concentration profile for different values of τ

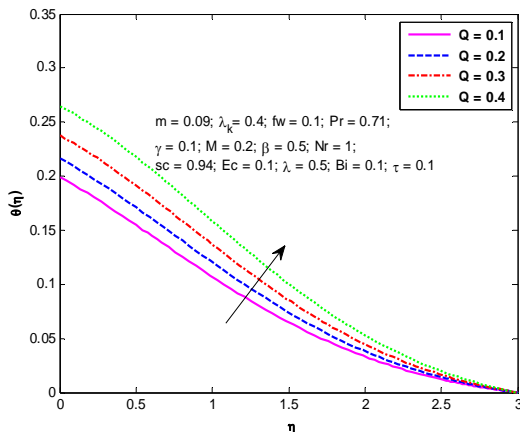


Fig.6 Temperature profile for different values of Q

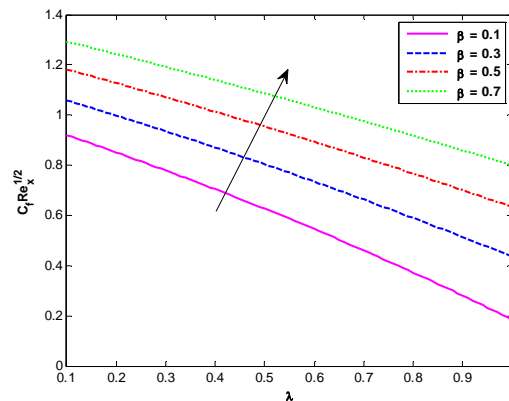


Fig. 9 Skin friction coefficient with λ for different values of β

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 6, Special Issue 13, July 2017

V. CONCLUSION

The analysis presents the effects of thermophoresis, heat generation/absorption, and viscous dissipation on the unsteady flow of a viscous incompressible fluid over a wedge. From computational results the following conclusions are drawn.

- Increasing values of wedge angle parameter enhances velocity.
- The velocity increases with increase in the strength of magnetic field
- The mass transfer rate is reduced with chemical reaction parameter
- The skin friction coefficient is reduced with λ while it experiences an enhancement with increasing values of β

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