

Flow of upper-convected Maxwell Micropolar fluid over a stretching sheet with slip effect

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ABSTRACT: This article is concerned with the flow of upper-convected Maxwell Micropolar fluid over a steady stretching sheet with slip effect. The governing partial differential equations of the problem are converted into ordinary differential equations using suitable transformations and then solved numerically using Runge-Kutta Fehlberg method. Influence of material parameter, Deborah number, magnetic parameter, velocity slip parameter on the flow is examined.

KEYWORDS: Upper-convected Maxwell Micropolar fluid, Stretching sheet, Slip effect

I. INTRODUCTION

The studies of boundary layer flows of Newtonian and non-Newtonian fluids over a stretching surface have received much attention because of their extensive applications in the field of metallurgy, chemical engineering and particularly, in the extrusion of polymer sheets from a die or in the drawing of plastic films. Maxwell fluid model belongs to the class of rate type fluids that predicts the stress relaxation. Several studies [1, 2, 3] etc., on Maxwell fluid flows on stretching surfaces are available in literature under different conditions. However, in all these studies the micro rotation of the fluid particles is not considered. The micro rotation of the fluid particles is essential as they spin of each particle is coupled with the microscopic velocity of the particle and hence it is of significance to understand its role in the dynamics of the fluid. The theory of Micropolar fluids was first introduced by two French brothers (Cosserat) in 1909. After the original work of Eringen [4, 5] in the 1960s, the micropolar fluids received great interest. Several authors [6, 7] etc., studied micropolar fluid flows over a stretching sheet. A very few authors studied upper-convected Maxwell Micropolar fluid flows. Recently Khan et al. [8] studied upper-convected Maxwell Micropolar fluid with Joule heating and thermal radiation using a hyperbolic heat flux equation.

In majority of studies of fluid flows no slip boundary condition is considered. In general the flow field is assumed to obey the no slip boundary condition. However, there are situations wherein the no slip condition at the boundary does not hold and hence requires that it is to be replaced by partial slip boundary condition. For instance, in non-Newtonian fluids such as polymer melts, suspensions, foams and emulsions, a macroscopic wall slip is observed. The non-adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances [9]. The fluids that exhibit boundary slip have important technological applications such as in the polishing of artificial heart valves and internal cavities. For some coated surfaces, such as Teflon, resist adhesion, the no slip condition is replaced by Navier's partial slip condition, where the slip velocity is proportional to the local shear stress.

In the present study we consider the upper-convected Maxwell Micropolar fluid flow over a stretching sheet with slip effect under the influence of a magnetic field.

II. MATHEMATICAL FORMULATION

We consider the steady laminar boundary layer flow of a upper-convected Maxwell (UCM-M) micro polar fluid past a stretching sheet with slip boundary condition. It is assumed that the sheet is stretched with a velocity

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$u_w = ax$, where a is a stretching constant. The flow is subjected to a constant transverse magnetic field $B = B_0$ which is applied in the positive y -direction, normal to the surface, where B_0 is magnetic field strength. The coordinate frame is selected in such a way that x -axis is extending along the stretching sheet and y -axis normal to it. The flow equations after boundary layer approximations can be defined as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left(u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) = \frac{(\mu+k)}{\rho} \left(\frac{\partial^2 u}{\partial y^2} \right) + \frac{k}{\rho} \frac{\partial N}{\partial y} - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\Omega}{\rho j} \frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j} \left(2N + \frac{\partial u}{\partial y} \right) \tag{3}$$

The boundary conditions are

$$u = u_w + A \frac{\partial u}{\partial y}, \quad v = 0, \quad N = -n \frac{\partial u}{\partial y} \quad \text{at } y = 0$$

$$u \rightarrow 0, \quad N \rightarrow 0 \quad \text{as } y \rightarrow \infty \tag{4}$$

In the above expressions, u and v are the fluid velocity components in the x and y directions respectively, μ is the dynamic viscosity, ρ is the fluid density, σ is the electrical conductivity, ν is the kinematic viscosity, λ is the relaxation time of the period, A is a constant, N is the micro rotation or angular velocity whose direction of rotation is normal to the $x - y$ plane, k is the vortex viscosity, j is the micro-inertia density, Ω is the spin-gradient viscosity and given by $\Omega = \left(\mu + \frac{k}{2} \right) j = \mu \left(1 + \frac{\beta}{2} \right) j$, where $\beta = \frac{k}{\mu}$. n is boundary parameter which is a constant such that $0 \leq n \leq 1$.

$$\text{We define } u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{5}$$

Where $\psi(x, y)$ is the stream function, which satisfies equation (1).

We define the following similarity transformations for dimensionless variables:

$$\eta = \sqrt{\frac{a}{\nu}} y, \quad \psi = \sqrt{a\nu} x f(\eta), \quad N = ax \sqrt{\frac{a}{\nu}} g(\eta) \tag{6}$$

Where f, g are dimensionless variables. Using the above similarity transformation and dimensionless variables, the governing equations (3) – (4) are reduced into the ordinary differential equations as follows:

$$(1 + \beta) f''' + f f'' - f'^2 + De(2f f' f'' - f^2 f''') + \beta g' - M f' = 0 \tag{7}$$

$$\left(1 + \frac{\beta}{2} \right) g'' - \beta(2g + f'') + f g' - f' g = 0 \tag{8}$$

With boundary conditions

$$f(0) = 0, f'(0) = 1 + h_1 f''(0), \quad g(0) = -n f''(0) \quad \text{at } \eta = 0$$

$$f'(\infty) \rightarrow 0, \quad g(\infty) \rightarrow 0 \quad \eta \rightarrow \infty \tag{9}$$

Where $\beta = \frac{k}{\mu}$ is the material parameter, $M = \frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter, $De = \lambda a$ is the Deborah number,

$h_1 = A \sqrt{\frac{a}{\nu}}$ is velocity slip parameter

The surface skin friction coefficient C_f , which is a measure of the frictional drag on the surface is defined by

$$C_f = \frac{\tau_w}{\rho u_w^2} \tag{10}$$

$$\text{where the wall shear stress } \tau_w \text{ is given by } \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \tag{11}$$

Using equation (11) in equation (10), we obtain $C_f \sqrt{Re_x} = -f''(0)$

The ordinary differential equations (7) and (8) are coupled and highly non – linear. These equations with the boundary conditions (9) are solved using Runge-Kutta Fehlberg method and obtained numerical solutions.

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Table 1: Comparison of values of $-f''(0)$ with slip factor γ when $\beta = De = M = n = 0$

h_1	Wang [10]	Sahoo and Do [11]	Present results
0.0	1.000	1.001154	1.000483
0.3	0.701	0.699738	0.701038
1.0	0.430	0.428450	0.430632
2.0	0.284	0.282893	0.284415

We validate the numerical scheme by comparing the present results viz., skin friction coefficient for various slip factors h_1 with those of Wang [10] and Sahoo and Do [11] in the absence of $\beta = De = M = n = 0$. It can be seen from that there is an excellent agreement with the previous results published in [10, 11]. The compared values of skin friction coefficient are presented in **Table 1**.

III. RESULTS AND DISCUSSIONS

The parametric analysis of the velocity components and the angular velocity is performed to understand the influence of the various governing parameters of the flow.

Fig. 1 shows the variation of the material parameter (β) on the axial velocity and its influence is seen to increase the axial velocity. From **Fig.2** the angular velocity is observed to diminish near the boundary till a specific point $\eta = 1.3$ and later it shows an increasing trend along the stretching surface eventually fulfilling the free stream condition. **Fig. 3** indicates that as Deborah number (De) increases the axial velocity is found to decrease. As Deborah number (De) which is the ratio of relaxation times to retardation times, increase in Deborah number (De) amounts to increase in relaxation time which causes the fluid to take more time to restore particles to attain its position. The angular velocity shows an increasing trend near the boundary while it reduces in the rest of the region for a similar variation in Deborah number (De) as illustrated in **Fig. 4**.

Fig.5 reveals that the presence of magnetic field diminishes the velocity due to the Lorentz force developed consequent on the application of magnetic field which has tendency to retard the flow of the fluid. Further, stronger strengths of the magnetic fields cause further reduction in the velocity due to stronger strengths of Lorentz force. **Fig.6** depicts that the magnetic field has an increasing influence on the angular velocity in the vicinity of the boundary while an exact opposite behaviour is seen in the rest of the region. The influence of slip parameter on both the velocity components is to reduce in their magnitude as depicted in **Fig. 7**.

Fig. 8 presents the effect of boundary parameter (n) on micro rotation velocity g . The micro rotation velocity g is seen to be an increasing function of n . When $n = 0$, the micro rotation velocity is zero. **Fig. 9** illustrates the behaviour of surface drag coefficient with velocity slip parameter for various values of the material parameter(β). It is noticed that skin friction coefficient decreases exponentially with increasing values of the velocity slip parameter. It is seen that the skin friction coefficient decreases with increasing material parameter(β). However, it is interesting to observe that the reduction in the values of skin friction is more for smaller values of the slip parameter i.e. ($h_1 = 0$).

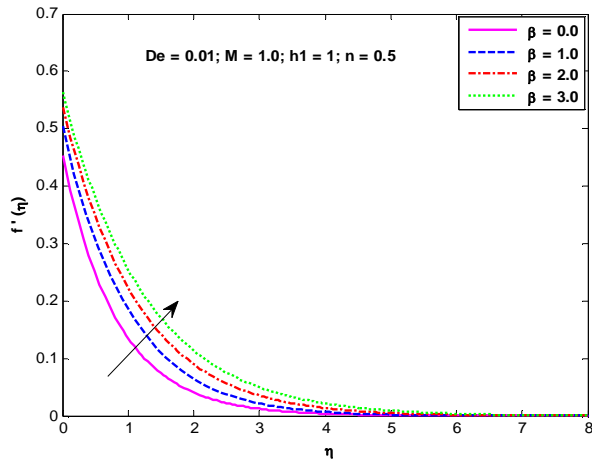


Fig. 1: Axial velocity Profiles for different values of β

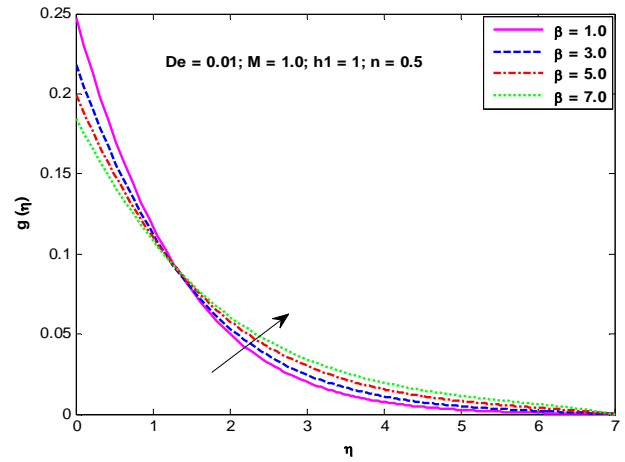


Fig. 2: Micro rotation Profiles for different values of β

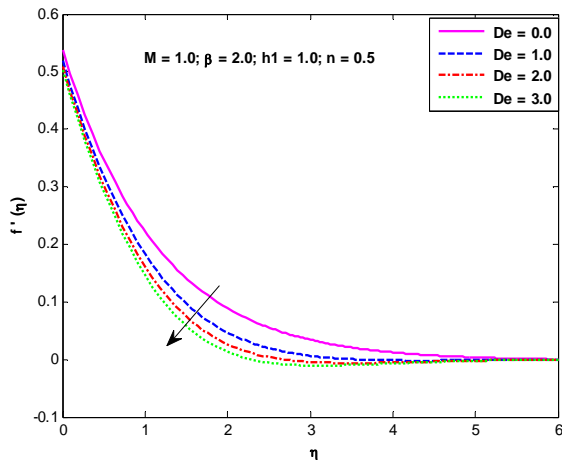


Fig. 3: Axial velocity Profiles for different values of De

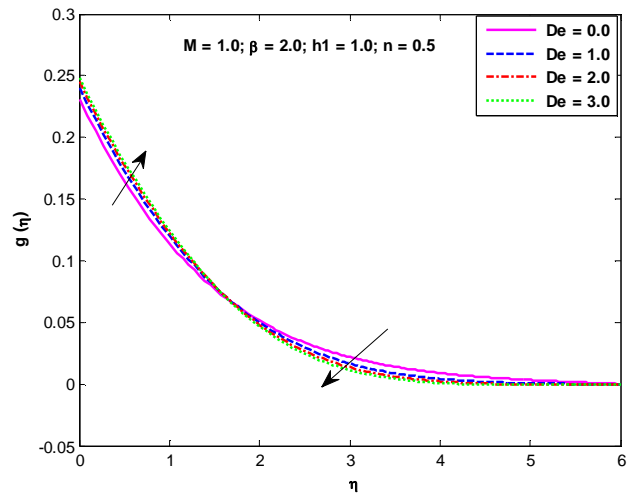


Fig. 4: Micro rotation Profiles for different values of De

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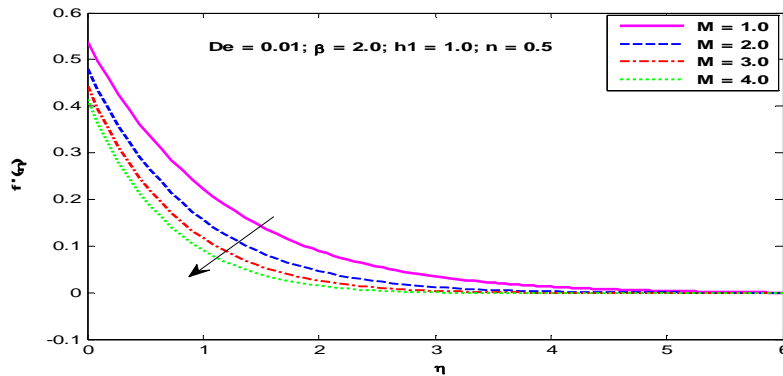


Fig. 5: Axial velocity Profiles for for different values of M

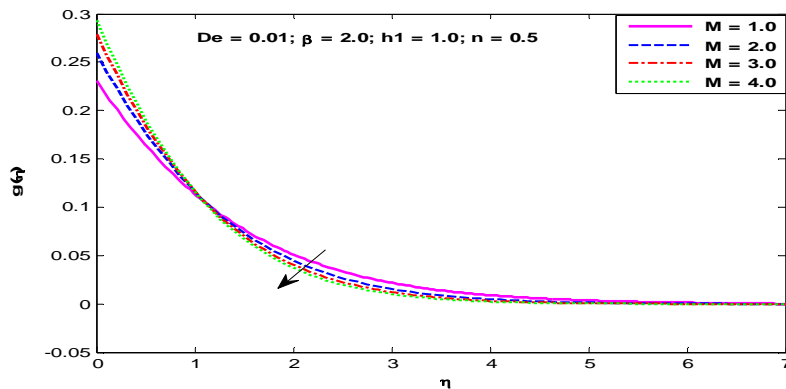


Fig. 6: Micro rotation Profiles for for different values of M

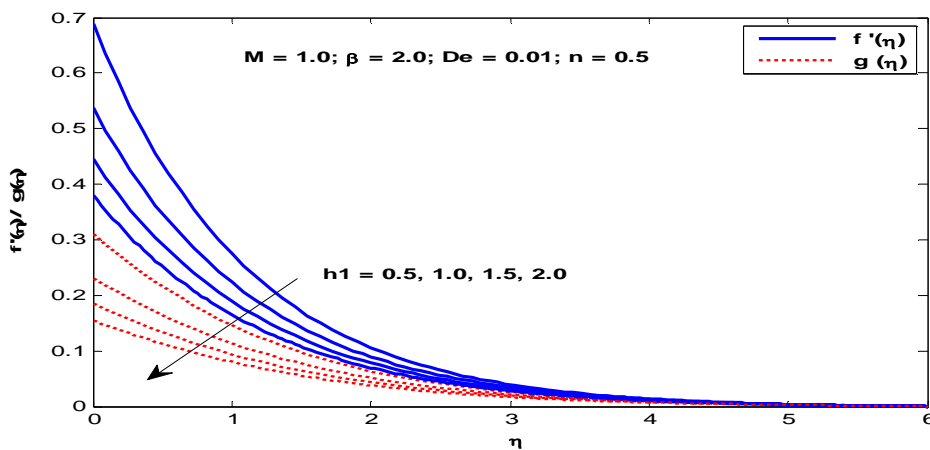


Fig. 7: Axial velocity and Microrotation Profiles for for different values of h_1

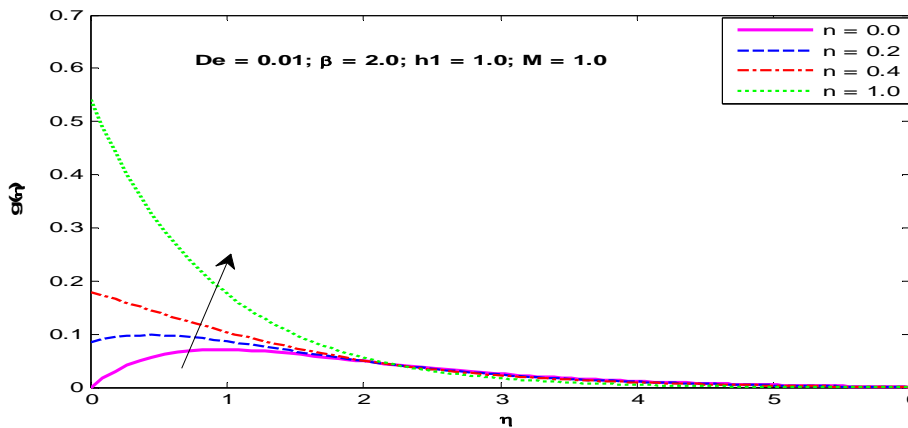


Fig. 8: Microrotation Profiles for different values of n

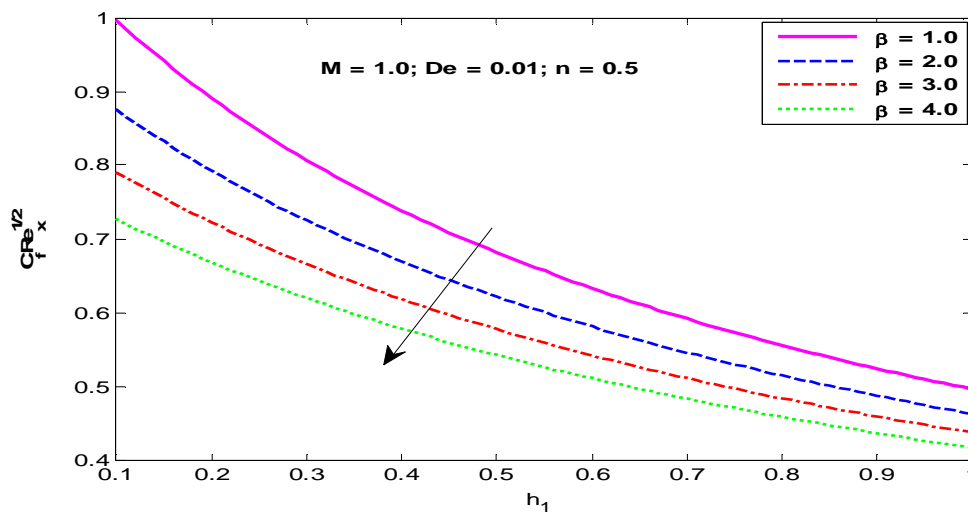


Fig. 9: Variation of skin friction coefficient with h_1 for different values of β

IV. CONCLUSIONS

In this paper, the effect of velocity slip and magnetic field on boundary layer flow of upper-convected Maxwell Micropolar fluid over stretching sheet is discussed. The boundary layer equations governing the flow are reduced into a couple of higher order non-linear ordinary differential equations using the similarity transformation. Then, the obtained differential equations are solved numerically using Runge-Kutta Fehlberg method. The effects of various governing parameters, microrotation velocity of the particles discussed using graphs and tables. Some of the important conclusions are listed below.

- The angular velocity shows an increasing trend near the boundary while it reduces in the rest of the region for increasing values of Deborah number (De).
- The velocity boundary layer thickness decreases as the absolute values of slip parameter increases.
- The micro rotation velocity is seen to be an increasing function of n .

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