

Influence of Thermal Radiation and Thermophoresis on Casson Fluid Flow over a Stretching Sheet Embedded in a Doubly Stratified Medium

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ABSTRACT: In this paper we investigated the effect of thermal and mass stratification on the MHD mixed convective boundary layer flow of a Casson fluid over a stretching sheet in the presence of thermophoresis, thermal radiation and first order chemical reaction. The boundary layer equations governing the flow, heat and mass transfer are transformed by applying similarity transformation into a set of coupled non-linear ordinary differential equations. The numerical solutions of these equations for velocity, temperature and concentration are obtained using the popular Runge-Kutta-Fehlberg method along with shooting technique. Parametric analysis of velocities, temperature and concentration for different variations in these parameters is performed graphically.

KEYWORDS: Casson fluid, MHD, Double stratified medium, thermophoresis.

I. INTRODUCTION

Stratification refers to the formation of layers that arise due to concentration differences, temperature variations, or the presence of fluids with different densities. Double stratification occurs when both heat and mass mechanisms take place simultaneously. Study of natural and mixed convection in a doubly stratified medium is interesting and of significant importance, because of its wide range of applications. Density differences in the presence of gravity are important on the dynamics and mixing of heterogeneous fluid. Stratification plays an important role in lakes and ponds as it controls temperature and concentration differences of hydrogen and oxygen in such environments which may affect the growth rate of various species. The study of thermal stratification has also applications in solar engineering as high energy efficiencies can be achieved with better stratification. Chen and Eichhorn [1] studied natural convection along an isothermal vertical plate in a thermally stratified medium. Kulkarni et al. [2] investigated the problem of natural convection from an isothermal flat plate suspended in a thermally stratified quiescent fluid and obtained similarity solutions. Takhar et al. [3] obtained the non-similar solution along a continuously moving vertical surface immersed in a thermally stratified medium. Saha and Hossain [4] examined the natural convection with mass transfer in a thermally stratified medium. Chang and Lee [5] investigated the free convection flow by a vertical plate with uniform and constant heat flux in a thermally stratified micropolar fluid. Srinivasacharya and Reddy [6] discussed the effect of double stratification in a mixed convection flow of a micropolar fluid. Mukhopadhyay and Ishak [7] obtained numerical solutions for the steady mixed convection boundary layer flow and heat transfer along a stretching cylinder in a thermally stratified medium. Hayat et al. [8] examined the steady and incompressible stagnation point flow of a double stratified Casson fluid over a stretching cylinder with chemical reaction.

In this paper we investigated the effect of thermal and mass stratification on the MHD mixed convective boundary layer flow of a Casson fluid over a stretching sheet in the presence of thermophoresis, thermal radiation and first order chemical reaction.

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II. MATHEMATICAL FORMULATION

We consider the steady two-dimensional doubly stratified mixed convection flow of an incompressible Casson fluid. The flow is caused by a linearly stretching surface at $y = 0$ with stretching velocity $u_w = bx$, where b is a positive constant with dimension per time. The flow occupies the domain $y > 0$. We assume that the surface has temperature T_w and concentration C_w and further T_∞ and C_∞ are the temperature and concentration of ambient fluid. x -axis is taken along the sheet and y -axis normal to the sheet. Magnetic field of uniform intensity B_0 is applied in y -direction. Thermal and concentration stratifications are taken into account in the presence of thermal radiation, thermophoresis and first order chemical reaction.

The governing boundary layer equations for momentum, energy, and concentration for mixed convection under Boussinesq approximation are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_0(C - C_\infty) - \frac{\partial}{\partial y} [V_T(C - C_\infty)] \quad (4)$$

where u and v are the fluid velocity components along x and y axes, respectively, ε is the porosity of the medium, ν is the kinematic viscosity, ρ is the density of the fluid, B_0 is the transverse magnetic field strength, $\beta = \mu_B \sqrt{2\pi_c} / P_y$ is the Casson parameter, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, P_y is the yield stress of the fluid, π_c is critical value of π based on non-Newtonian model, g is the acceleration due to gravity, β_T is the coefficient of thermal expansion, β_C is the coefficient of concentration expansion, T is the temperature of the fluid inside the thermal boundary layer, C is the fluid concentration, c_p is the specific heat at constant pressure, k is the thermal conductivity of the medium, σ^* is the Stefan-Boltzman constant, k^* is the absorption coefficient, Q_0 is heat generation ($Q_0 > 0$) or absorption ($Q_0 < 0$) coefficient, D is the mass diffusivity, k_0 is the chemical reaction and V_T is the thermophoresis velocity, following Talbot *et al.* (1980), given by $V_T = -\frac{k_t v}{T_{ref}} \frac{\partial T}{\partial y}$, where T_{ref} is some reference temperature and the value of $k_t v$ represents the thermophoretic diffusivity.

Boundary conditions:

$$u = u_w, v = 0, T = T_w = T_0 + m_1 x, C = C_w = C_0 + n_1 x \quad \text{at } y = 0 \quad (5)$$

$$u \rightarrow 0, T \rightarrow T_\infty = T_0 + m_2 x, C \rightarrow C_\infty = C_0 + n_2 x \quad \text{as } y \rightarrow \infty \quad (6)$$

where m_1, m_2, n_1, n_2 are dimensional constants, T_w is the uniform temperature and T_∞ is the fluid temperature in the free stream with $T_w > T_\infty$, C_w is the uniform concentration and C_∞ is the fluid concentration in the free stream with $C_w > C_\infty$ and T_0, C_0 are the reference temperature and reference concentration, respectively.

III. METHOD OF SOLUTION

To solve the governing boundary layer equations (2) – (4), the following similarity transformations are introduced:

$$u = bxf'(\eta), v = -\sqrt{bv}f(\eta), \eta = \sqrt{\frac{b}{\nu}}y$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_0}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_0} \quad (7)$$

Substitution of equation (7) into the governing equations (2) – (4) we obtain the following system of non-linear ordinary differential equations:

$$\left(1 + \frac{1}{\beta} \right) f''' + ff'' - f'^2 - Mf' + \lambda(\theta + N\phi) = 0 \quad (8)$$

$$\left(1 + \frac{4}{3}Nr \right) \theta'' + Pr(f\theta' - f'\theta - \varepsilon_1 f' + Q\theta) = 0 \quad (9)$$

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$$\phi'' + Sc(f\phi' - f'\phi - \varepsilon_2 f' - \gamma\phi - \tau(\theta'\phi' + \theta''\phi)) = 0 \quad (10)$$

The boundary conditions (5) and (6) become:

$$f(0) = 0, f'(0) = 1, \theta(0) = 1 - \varepsilon_1, \phi(0) = 1 - \varepsilon_2 \quad (11)$$

$$f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0 \quad (12)$$

where primes denote differentiation with respect to η ,

$M = \sigma B_0^2 / \rho b$ is the magnetic field parameter, $\lambda = Gr_x / Re_x^2$ is the mixed convection parameter, $N = Gr_x^* / Gr_x$ is the ratio of thermal to concentration buoyancy forces, $Gr_x = g\beta_T(T - T_\infty)x^3/\nu^2$ is the Grashof number due to temperature, $Gr_x^* = g\beta_C(C - C_\infty)x^3/\nu^2$ is the Grashof number due to Concentration, $Pr = \frac{\rho c_p \nu}{k}$ is the Prandtl number, $Nr = 16\sigma^* T_\infty^3 / 3kk^*$ is the thermal radiation parameter, $\varepsilon_1 = m_2/m_1$ is the thermally stratified parameter, $\varepsilon_2 = n_2/n_1$ is the solutal stratified parameter, $Q = Q_0/\rho c_p b$ is the heat source/sink parameter, $Sc = \nu/D$ is the Schmidt number, $\gamma = k_0/b$ is the chemical reaction parameter and $\tau = -\frac{k_t(T_w - T_\infty)}{T_r}$ is the thermophoretic parameter.

The physical quantities of engineering interest in this problem are the skin friction coefficient C_f , local Nusselt number Nu_x and local Sherwood number Sh_x which are defined by

$$\frac{1}{2}C_f\sqrt{Re_x} = \left(1 + \frac{1}{\beta}\right)f''(0), Nu_x/Re_x^{1/2} = -\left(1 + \frac{4}{3}Nr\right)\left(\frac{1}{1-\varepsilon_1}\right)\theta'(0), Sh_x/Re_x^{1/2} = -\left(\frac{1}{1-\varepsilon_2}\right)\phi'(0) \quad (13)$$

where $Re_x = \frac{u_w x}{\nu}$ is the local Reynolds number.

The ordinary differential equations (8)-(10) are coupled and highly non-linear. These equations with the boundary conditions (11) and (12) are solved using Runge-Kutta-Fehlberg method with shooting technique and obtained numerical solutions. The accuracy of numerical scheme is validated by comparing the results, that is $f''(0)$ of the present analysis with those calculated by Ibrahim and Shanker [9], Hayat et al. [10] in the limiting case for a Newtonian fluid when $\lambda = N = Pr = Nr = Q = Sc = \gamma = \tau = \varepsilon_1 = \varepsilon_2 = 0$. **Table 1** confirms that the results are in good agreement.

Table 1: Comparison of skin friction coefficient $f''(0)$ with Ibrahim and Shankar (2013) and Hayat *et al.* (2016) when $\lambda = N = Pr = Nr = Q = Sc = \gamma = \tau = \varepsilon_1 = \varepsilon_2 = 0$ for Newtonian fluid.

M	Ibrahim and Shankar [9]	Hayat <i>et al.</i> [10]	Present results
0.0	-1.2808	-1.2808	-1.2808
1.0	-1.4142	-1.4142	-1.4142
5.0	-2.4494	-2.4494	-2.4494

IV. RESULTS AND DISCUSSION

The objective of this investigation is to know the effect of double stratification on the Casson fluid flow past a stretching surface under the influence of a transverse magnetic field, thermal radiation in the presence of thermophoresis, heat source/sink and first order chemical reaction. To get a physical insight of the problem, the velocity, temperature and species concentration are computationally evaluated and presented through graphs. Physical interpretation of the results is discussed in detail.

Fig. 1 indicates the effect of non-Newtonian rheology parameter/ Casson parameter (β) on velocity. It is clear that velocity decreases with increasing β as expected since increasing values of β correspond to higher values of plastic dynamic viscosity of the fluid. **Fig. 2** shows the influence of magnetic field (M) on velocity. It is clear that increase in the strength of magnetic field is to diminish velocity near the plate ($0 \leq \eta \leq 6$) and outside this range its influence is negligible. It is observed that velocity decreases with increase in M accompanied by a reduction in the thickness of the boundary layer. This reduction can be attributed to the fact that the transverse magnetic field gives rise to a resisting type of force known as Lorentz force. This force tends to slow down the motion of the fluid and consequently velocity depreciates. It is noted from **Fig. 3** that increasing values of thermal buoyancy parameter (λ) have an enhancing effect

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on velocity since buoyancy force acts as a favorable pressure gradient which accelerates the fluid within the boundary layer. We may conclude that enhanced thermal buoyancy leads to formation of thicker boundary layers. From **Fig. 4** it is observed that velocity rises with increasing values of solutal buoyancy parameter (N) in the region ($0 \leq \eta \leq 6$) and is negligible outside the region.

It is seen from **Fig. 5** that in the presence of thermal stratification (ε_1), the temperature gradient between boundary surface and ambient fluid will decrease and hence temperature gets reduced. **Fig. 6** is the plots of temperature field for a variation in thermal radiation parameter (Nr). Temperature and thickness of thermal boundary layer are enhanced with an increase in the thermal radiation parameter. Larger values of Nr provide more heat to the fluid which results in an enhancement in temperature and thickness of thermal boundary layer.

It is clear from **Fig. 7** that the non-dimensional concentration of the fluid decreases with increase of solutal stratification parameter (ε_2). Variation of thermophoretic parameter (τ) on concentration is plotted in **Fig. 8**. It is observed that increasing values of thermophoretic parameter results in reduction of species concentration. Concentration of the fluid steadily changes from higher value to the lower value and ultimately fulfilling the free stream boundary condition as $\eta \rightarrow \infty$.

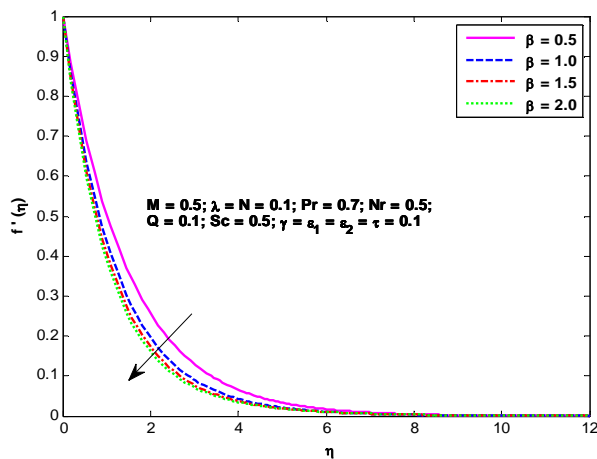


Fig. 1: Variation of β on f'

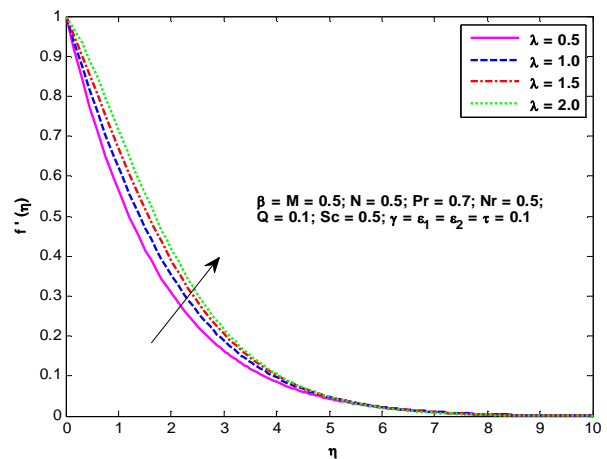


Fig. 3: Variation of λ on f'

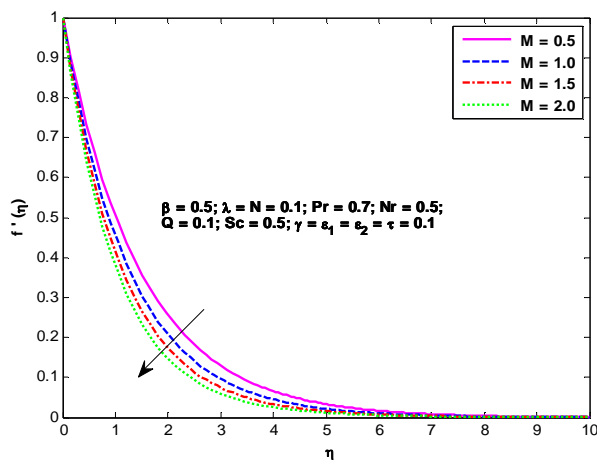


Fig. 2: Variation of M on f'

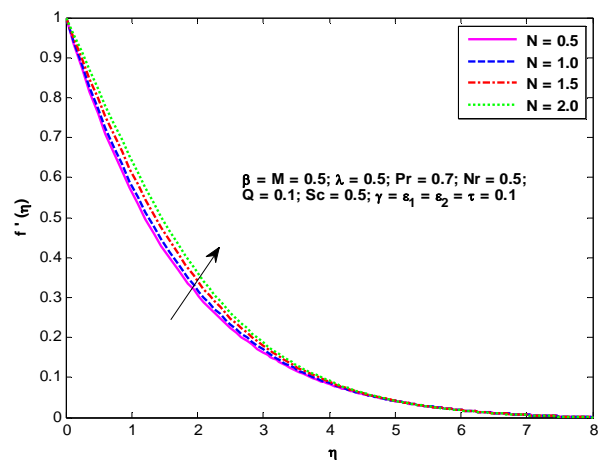


Fig. 4: Variation of N on f'

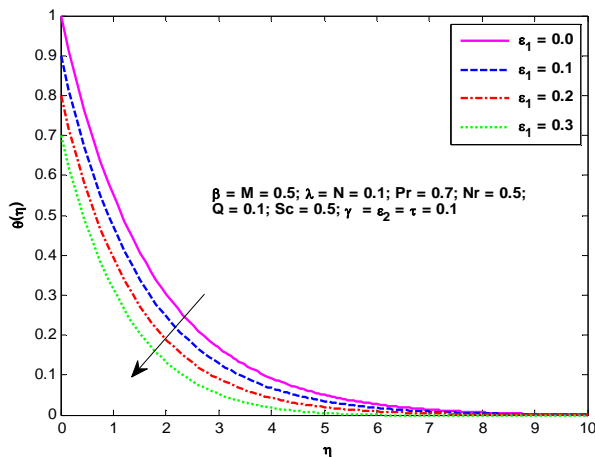


Fig. 5: Variation of ε_1 on θ

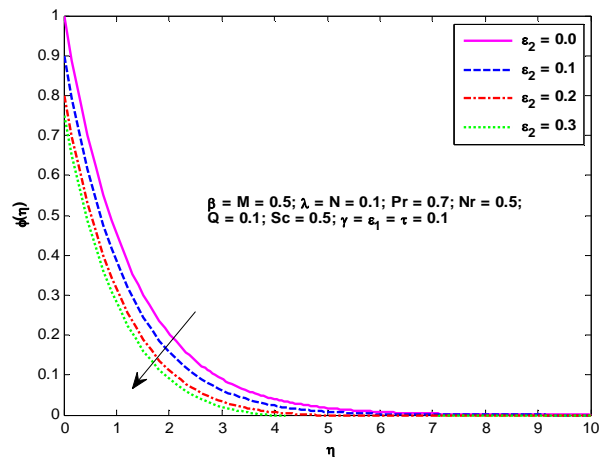


Fig. 7: Variation of ε_2 on ϕ

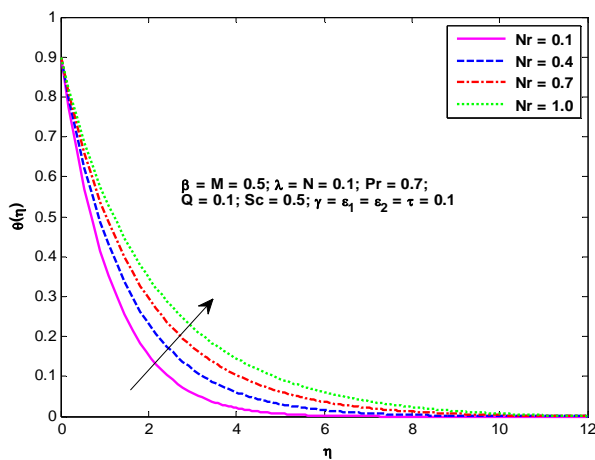


Fig. 6: Variation of Nr on θ

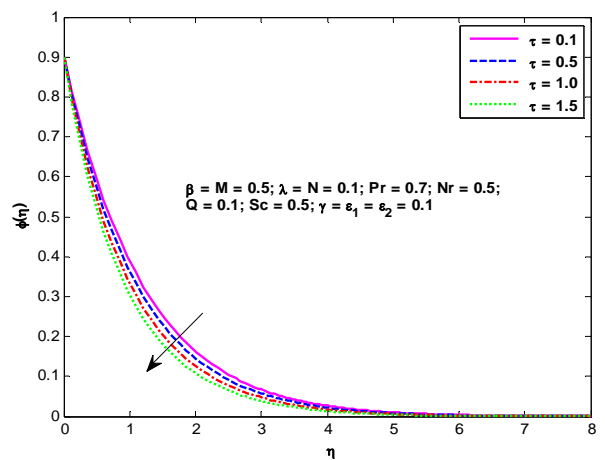


Fig. 8: Variation of τ on ϕ

From **Table 2** it is observed that increase in Casson parameter (β) decreases the absolute values of surface drag coefficient whereas the magnetic field has a reverse effect. The Nusselt number and Sherwood number decrease for an increase in β as well as magnetic field parameter. Thermal and solutal buoyancy parameters are observed to decrease the magnitude of the skin friction coefficient whereas Nusselt number and Sherwood number are seen to enhance. The effect of Prandtl number and thermal radiation parameter has an enhancing influence on Nusselt number. Rate of heat transfer is observed to enhance with increasing intensity of the heat source/absorption parameter. Thermal and solutal stratification parameters are found to enhance the magnitude of the surface drag coefficient. Nusselt number is seen to increase with increase in ε_1 while Sherwood number has an opposite influence. Nusselt number is found to be a decreasing function of solutal stratification parameter whereas Sherwood number is an increasing function. Sherwood number is observed to be an increasing function of Schmidt's number as well as chemical reaction parameter. Effect of thermophoresis parameter on Sherwood number is found to be the same as that of γ and Sc .

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Table 2: Skin friction coefficient, Nusselt number and Sherwood number for various values of pertinent parameters

M	β	λ	N	Pr	Nr	Q	ϵ_1	ϵ_2	Sc	γ	τ	$C_f\sqrt{Re_x}$	$Nu_x Re_x^{-1/2}$	$Sh_x Re_x^{-1/2}$
0.5												-2.048187	1.069412	0.823938
1.0	0.5	0.1	0.1	0.7	0.5	0.1	0.1	0.1	0.5	0.1	0.1	-2.377707	1.009082	0.795713
1.5												-2.668774	0.956308	0.771695
2.0												-2.932225	0.909420	0.750789
	0.5											-2.047974	1.068689	0.824026
0.5	1.0	0.5	0.1	0.7	0.5	0.1	0.1	0.1	0.5	0.1	0.1	-1.660390	0.988777	0.787343
	1.5											-1.510661	0.950302	0.769972
	2.0											-1.430349	0.927676	0.759775
		0.5										-1.773799	1.114798	0.846185
0.5	0.5	1.0	0.1	0.7	0.5	0.1	0.1	0.1	0.5	0.1	0.1	-1.456239	1.156679	0.867936
		1.5										-1.156389	1.190292	0.886047
		2.0										-0.869167	1.218925	0.901844
			0.5									-2.026542	1.074826	0.825408
0.5	0.5	0.1	1.0	0.7	0.5	0.1	0.1	0.1	0.5	0.1	0.1	-1.998565	1.078916	0.827487
			1.5									-1.970768	1.082898	0.829522
			2.0									-1.943143	1.086786	0.831516
				0.7								-2.048187	1.069412	0.823938
0.5	0.5	0.1	0.1	1.0	0.5	0.1	0.1	0.1	0.5	0.1	0.1	-2.058556	1.350922	0.828069
				1.5								-2.068933	1.741469	0.835041
				2.0								-2.075318	2.071306	0.841648
					0.1							-2.059314	0.935986	0.828516
0.5	0.5	0.1	0.1	0.7	0.4	0.1	0.1	0.1	0.5	0.1	0.1	-2.050541	1.039603	0.824820
					0.7							-2.043295	1.120983	0.822833
					1.0							-2.037272	1.188537	0.821685
						-0.2						-2.055062	1.257758	0.826742
0.5	0.5	0.1	0.1	0.7	0.5	-0.1	0.1	0.1	0.5	0.1	0.1	-2.053159	1.200542	0.825842
						0.1						-2.047974	1.068689	0.824026
						0.2						-2.043987	0.987184	0.823157
							0.1					-2.047974	1.068689	0.824026
0.5	0.5	0.1	0.1	0.7	0.5	0.1	0.2	0.1	0.5	0.1	0.1	-2.058934	1.163653	0.822142
							0.3					-2.069884	1.285439	0.820247
							0.4					-2.080825	1.447450	0.818340
								0.1				-2.048187	1.069412	0.823938
0.5	0.5	0.1	0.1	0.7	0.5	0.1	0.1	0.2	0.5	0.1	0.1	-2.049106	1.069196	0.883850
								0.3				-2.050025	1.068978	0.960868
								0.4				-2.050944	1.068760	1.063546
									0.5			-2.049059	1.071482	0.823713
0.5	0.5	0.1	0.1	0.7	0.5	0.1	0.1	0.1	1.0	0.1	0.1	-2.050604	1.071025	1.251843
									1.5			-2.051356	1.070864	1.586461
									2.0			-2.051825	1.070782	1.872003
										0.1		-2.049059	1.071482	0.823713
0.5	0.5	0.1	0.1	0.7	0.5	0.1	0.1	0.1	0.5	0.4	0.1	-2.049471	1.071349	0.924668
										0.7		-2.049770	1.071262	1.012028
										1.0		-2.050004	1.071198	1.090551
											0.1	-2.049059	1.071482	0.823713
0.5	0.5	0.1	0.1	0.7	0.5	0.1	0.1	0.1	0.5	0.1	0.5	-2.049301	1.071414	0.908271
											1.0	-2.049586	1.071338	1.016337
											1.5	-2.049850	1.071269	1.126881

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V. CONCLUSION

Some of the significant observations of the study are the following:

- ❖ An increasing thermal stratification parameter reduces velocity, temperature while the concentration increases. The local heat transfer coefficient enhances while the mass transfer coefficient decreases.
- ❖ Larger values of solutal stratification parameter lead to a depreciation in the flow and concentration where as the temperature increases. The Nusselt number is seen to be a decreasing function of the solutal stratification parameter while mass transfer coefficient has a reversal effect.
- ❖ The presence of thermophoresis reduces the heat transfer coefficient while the mass transfer coefficient has an opposite behaviour.

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