

# Radiation Effect on Finite Element Analysis of Convective Heat Transfer through a Porous Medium in a Horizontal Porous Channel

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**ABSTRACT:** The Finite Element analysis of viscous incompressible fluid through a porous medium in a Horizontal channel bounded by porous walls with Radiation effect. The momentum and energy equations are solved by employing Galerkin Finite Element analysis with Eight noded Serendipity elements. The velocity and the Temperature are analysed for different values of  $G, D^{-1}$ , and  $\alpha$  at different levels the shear stress and the rate of Heat Transfer on the walls  $z=0$  and  $z=1$  are evaluated numerically for different values of  $G, D^{-1}$ , and  $\alpha$ .

**KEYWORDS:** Heat Transfer, Porous medium, Horizontal channel, FEM

## I. INTRODUCTION

Natural convection flows due to the combined buoyancy effects of thermal and species diffusion in a fluid saturated porous medium have many applications, such as geothermal fields, soil pollution, fibrous insulation and nuclear – waste disposal.

The flow of a viscous incompressible fluid bounded by one or two infinite planes with porous walls has gained considerable importance in view of its applications to reduce boundary layer to a turbulent may be suppressed is to reduce mass from the boundary layer through pores or slits on the boundary. To obtain any desired reduction in the drag increasing suction along is uneconomical as the energy consumption of the suction pump will be more. Therefore the method of “cooling of the wall” in controlling the laminar flow together with applications of suction has become more useful.

In fluid flows through porous media the finite element method has been used extensively and found to be highly successful. Sugunamma [6] has analysed the free convective heat transfer flow of a viscous fluid through a porous medium confined in horizontal channel by using the Galerkin finite element method. In all these studies the thermal diffusion is not considered. This assumption is true only when the flow takes place at low concentration level. There are, however some exceptions. In view of the importance of this diffusion thermo effect Jha and Singh [4], Ajay Kumar Singh [2], and Riah and Hsui [7] have analysed the convective heat and mass transfer with Soret effect under different conditions.

## II. MATHEMATICAL FORMULATION

We analyse the convective flow of an incompressible viscous fluid in a horizontal channel filled with a porous matrix bounded by parallel porous walls. The flow takes place along the axis of the channel. The surface of the walls are kept at constant heat and mass flux in the direction of the flow. The momentum equations for the fully developed free convection flow. It is assumed that the fluid are in local thermal equilibrium. Boussinesq approximation is used so

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that the density variation will be retained on the buoyancy term. We choose the Cartesian frame of reference  $O(x,y,z)$ , such that the imposed pressure gradient is along  $x$ -axis and  $y = \pm h$  are the boundary planes (Fig.(a)). In the absence of extraneous forces the flow is unidirectional along  $x$ -axis.

Under these assumptions the equations governing the flow and heat transfer are

$$\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial y^2}\right) - \left(\frac{\mu}{k}\right) v_0 \frac{\partial u}{\partial y} = \frac{1}{\rho} \left(\frac{\partial p}{\partial x}\right) \quad (1.1)$$

$$0 = \left(\frac{\partial p}{\partial y}\right) - \rho g \quad (1.2)$$

$$-v_0 \frac{\partial T}{\partial y} = k_f \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2}\right) + Q(T - T_e) - \left(\frac{\partial(q_r)}{\partial y}\right) \quad (1.3)$$

$$\rho = \rho_0 [1 - \beta(T - T_0)] \quad (1.4)$$

By using Rosseland approximation (Brewster\*) the radiative heat flux is given by

$$q_r = -\left(\frac{4\sigma^*}{3\beta_R} \frac{\partial(T'^4)}{\partial y}\right) \quad (1.5)$$

and expanding  $T'^4$  by Taylor's series and neglecting terms of higher order we get

$$T'^4 \cong 4T_e^3 T - 3T_e^4 \quad (1.6)$$

where  $\sigma^*$  is the Stefan -Boltzman constant and  $\beta_R$  is mean absorption coefficient.

The flow being unidirectional, in view of the equation of continuity  $u = u(y, z)$ .

The heat mass flux being constant along the channel

$$\frac{dT}{dx} = A, \text{ on the wall } y = h$$

where  $A$  is the uniform temperature gradients respectively. Hence the temperature in the flow field may chosen to be

$$T = Ax + T_1(y, z)$$

The boundary conditions are

$$u = 0 \quad \text{on } y = h$$

$$T = T_1 \quad \text{on } y = h \text{ at the entry } x = 0 \text{ and} \quad (1.7)$$

$$T = Ax + T_1 \quad \text{on } y = h, x \neq 0$$

In view of the symmetry w.r.t. the central line  $y = 0$

$$\frac{\partial u}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = 0 \quad \text{on } y = 0 \quad (1.8)$$

We introduce the following non-dimensional variables as follows.

$$z = z^* b, \quad y = y^* b, \quad T = T_0 + \theta^* (T_1 - T_0) \quad u^* = \frac{v u}{\beta g b^2 (T_1 - T_0)}$$

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Substituting these Non-dimensional variables in equations (1.1)-(1.4)  $\frac{\partial}{\partial y} \left( \frac{\partial^2 u}{\partial z^2} \right) + \frac{\partial^3 u}{\partial y^3} - D^{-1} + s \left( \frac{\partial^2 u}{\partial y^2} \right) = N_t$

$$(1.9) \quad \left( 1 + \frac{4}{3N} \right) \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} + ps \frac{\partial \theta}{\partial y} - \alpha \theta = N_t PGU \quad (1.10)$$

Where

$$Nt = \frac{Ab}{T_1 - T_0} \quad (\text{Non-dimensional temperature gradient})$$

$$P = \frac{\mu C_p}{k_1} \quad (\text{Prandtl number}), \alpha = \frac{QL^2}{k_f} \quad (\text{Heat source parameter})$$

$$G = \frac{\beta g b^3 (T_1 - T_0)}{\nu^2} \quad (\text{Grashof number.}), Ec = \frac{\beta g b}{C_p} \quad (\text{Eckert number})$$

$$N = \frac{\beta_R K_f}{4\sigma^* T_e^3} \quad (\text{Radiation parameter})$$

The corresponding boundary conditions in the non-dimensional form are

$$u = 0 \quad \text{on } y = 1 \quad \theta = 1, \text{ at } x = 0 \quad \text{on } y = 1 \quad (1.11)$$

$$\theta = 1 + N_1 x, \quad x \neq 0 \quad \text{on } y = 1 \quad (1.12)$$

$$\frac{\partial u}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = 0 \quad \text{on } y = 0 \quad (1.13)$$

In view of the two dimensionality and symmetry of the flow w.r.t. the midplane of the channel we analyse the flow features in a domain in the upper half of the channel bounded by the impermeable wall lying between two parallel planes normal to the wall at unit distance apart. The finite element analysis with quadratic approximation functions is carried out using eight noded serendipity elements.

### III. FINITE ELEMENT ANALYSIS

The finite element method was initially developed as an adhoc engineering procedure for constructing matrix solutions to stress and displacement calculations in structural analysis. Very few fluid dynamic problems can be expressed in a variation form. Consequently most of the finite element applications in fluid dynamics have used the Galerkin finite element formulation. A traditional engineering interpretation of finite element method and its applications to fluid mechanics are treated by Thomas set [8] and Baker[1].A mathematical perceptive of this method is provided by Strang and Oden and Reddy [5], and Jain et al [3] .The Galerkin finite element method has two important features. Firstly the approximate solution is written directly in terms of the nodal unknowns. Secondly the approximately functions or the shape functions are chosen exclusively from low order piecewise polynomials restricted to contiguous elements. We now make use of Galerkin finite element method to solve the coupled governing equations and thereby obtain the velocity  $u$  and the temperature  $\theta$

If  $u^i$  and  $\theta^i$  are the approximations of  $u$  and  $\theta$  we define the errors (residual)  $E_1^i$  and  $E_2^i$  as

$$E_1^i = \frac{\partial}{\partial y} \left( \frac{\partial^2 u^i}{\partial z^2} \right) + \frac{\partial^3 u^i}{\partial y^3} - D^{-1} \frac{\partial u^i}{\partial y} + S \frac{\partial^2 u^i}{\partial y^2} - N_1 \quad (3.1)$$

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$$E_2^i = \left(1 + \frac{4}{3N}\right) \frac{\partial^2 \theta^i}{\partial y^2} + \frac{\partial^2 \theta^i}{\partial z^2} + ps \frac{\partial \theta^i}{\partial z} - \alpha \theta^i \quad (3.2)$$

Where

$$u^i = \sum_{k=1}^8 u_k^i N_k^i \quad (3.3)$$

$$\theta^i = \sum_{k=1}^8 \theta_k^i N_k^i \quad (3.4)$$

Using Green 's theorem over the surface integral we obtain

$$\int_{\Omega_i} \left( \frac{\partial^2 u^i}{\partial y^2} \frac{\partial N_j^i}{\partial y} + \frac{\partial^2 u^i}{\partial z^2} \frac{\partial N_j^i}{\partial z} + M^2 u^i \frac{\partial N_j^i}{\partial y} + N_1 \right) N_j^i d\Omega = \int_{\Gamma} \left( N_j^i \frac{\partial^2 u^i}{\partial y^2} n_y + N_j^i \frac{\partial^2 u^i}{\partial z^2} n_z + N_j^i \frac{\partial u^i}{\partial z} n_z \right) d\Gamma_i \quad (3.5) \text{ where}$$

$$\int_{\Omega_i} \left( \frac{\partial \theta^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial \theta^i}{\partial z} \frac{\partial N_j^i}{\partial z} \alpha_1 \theta N_j^i - N_1 P_1 G U \right) N_j^i d\Omega_i = \int_{\Gamma_i} \left( N_j^i \frac{\partial \theta^i}{\partial y} n_y + N_j^i \frac{\partial \theta^i}{\partial z} n_z \right) d\Gamma_i \quad (3.6)$$

$\Omega_i$  is the serendipity element bounded by  $\Gamma_i$ ,  $n_y$  is the direction cosine normal to  $\Gamma_i$ .

Substituting (2.3) and (2.4) in L.H.S of (2.5) and (2.6), we get

$$\int_{\Omega_i} \left( \sum_{k=1}^8 u_k^i \left\{ \frac{\partial^2 N_k^i}{\partial y^2} \frac{\partial N_j^i}{\partial z} + \frac{\partial^2 N_k^i}{\partial z^2} \frac{\partial N_j^i}{\partial y} + M^2 N_k^i \frac{\partial N_j^i}{\partial y} + N_1 \right\} N_j^i d\Omega_i = Q_j^i \quad (3.7)$$

$$\int_{\Omega_i} \left( \sum_{k=1}^8 \theta_k^i \left\{ \frac{\partial N_k^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z} \frac{\partial N_j^i}{\partial z} - \alpha N_j^i N_k^i \right\} + \sum_{k=1}^8 u_k^i \left\{ P E c G \left( \left( \frac{\partial N_k^i}{\partial y} \right)^2 + \left( \frac{\partial N_k^i}{\partial z} \right)^2 \right) + M^2 (N_k^i)^2 - N_1 P G N_k^i \right\} N_j^i \right) d\Omega_i = (Q^T)^i_j \quad (3.8)$$

where  $Q_j^i = \int_{\Gamma_i} [N_j^i \frac{\partial^2 u^i}{\partial y^2} n_y + N_j^i \frac{\partial u^i}{\partial y} n_y + N_j^i \frac{\partial u^i}{\partial z} n_z] d\Gamma_i$

$$(Q^T)^i_j = \int_{\Gamma_i} [N_j^i \frac{\partial \theta^i}{\partial y} n_y + N_j^i \frac{\partial \theta^i}{\partial z} n_z] d\Gamma_i \quad j=1,2,\dots,8.$$

Choosing different  $N_k^i$ 's corresponding to each element  $e_i$  the equation (3.7) and (3.8) results in sixteen equations for three sets of unknown

$(u_k^i), (\theta_k^i)$  viz

$$(a_{kj}^i) (u_k^i) = (Q_j^i) \quad (3.9)$$

$$(b_{kj}^i) (\theta_k^i) + (d_{kj}^i) (u_k^i) = (Q_j^T)^i \quad (3.10)$$

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where  $(a_{kj}^i), (b_{kj}^i)$  and  $(d_{kj}^i)$ , are  $8 \times 8$  stiffness matrices and  $(Q_j^i), (Q_j^T)^i$  are  $8 \times 1$  column matrices. Repeating the process with each of mn elements and making use of global coordinates and inter element continuity conditions as well as the boundary conditions to assemble the element matrices, we obtain global matrices for the unknown  $u$  and  $\theta$  at the respective global nodes which are ultimately determined on solving the matrix equation.

## IV. RESULTS AND DISCUSSION

The Fintie Element analysis is used to investigate the mixed convective Heat Transfer flow of a viscous incompressible fluid through a saturated porous medium in a Horizontal channel through a flow with in the porous medium is taken into account based on the Brinkaman extended Darcy model the flow is uni directional and the behaviour of velocity temperature and Nusselt number is discussed for different variations of the governing parameters . The variation of  $u$  at different vertical levels  $Z=0$  and  $1$  are drawn in Figs.1and 2. these profiles are asymmetric curves exhibiting a reversal flow in the region  $0 \leq y \leq 0.6$  at  $Z=0$  and  $Z=1$ . Lesser the permeability of the porous medium larger  $u$  at  $Z=0$  level reversal flow in the vicinity of  $y=1$ . Lesser the permeability of the porous medium larger  $u$  at  $Z=0$  level Larger  $u$  in the flow region and for further lowering of the permeability smaller the velocity in the flow region (Fig.1). Fig.2 shows that at  $Z=1$  level a reversed behaviour is noticed in  $u$  with  $D^{-1}$ . Also the reversal zone shrinks with  $D^{-1} \leq 5 \times 10^3$  and enhances with  $D^{-1} \geq 10^4$ . The non-dimensional temperature  $\theta$  is shown in Figs.3 and 4 at different horizontal and vertical levels. We follow the convention that the non-dimensional temperature positive or negative according as the actual temperature is greater/lesser than in mean temperature. We notice that the non-dimensional temperature is negative for all variations this indicates that actual temperature is lesser than the mean temperature every where in the fluid region lesser the porous permeability smaller the actual temperature in the every flow region at all vertical and horizontal levels(Figs.3 and 4)

The shear stress has been evaluated for variations in  $D^{-1}$  and  $S$ . At three different axial positions  $Z=0$ , and  $1$  of the channels and is exhibited in Tables. 1and 2). It is observed from these tables that lesser the permeability of the porous medium larger the shear stress at all axial positions (Tables.1 and 2). The Nusslet Number (Nu) which represents the rate of Heat Transfer has been evaluated for variations in  $G, D^{-1}$ , and  $S$ . The Nusselt Number at two different position at  $Z=0$  and  $1$  across the boundary plane has been shown in Tables.3 and 4 for different variations. The analysis has been carried out with Pranditl Number  $P=0.71$ . It is noticed that the Nusselt Number (Nu) decreases as we move across the boundary plane at all axial positions where as its magnitude decreases with increase in the axial distance starting from the entry position  $Z=0$ . Fixing the other parameters, in general the rate of Heat Transfer enhances with  $D^{-1}$  and  $|G|$  at all axial distances. An increasing the suction parameter  $S \leq 0.3$  enhances  $|Nu|$  and depreciates with higher  $S \geq 0.3$  (Table.3 ).

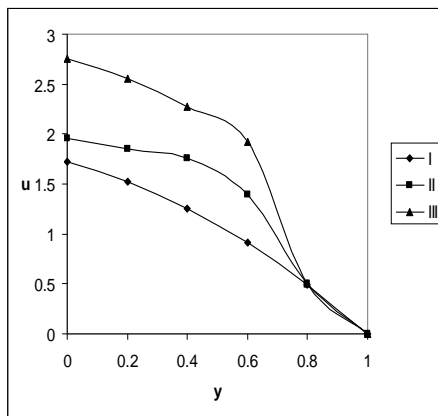


Fig. 1 : u with  $D^{-1}$  at  $z = 0$

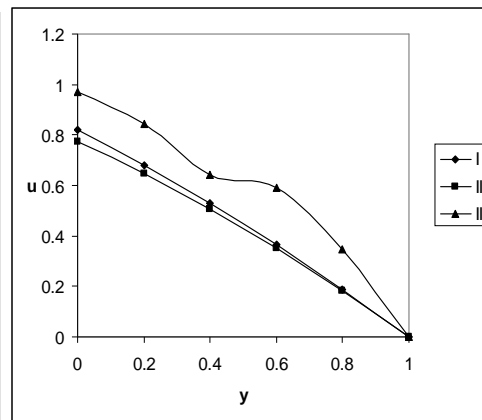


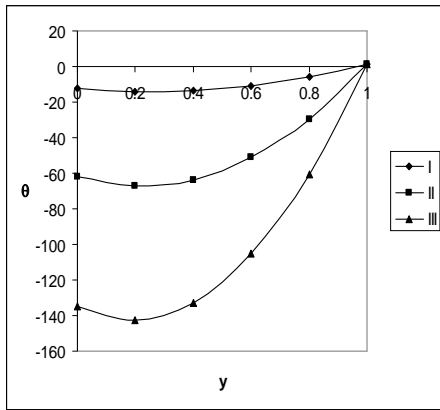
Fig. 2 : u with  $D^{-1}$  at  $z = 0$

$D^{-1}$  I II III  $10^3$   $5 \times 10^3$   $10^4$   $D^{-1}$  I II III  $10^3$   $5 \times 10^3$   $10^4$

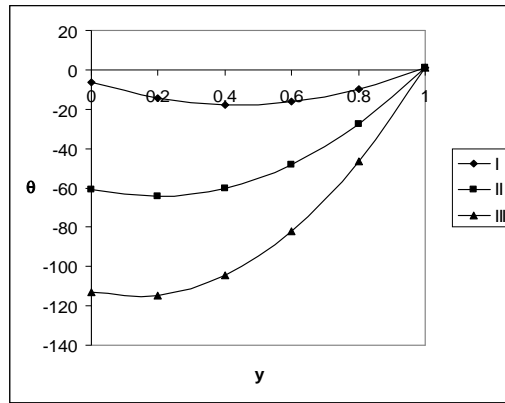
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**Fig. 3 :  $\theta$  with  $D^{-1}$  at  $z = 0$**



**Fig. 4 :  $\theta$  with  $D^{-1}$  at  $z = 0$**

$D^{-1}$ 
I    II    III
 $10^3$      $5 \times 10^3$      $10^4$

**Table -1 Stress ( $\tau$ ) at  $Z=0$**

$D^{-1}$	I	II	III
$10^2$	77.0146	76.0663	75.1302
$3 \times 10^2$	231.335	228.973	226.64
$5 \times 10^2$	386.013	382.283	378.599
S	0.1	0.3	0.5

**Table -2 Stress ( $\tau$ ) at  $Z=1$**

$D^{-1}$	I	II	III
$10^2$	55.8535	55.1435	54.4468
$3 \times 10^2$	131.845	130.276	128.733
$5 \times 10^2$	207.136	204.711	202.325
S	0.1	0.3	0.5

**Table -3 Nusselt Number (Nu) at  $Z=0$  and  $Z=1$**

$D^{-1}$	I	II	III	IV	V	VI	VII	VIII
$10^2$	16.962	46.9883	77.0146	-13.0642	-43.0905	-73.1168	150.19	75.1302
$3 \times 10^2$	47.826	139.58	231.335	-43.9282	-135.683	-227.437	456.002	226.64
$5 \times 10^2$	78.761	232.389	386.013	-74.8639	-228.489	-382.115	762.622	378.599
G	10	30	50	-10	-30	-50	100	100
S	0.1	0.1	0.1	0.1	0.1	0.1	0.3	0.5

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