

Relevance of Comparative Approaches Plan Critical and Integral

N. Ngarmaim¹, B. Tikri², B. Bassa¹, N. Kimtanger¹, F. Penne³, J-L. Robert³.

University of Djaména-Chad¹

University Polytechnic Institute of Mongo-Chad²

Institute Pascal, IUT Montluçon-France³

ABSTRACT: The work in this paper is to compare relatively recent fatigue criteria with already proven and recognized formalisms used for fatigue design calculations in industry. We dispose, to do this, of a large number of multiaxial tests corresponding to solicitations very diverse put to contribution to validate the criteria. Three criteria of integral approach (Zenner, Fogue and Papuga I), three criteria of approach critical plan (Dang Van, Robert and Papuga II) and two criteria based on the invariants of the constraints (Crossland and Vu) are retained. Two analyzes were carried out:

- The one on the accuracy of the predictions of the criteria, from a general point of view on all the tests used in this work. It emerges from this analysis that the integral approach criteria Zenner and Fogue are practically the most accurate;
- The second analysis consists in showing the comparative relevance of the critical plan and integral approach to translate the behavior into multiaxial fatigue. A representative of each approach is chosen, the criterion of Dang Van, for the critical plan approach, and that of Fogue, for the integral approach. The analysis is based on the distribution of their damage indicator by plane, Eh, and on the value of their fatigue function, E, for the test considered. The selected tests are representative of the main cases of solicitations that can be encountered. This study allowed to show the influence of the middle part and the phase shift of the solicitations as to the forecasts of the two criteria (fatigue functions). This is the integral approach that takes precedence over other approaches developed.

KEYWORDS: Criteria, Multiaxial fatigue, Damage indicator, Fatigue function, Average stress influence, Phase shift influence.

I. INTRODUCTION

One of the specific problems of the fatigue design of the mechanical components is taken into account of the character multiaxial. The tool adapted to deal with this aspect is a criterion of multiaxial fatigue, which establishes a relationship between the stresses of multiaxial cycle analysed and fatigue characteristics of the material, relationship that allows to adjudicate on the fatigue strength of the material when subjected to stress cycle. Practically, the fatigue analysis is carried out in all the points where of the component are known the stresses. If the critical point of the component is located in advance, fatigue calculations are performed only in this point. In the opposite case, the calculations are carried out fatigue at all the points to determine the critical area of the component from the point of view of fatigue and thus establish a mapping life of this component. The work presented here have had the ambition provide the Design Office elements decision as to regarding criteria multiaxial fatigue. New proposed criteria have appeared in recent years [1-11]. Some cycles of specific constraints prove discriminant overlooked the most efficient criteria and thus allowing to better perceive the specificity of the criteria [12] and [11].

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 6, Issue 11, November 2017

II. GENERAL FORMALISM CRITERIA MULTIAXIAL FATIGUE

Order to rule on a given compared to the endurance limit of the material cycle multiaxial, the principle implemented by a fatigue criterion is the constitution of a fatigue function denoted E, which connects the parameters of stresses specific to multiaxial cycle analyzed and endurance limits of the material as σ_{-1} , τ_{-1} , σ_0 , etc. The function of fatigue is equal to unity when the endurance limit of the material is attained through analyzed the cycle; this relationship is written as:

$$E(t) = E([\sigma(t)], \sigma_{-1}, \sigma_0, \tau_{-1}) \quad (1)$$

Where $[\sigma(t)]$ is the stress tensor of the cycle during a period, Table 1 gives the different expressions of the indicator of damage and functions studied fatigue of the criteria [5, 7, 11-13].

TABL 1 - FORMALISM AND WEDGING OF CRITERIA

Author	Formalism	Wedging
Based on invariant stress criteria		
Crossland	$E_{CR} = \frac{\sqrt{J_{2a}} + A.I_{1max}}{B}$	σ_{-1}, τ_{-1}
Vu	$E_{VU} = \frac{1}{\beta} \text{Max}_t \left(\sqrt{\gamma_1 J_{2a}^2(t) + \gamma_2 J_{2m}^2 + \gamma_3 I_f(I_{1a}, I_{1m})} \right)$	σ_{-1}, τ_{-1}
Criteria for critical plan approach		
Dang Van	$E_h(t) = \frac{\ \vec{\tau}_{ha}(t)\ + \alpha.p(t)}{\theta} \quad E_{DV} = \text{Max}_{t,h} [E_h(t)]$	σ_{-1}, τ_{-1}
Robert	$E_h(t) = \text{Max}_t \left\{ \frac{\tau_{ha}(t) + \alpha.\sigma_{hha}(t) + \beta.\sigma_{hhm}}{\theta} \right\} \quad E_{RB} = \text{Max}_h [E_h(t)]$	$\sigma_{-1}, \tau_{-1}, \sigma_0$
Papuga II	$E_h = \frac{1}{\sigma_{-1}} \sqrt{\alpha\tau_{ha}^2 + \beta \left(\sigma_{hha} + \frac{\tau_{-1}}{\sigma_0} \sigma_{hhm} \right)} \quad E_{PAII} = \text{Max}_h (E_h)$	σ_{-1}, τ_{-1}
Criteria Integral approach		
Fogge	$E_h = \frac{a\tau_{ha} + b\sigma_{hha} + d\sigma_{hhm}}{\sigma_{-1}} \quad E_{FG} = \sqrt{\frac{1}{S} \int_S E_h^2 ds}$	$\sigma_{-1}, \tau_{-1}, \sigma_0$
Papuga I	$E_h = \alpha\tau_{ha}^2 + \beta \left(\sigma_{hha} + \frac{\sigma_{-1}}{\sigma_0} \sigma_{hhm} \right) \quad E_{PAI} = \sqrt{\frac{1}{S} \int_S E_h ds}$	σ_{-1}, τ_{-1}
Zenner and Liu	$E_h = a\tau_{ha}^2 (1 + m\tau_{hm}^2) + b\sigma_{ha}^2 (1 + n\sigma_{hhm}) \quad E_Z = \frac{1}{\sigma_{-1}} \sqrt{\frac{1}{S} \int_S E_h ds}$	σ_{-1}, τ_{-1}

III. ANALYSIS PREDICTIONS MULTIAXIAL FATIGUE CRITERIA

We go here compare the proposals of relatively recent formalisms already proven and accepted criteria for fatigue, and used for the calculations of the fatigue design in industry (Dang Van criteria and Zenner). We have to do this, a large number of tests multiaxial corresponding to very different solicitations enlisted to validate the criteria. This validation is therefore of a very general character since it the criteria allows confront the criteria of fatigue to a large panel of different conditions of use in terms of solicitations with or without phase shifts, with or without the presence

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 6, Issue 11, November 2017

of mean stresses at levels loading corresponding to lifetimes variant of 10^5 to 2×10^6 cycles and quite different from each other steel grades. An initial survey of tests are listed into 17 series had been carried out by B. Weber, a 18th series of tests came to complete this database, a total of 259 trials multiaxial [NGA10].

III.1. PRINCIPLE OF VALIDATION THE CRITERIA MULTIAXIALS

Each series of tests corresponds to the multiaxial fatigue limit of the material for a given number of cycles. In this life are also known fatigue limits of the material under simple sollicitations ($\sigma_1, \tau_1, \sigma_0$). For these simple calibration tests, the criterion is verified by definition ($E=1$). To adjudicate the validity of a criterion, one examines the gap recorded between the actual function fatigue E of the criterion and its theoretical value equal to unity. The validity of the criterion for a given test is judged by the amount denoted ΔE and equal to $E-1$. The value of ΔE allows to the passage to situate the multiaxial cycle tested compared with the fatigue limit of the material:

1. if $\Delta E < 0$, the cycle multiaxial within the meaning of the criterion is less than the fatigue limit of the material, which means that the test provides for a longer life greater than that obtained experimentally. Such cases penalize the Study Office, as it means a non-conservative prediction of service life, thus keeping the component;
2. if $\Delta E = 0$, the test correctly predicts that the fatigue limit is reached. It turns out in this case therefore exact;
3. if $\Delta E > 0$, the fatigue criterion perceives the multiaxial cycle analyzed as superior capabilities fatigue resistance of the material, it provides for therefore a shorter service life than is actually obtained for this cycle. That is to say an earlier crack initiation than it is in reality. It proves so in this case conservative for the design office.

III.2. RESULTS

To give the synthesis of all the results obtained by a fatigue criterion on the database of the tests in its totality, we construct a histogram of the results reported in which the ordinate, the number of tests with a standard ΔE belonging to a given interval (class of belonging). If the criterion was perfect all tests should be in the middle of the histogram (for a difference of validity ΔE zero). The trends to a conservative or non-conservative forecast translate a displacement of the histogram in areas where $\Delta E > 0$ and $\Delta E < 0$, respectively. Figure 1 shows the histograms obtained for the 3 criteria of integral (Zenner, Fogue and Papuga I), the 3 criteria critical plane approach (Van Dang, Robert and Papuga II) and 2 criteria based on invariants stresses (Crossland and Vu).

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 6, Issue 11, November 2017

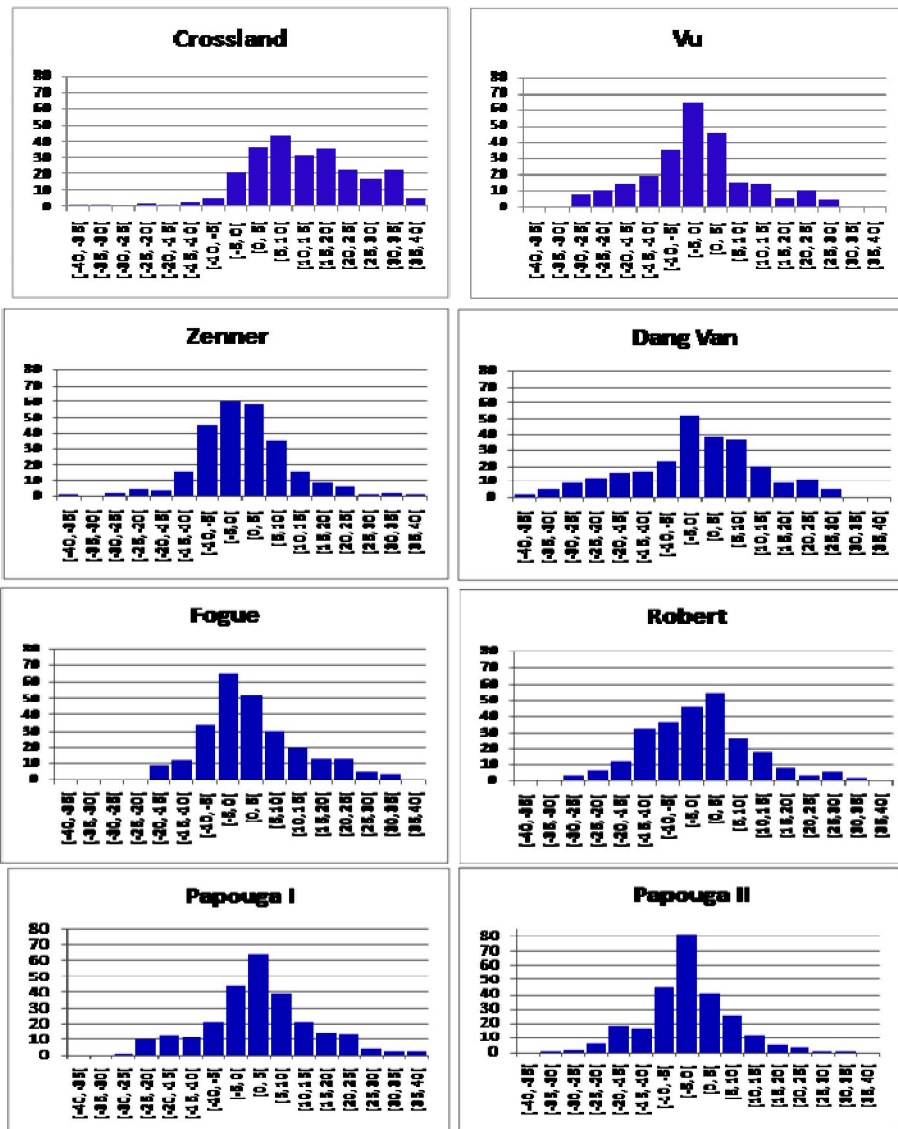


Fig.1 -Histogramsofvalidity of the criteriastudied

III.2.1. ANALYSIS OF RESULTS

The analysis will focus on the prediction accuracy of the criteria, this from a general point of view on all tests contribution set to in this work. For this purpose, Tab.2 and Fig.2 provides a summary of the results achieved with the 8 fatigue criteria studied, confronted with the databank of 259 tests multiaxials fatigue.

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 6, Issue 11, November 2017

TAB.2 -SUMMARY OF RESULTS

Authors	Character no conservative $\Delta E < 0 \%$		Character conservative $\Delta E > 0 \%$		Appropriateness of the criterion $-5\% \leq \Delta E \leq +5 \%$	
	Number	Percentage	Number	Percentage	Number	Percentage
Zenner (ZN)	131	50,58 %	128	49,42 %	118	45,56%
Papuga-I (PAI)	68	37,84 %	161	62,16 %	108	41,70%
Papuga-II(PAII)	170	65,64 %	89	34,36 %	122	47,10%
VU (VU)	156	60,23 %	103	39,77 %	111	42,86%
Fogue (FG)	121	46,72 %	138	53,28 %	117	45,17%
Crossland (CR)	35	13,51 %	224	86,49 %	58	22,39%
Dang Van (DV)	138	53,28 %	121	46,72 %	91	35,14%
Robert (RB)	139	53,67 %	120	46,33 %	101	39,00%

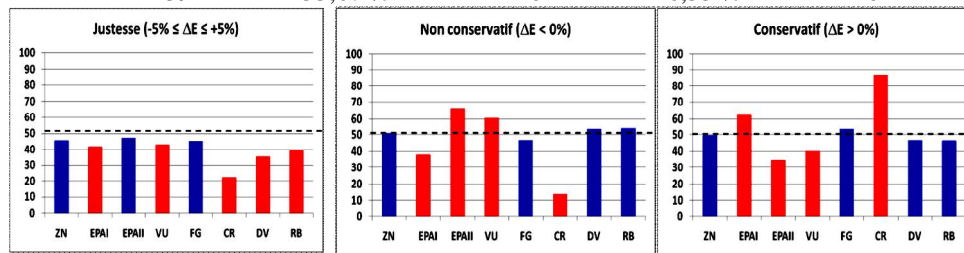


Fig.2 -Histogramthe synthesis ofcriteria studied

As fromsummary table (Tab.2) and histogramsvaliditycriteria(Fig.2) above, the analysis and evaluation ofresults obtained areas follows :

- The integral approach Zenner and Fogue criteria are practically the most accurate. They feature formalisms that predispose to better detect whether endurance of metallic materials threshold is reached or not. The forecasts are quite balanced and practically no uncommanded or opposite excess conservatism.
- The third criterion integral approach, that of Papuga I, is a little less efficient but proves to be mostly quite strongly conservative.
- The criterion of critical plane approach Papuga II gives excellent predictions (slightly better than integral approaches Zenner and Fogue) but it turns out he is very strongly non-conservative.
- The other two approaches critical plane type studied (Dang Van and Robert) are less efficient than the integral approach and conservative trend rather not.
- Given the criterion (based on invariants) is almost on a par with the integral approach in terms of the prediction, but it turns out to be non-conservative.
- The criterion of Crossland (invariant constraints) turns out to be rather mediocre compared to other criteria. It reveals more conservative.

IV. PARTICULARITY APPROACHES CRITICAL PLANE AND INTEGRAL

With the aim to analyze the relevance of comparative approaches critical and integral plane translate behavior multiaxial fatigue, we chose a representative of each approach, the Dang Van criterion for the approach to critical level and that of Fogue for the integral approach. The analysis is made on the distribution of the indicator of damage by plane E_h and the function of the fatigue test considered. Successful tests are representative of major cases of solicitations may be encountered. Five stress cycles in total are used for this analysis [11]. The five stress cycles used correspond either to the state of constraint proportional either to the non-proportional stress state. In Figures 3 to 7, are shown on the left, the components of the selected stress cycles. To the right of these figures, for the purpose of comparing the two multiaxial fatigue approaches, are shown the distributions of the damage indicator by E_h plane. E_{DV} and E_{FG} are respectively the damage indicators Dang Van criteria and Fogue for the successful stress cycle.

IV.1. CYCLES CORRESPONDING TO STATES PROPORTIONAL STRESS

Two cyclesto proportional stress states were selected for this analysis.

- **Test 1 (I04):** traction balance biaxial phase, fixed principal directions

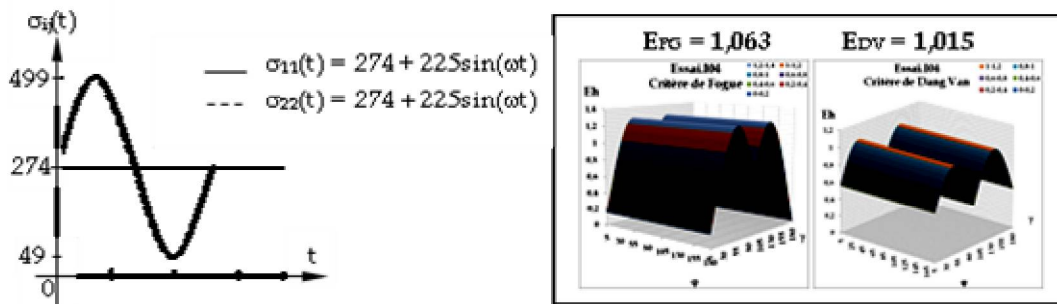


Fig.3- CycleTest.1and distributions of damage indicators

The two criteria are more accurate from the perspective of the value of their respective function of fatigue. The Dang Van criterion however better than that of Fogue for this test, its function fatigue being closer to the unit value. The distribution of the indicator Eh for these two criteria a multitude of critical planes defined by $\gamma=45^\circ$ or $\gamma=135^\circ$ to the Dang Van criterion and $\gamma=55^\circ$ or $\gamma=125^\circ$ for those of Fogue.

- **Test 2 (B11):** Traction-torsion phase, fixed principal directions

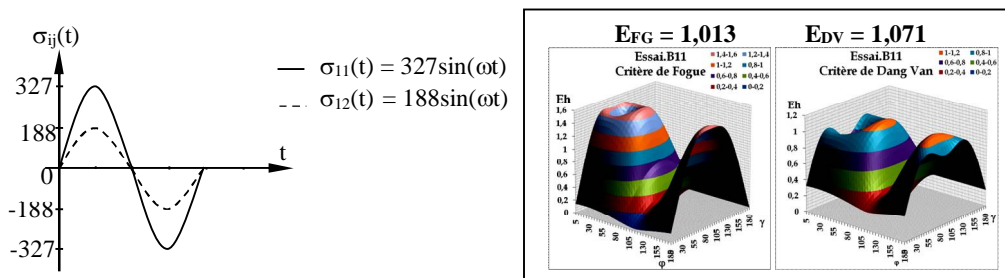


Fig.4- CycleTest 2and distributions of damage indicators

The two criteria shall rule in a way almost similar to the severity of the stress state in terms of the fatigue resistance, but the criterion Fogue is however more accurate (closer to the unit value) than that of Dang Van. The distribution of this indicator plan critical coordinates ($\gamma=90^\circ$, $\varphi=55^\circ$) to the criterion Fogue, and two critical coordinate planes ($\gamma=90^\circ$; $\varphi=70^\circ$ or 160°) for the Dang Van criterion.

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 6, Issue 11, November 2017

IV.2. CYCLES CORRESPONDING TO STATES OF NO-PROPORTIONAL STRESS

The same type of analysis is conducted here for three cycles states stress non-proportional, fixed or mobile principal directions.

Test 3(I07): Traction biaxialphase, fixedprincipal directions

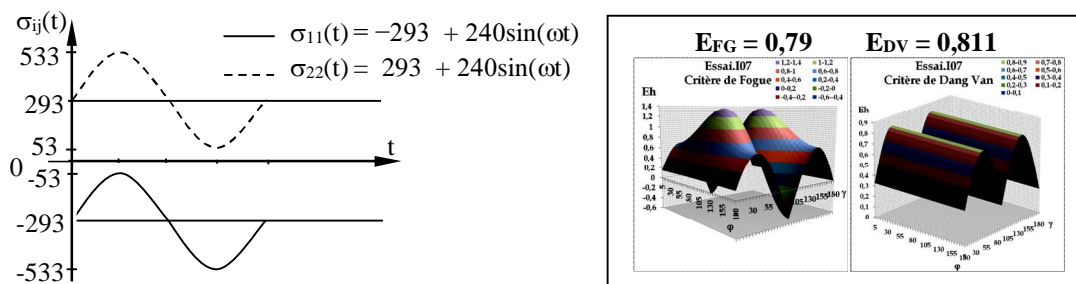


Fig.5- CycleTest 3and distributions of damage indicators

The two criteria are very fatiguedistant from real behavior of the material for this test (non-conservative predictions). The distribution of the indicator criterion Fogue presents two critical coordinate planes ($\gamma=55^\circ$ or 125° ; $\phi=90^\circ$), while that obtained for by Dang Van presents a multitude of coordinates $\gamma=45^\circ$ and $\gamma=135^\circ$.

• **Test 4 (B04):** Traction-compression out of phase, fixed principal directions

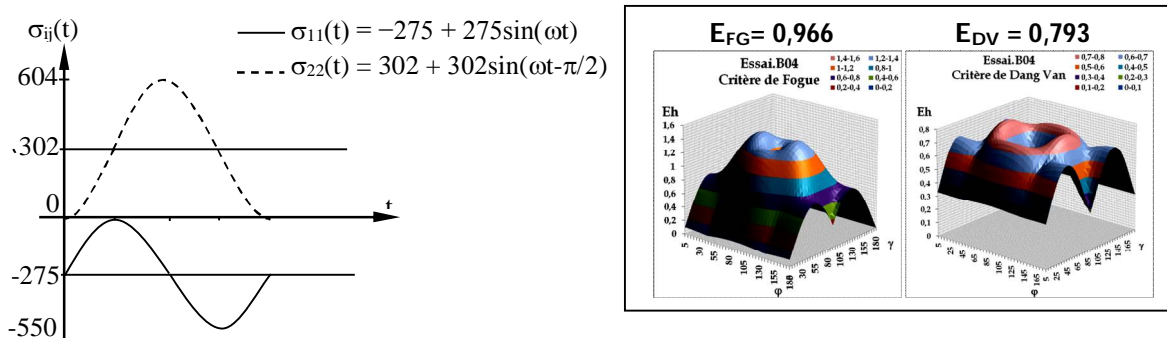


Fig.6 -CycleTest 4and distributions of damage indicators

In view of function values of fatigue, the criterion of Fogue is much better in this test than that of Dang Van. The distribution of the indicator presents two critical planes for each of two criteria, the coordinate ($\gamma=90^\circ$; $\phi=60^\circ$ or 120°) and those of Fogue ($\gamma=90^\circ$; $\phi=45^\circ$ or 135°) to those of Dang Van.

Test 5 (C15) : Traction torsion out phase, mobile principal directions

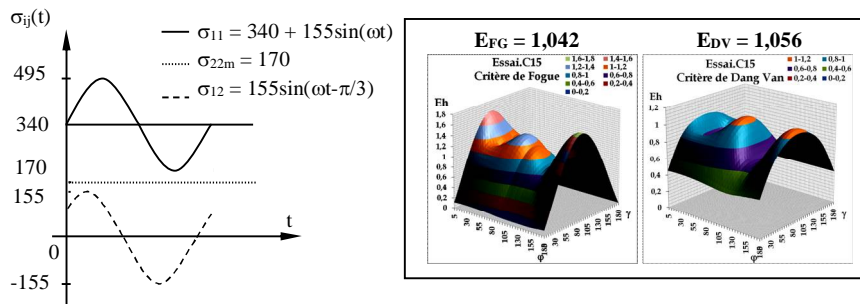


Fig.7- CycleTest5 and distributions of damage indicators

Both criteria are rather efficient for this test, the criterion of Fogue being somewhat better than that of Dang Van. The distribution of the indicator has a single critical plane for both criteria whose coordinates are $(\gamma = 90^\circ; \varphi = 175^\circ)$ according to the criterion Fogue and $(\gamma = 90^\circ; \varphi = 80^\circ)$ according to the Dang Van.

Fig.8 shows the EFG and EDV fatigue function values of the criteria for the five tests.

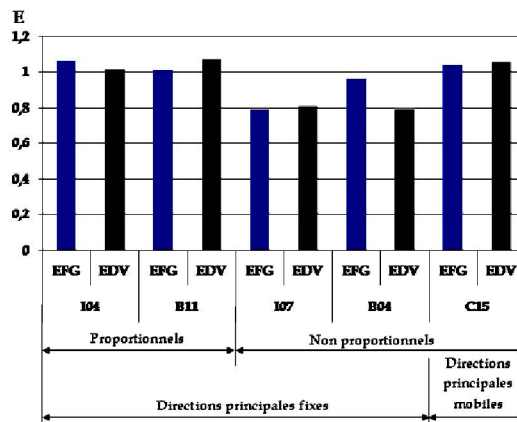


Fig.8 – Recapitulation of the functions fatigue criteria Fogue and Dang Van for five tests

V. INFLUENCE OF MEAN STRESS AND PHASE DISPLACEMENT ON THE FORECASTS OF THE TWO CRITERIA

An additional study on the influence of the mean stress and phase displacement solicitations regarding forecasts of the two criteria (functions fatigue) has identified the following observations:

V.1. SENSITIVITY TO MEAN STRESS

The average tensile stress has a detrimental effect on the fatigue strength of the material according to two criteria. Sensitivity to mean stress is more pronounced for the criterion Fogue compared to the Dang Van criterion. This sensitivity is translated by the value of the function of fatigue (for the test 1) passing from 0.60 to 1.063 for the criterion Fogue and 0.76 to 1.015 for the Dang Van, respectively without and with biaxial average constraints of the test. The same trend is observed for the test 3 but is less pronounced because an average tensile strain is superimposed on an average compressive stress in a perpendicular direction.

V.2. SENSITIVITY TO THE PHASE SHIFT

The two criteria are also sensitive to the phase shift. For test 4, the value of the function of fatigue without phase shift passes from 0.867 to 0.966 with phase shift for the criterion Fogue and 0.902 without

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 6, Issue 11, November 2017

phase shift to 0.793 with phase shift for the Dang Van criterion. The phase shift therefore renders more solicitation endomamageantes within the meaning of fatigue resistance for the criterion Fogue but it is the opposite effect for the Dang Van criterion. For tension-torsion test 5 out of phase, the deletion of phase shift between the components for the solicitation function increases the fatigue of the two criteria. That of Fogue passes from 1.042 to 1.055 and the Dang Van varies from 1.056 to 1.142. For this type of solicitation, therefore the phase shift has a beneficial effect fatigue where a traction-compression this is true only for the Dang Van criterion.

VI. DISCUSSION

The two points of discussion of the results obtained carry firstly on the character overall conservative or non-conservative criteria on all of tests of database, independently of any consideration of the nature of the tests, and secondly the accuracy of the predictions criteria there too a general point of view on all put to test in this work. For this purpose, Tab.2 and Fig.2 providing a review of results obtained with 8 fatigue criteria studied, confronted with the database of 259 multiaxial fatigue testing.

From previous table summary and histograms validity criteria that were presented, the analysis and stock of the results obtained are as follows:

- The integral approach Zenner and Fogue criteria are practically the most accurate. Formalisms predispose their best to detect if the of metallic materials endurance threshold is reached or not. Forecasts are quite balanced and practically no very marked conservatism or reverse excess.
2. The third criterion integral approach, that of Papuga I, is a little less efficient but proves to be mostly quite strongly conservative.
- The criterion of critical plane approach of Papuga II gives excellent predictions (slightly better than integral approaches Zenner and Fogue) but it turns out it is strongly non-conservative.
- The other two approaches critical plane type studied (Dang Van and Robert) are less efficient than the integral approach and conservative trend rather not.
- The criterion VU (based on invariants) is almost at par with the integral approach in terms of prediction, but it turns out to be non-conservative.
- The criterion of Crossland (invariant constraints) turns out to be rather mediocre compared with the other criteria. It reveals more conservative.

VII. CONCLUSION

The integral approach seems to take precedence over other approaches developed for the criteria of multiaxial fatigue, and seems to justify industrial practice most frequently observed in Germany. The benefit a little back of the Dang Van criterion, though very commonly used in France, may be to find in the fact that his "critical plane" character is not respected quite at the foot of the letter : the combination of the hydrostatic pressure which is the normal stress on the existing plane octahedral, with the alternating shear stress acting on a given plane, in fact associated constraint parameters that are not related to the same physical plane (unless, and only in this case, when the plan is analyzed the octahedral plane, and that the latter "turns" not during the cycle). Assuming that the severity of loading on the material plane or facet is related to the constraints that actually generates the load on this plan it seems more logical to use the normal and tangential stresses specific to the relevant plane. Deficiency of the Dang Van criterion vis-a-vis the effect of the mean stress is related to the fact that the mean and alternating parts of the hydrostatic stress involved in the same proportion (coefficient α) since calibrated only on two fatigue limits in alternating tension and alternating torsion symmetrical, which do not translated the actual particular influence of mean stress.

VIII. ACKNOWLEDGEMENTS

We thank the commission COCOFOR of NRAC (National Centre for Support to Research in Chad) to which financed our trip in France for the finalization of this article. Faculty of Exact and Applied Sciences at the University of N'Djamena (Chad) and the Universitaire Polytechnic Institute of Mongo to have allowed us to release this work. All of

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 6, Issue 11, November 2017

our gratitudes to the responsible for of the Laboratory MMS Pascal Clermont II Institute's website Montlucon for their supports techniques.

REFERENCES

- [1] Bastien Weber. Fatigue multiaxiale des structures industrielles sous chargement quelconque. Thèse de l'INSA de Lyon. N° d'ordre 99ISAL0056. 1999.
- [2] Andrea Carpinteri, Andrea Spagnoli, Multiaxial high-cycle fatigue criterion for hard metal, International Journal of Fatigue 23(2001) pp135-145
- [3] C. Han, X. Chen, K.S. Kim Evaluation of multiaxial fatigue criteria under irregular loading. International Journal of Fatigue 24 (2002) 913–922.
- [4] A. Fatemi and L. Yangt. Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials. PII: S0142 1123(97)00081
- [5] Jean-Louis Robert. Contribution à l'étude de la fatigue multiaxiale sous sollicitations périodiques ou aléatoires. Thèse de l'INSA de Lyon :N°d'ordre : 92 ISAL 004. 1992, 228 p.
- [6] Gough H.J., Pollard H,V, Clenshow W.J., Some experiments on the resistance of metals to fatigue under combined stresses, Aeronautical Research Concl. Report and Memoranda N° 2522. London : HMSO. 1951. 141p.
- [7] Kenneugne B., Contribution à la modélisation du comportement en fatigue sous sollicitation multiaxiale d'amplitude variable, Thèse de l'INSA de Lyon, N° d'ordre 96 ISAL 0064 (1996).
- [8] Jan Papuga, Milan Ruzicka, Two new multiaxial criteria for high cycle fatigue computation ; International Journal of Fatigue 30 (2008) 56-66
- [9] Quoc Huy VU. Fatigue polycyclique multiaxiale de l'acier C35 : caractérisation et modélisation des mécanismes d'endommagement. Thèse de l'ENSMA, Poitiers 2009, 237p.
- [10] Cécile KREBS. Endurance limitée sous chargement multiaxial à amplitude variable. Thèse de doctorat de l'Ecole Polytechnique Paris Tech, 2010, 156 p.
- [11] Ngarmaïm NADJITONON. Contribution à la Modélisation des lois endommagement par fatigue. Thèse N° 99ISAL0056. 2010 - Université Blaise Pascal – Clermont II.
- [12] Bienzeube TIKRI. Etude de l'influence de surcharges sur le comportement en fatigue d'aciers soudés par points. Thèse N° EDSPIC : 553 (2011).
- [13] B.SohFotsing, E. Maronne, N. Nadjitonon, J.-L. Robert. Intégration d'une démarche fiabiliste dans l'exploitation des critères de fatigue multiaxiaux. 23ième Journées de Printemps de la Commission de Fatigue. (25 au 26 mai 2004)