

A Decision Model for Solving Pick-up and Delivery Heterogeneous Vehicle Routing Problem

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ABSTRACT: The Pickup and Delivery (PD) problem of Vehicle Routing is a further generalization of the Vehicle Routing Problem, in which each customer demand is associated with two locations: an origin location where a certain demand must be picked up and a destination where this demand must be delivered. This paper considers a logistic distribution problem in which that there are several type of vehicles with different capacity operated to meet the PD conditions for customers. We use mixed integer nonlinear program to get the optimal decision of the problem. A neighborhood search approach is developed to solve the model.

KEYWORDS: Vehicle Routing Problem, Logistic, Neighborhood search, Modelling.

I. INTRODUCTION

In the Vehicle Routing Problem (VRP), as a crucial element in logistic-distribution system, the set of customer along with a set of vehicles are given. The aims are to determine the optimal set of routes to be served by these vehicles, so that the total cost of used vehicles is minimized [1]. In other words, the problem is how to design the least-cost vehicle routes which starts and terminates at the service center (depot) in order to fulfill customers demand, subject to vehicle capacity constraints and other technical constraints. [2] describes an interesting compact review on VRP literature.

In the Vehicle Routing Problem with Time Windows (VRPTW), a time window is associated with each customer and the vehicle visiting a given customer cannot arrive after the end of the time window. The Pickup and Delivery Problem with Time Windows (PDVRPTW) is a further generalization of the VRPTW in which each customer request is associated with two locations: an origin location where a certain demand must be picked up and a destination where this demand must be delivered. Each route must also satisfy pairing and precedence constraints: for each request, the origin must precede the destination, and both locations must be visited by the same vehicle. The VRPTW can be seen as a special case of the PDVRPTW in which all requests have a common origin which corresponds to the depot [3].

The PDVRPTW has applications in various contexts such as urban courier services, less than truckload transportation, and door-to-door transportation services for the elderly, the disabled and other special conditions. In the latter case, narrow time windows are often considered and ride time constraints are imposed to control the time spent by a passenger in the vehicle. The resulting problem is called the Dial-a-Ride Problem (DARP).

The service company focused in this paper has various type of vehicle in its operation to deliver goods for customers. Each type of vehicle has different capacities. The variant of VRP which considers mixed fleet of vehicles is called Heterogeneous VRP (HVRP), introduced firstly by [19]. This generalization is important in practical terms, for most of customers demand are served by several type of vehicles [20], [21]. The objective of the HVRP is to find fleet composition and a corresponding routing plan that minimizes the total cost.

As this is an NP-hard problem it is reasonable that the most approach addressed for solving the HVRP is heuristics. [22] were the first to tackle the HVRPTW and developed a number of parallel insertions heuristics and embedding in the

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calculations of the relevant criteria the acquisition costs of [19]. [23] presented a deterministic simulated annealing meta heuristic for the HVRPTW, outperforming the results of [22], and then [24] developed a linearly scalable hybrid threshold-accepting and guided local search meta heuristic for solving large scale HVRPTW instances. [25] designed a trajectory search heuristic to solve a large-scale HVRPTW in which the vehicles are allowed to have multi-trips. [26] presented an Adaptive Memory Programming solution approach for the HVRPTW that provides a promising very good results in the majority of the benchmark instances examined. [27] developed a tabu search for solving HVRPTW, and they conducted the approach on a large set of test cases. [28] proposed a hybrid algorithm for the problem. Their algorithm is composed by an Iterated Local Search based heuristic and a Set Partitioning formulation. [29] described a modified column generation to solve HVRPTW.

This paper deals with PDHVRP with time windows. The basic framework of the vehicle routing part can be viewed as a Vehicle Routing Problem with Time Windows (VRPTW) in which there are several number of vehicles, characterized by different capacities are available for services. Another conditions considered are the routes of vehicle have a node for pickup demands and a node for delivering demands We address a large-scale mixed integer nonlinear programming formulation to model the problem. A feasible neighbourhood heuristic search is proposed to get the sub-optimal integer feasible solution based on the notion of superbasic variables.

II. RELATED WORK

Since it generalizes the VRPTW, the PDVRPTW is clearly NP-hard. Two main approaches have been used to solve the PDVRPTW exactly: branch-and-price and branch-and-cut. Branch-and-price methods (see, e.g., [4]). [5] use a branch-and-bound scheme in which lower bounds are computed by column generation. The first branch-and-price algorithm for the PDVRPTW was proposed by [6] who considered a set partitioning formulation of the problem in which each column corresponds to a feasible vehicle route and each constraint is associated to a request that must be satisfied exactly once. The resulting pricing subproblem is a shortest path problem with time window, capacity, pairing and precedence constraints. This problem is solvable by dynamic programming and the authors used an algorithm similar to the one developed by [7] for the single-vehicle pickup and delivery problem with time windows. Several label elimination methods were proposed to accelerate the dynamic programming algorithm, and arc elimination rules were used to reduce the size of the problem. The authors pointed out that their approach works well when the demand of each customer is large with respect to vehicle capacity.

Another branch-and-price approach for the PDVRPTW was later described by [8]. Their approach differs from that of [7]. in several respects: construction and improvement heuristics are used to solve the pricing subproblem; a column management mechanism is used to reduce the size of the column generation master problem; columns are selected with a bias toward increasing the likelihood of identifying feasible integer solutions; branching is performed on additional variables representing the fraction of a request that is served by a given vehicle; and a primal heuristic is used at each node of the search tree to compute upper bounds. Column generation was also used recently by [9] and [10] to address variants of the PDVRPTW arising in long-haul transportation planning and in the transportation of live animals, respectively.

The second family of exact approaches for the PDVRPTW is branch-and-cut. In branch and-cut, valid inequalities (i.e., cuts) are added to the formulation at each node of the branch-and-bound tree to strengthen the relaxations which are usually solved by the simplex algorithm. Relying on the previous work of [11] and [12] on the Precedence-Constrained Traveling Salesman Problem (PCTSP) and the TSP with Pickup and Delivery (TSPPD), [13] developed a branch-and-cut algorithm for the DARP based on a three-index formulation of the problem. This algorithm was able to solve instances with four vehicles and 32 requests. [14] introduce a new branch-and-cut-and-price algorithm for solving PDVRPTW. They generate lower bound needed in the algorithm through column generation the linear programming relaxation. Another branch-and-cut approach, based on a two-index formulation, was proposed by [15]. This formulation contains a polynomial number of constraints, but relies on extra variables to impose pairing and precedence constraints. Instances with up to five vehicles and 25 requests were solved optimally with this approach. For reviews on pickup and delivery problems, the reader is referred to the works of [16], [17], and [18].

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III. PROBLEM FORMULATION

The logistic problem considered in this paper was faced by a service company located in Medan city, North Sumatra Province, Indonesia. In terms graph theory, the basic frame work of PDHVRP for servicing customers can be defined as follows. Let $G = (V, A)$ be a directed graph, where $V = \{0, 1, \dots, n\}$ is the vertex set and $A = \{(i, j) : i, j \in V, i \neq j\}$ is the set of route. For each route $(i, j) \in A$ a distance (or travel) cost c_{ij} is defined. Vertex 0 ($i = 0$) and vertex n ($i = n$) are the depot vertex, center of service, where the vehicle fleet is located. Define V_c is the set of customers' vertex. Each vertex $i \in V_c$ has a known fixed daily nonnegative demand q_i , a nonnegative service time s_i , and a service time windows $[a_i, b_i]$. The subsets $P = \{1, \dots, p\}$ and $D = \{p+1, \dots, n\}$, respectively, denote the pick-up node and delivery node, Note that $P \subset V$, and $D \subset V$.

In particular, at depot the demand $q_0 = q_n = 0$, $q_i = -q_{p+1}$, ($i = 1, \dots, p$) and service time $s_0 = s_n = 0$. As this is a heterogeneous problem, we define a fleet of K vehicles composed by m different type of vehicles, each with capacity Q_m . The number of vehicles available for vehicle type m is n_m . Define K_m as the set of vehicle type m . Each customer is served by exactly one vehicle with the associated type. At the depot ($i = 0$), a time window for vehicles to leave and to return to depot is given by $[a_0, b_0]$. The arrival time of a vehicle at customer i is denoted by a_i and its departure time is b_i . Vehicle routes are restricted to a maximum duration of H_k , $k = 1, \dots, K$. Each type of vehicle is associated with a fixed cost, f_m . Another cost occurs for travelling through route $(i, j) \in A$, defined as $\alpha_{ij}^m = d_{ij}g_{ij}$, where d_{ij} is the distance travelled between route i to route j and g_{ij} a factor cost for travelling, for m type of vehicle. Further more, a fixed acquisition cost f_k is incurred for each of vehicle k in the routes. Each route originates and terminates at the central depot and must satisfy the time window constraints, i.e., a vehicle cannot start servicing customer i before a_i and after b_i however, the vehicle can arrive before a_i and wait for service. We impose travel time as a function of departure time b_i ($t_{ij} \geq 0$)

IV. MATHEMATICAL MODELING

Firstly we define the decision variables as follows.

- $x_{0j}^k = \begin{cases} 1 & \text{if vehicle type } k \in K \text{ to deliver from depot to customer } j \in V_c; \\ 0 & \text{otherwise;} \end{cases}$
- $x_{ij}^m = \begin{cases} 1 & \text{if vehicle type } m \in K_m \text{ to deliver for } (i, j) \in V_c, i \neq j; \\ 0 & \text{otherwise;} \end{cases}$
- $z_0^m = \begin{cases} 1 & \text{if vehicle type } m \in K \text{ is available and active at depot;} \\ 0 & \text{otherwise;} \end{cases}$
- l_i^m Arrival time for vehicle type $m \in K_m$ at customer $i \in V_c$ (non-negative continuous variable)
- u_i^m Duration of service of vehicle type $m \in K_m$ at customer $i \in V_c$ (non-negative continuous variable)

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- W_j^k Load of vehicle k upon leaving customer node j (non-negative continuous variable).

In this basic framework the manager of the service company wants to use the available vehicle for each type efficiently, such that the total cost is minimized, and the pick-up and delivery conditions are met. The total cost consists of traveling cost of all vehicle used and the cost for the availability of vehicle in the planning horizon time of a day.

$$\text{Minimize } \sum_{j \in V_c} c_{oj} \sum_{k \in K} x_{0j}^k + \sum_{(i,j) \in V_c} \sum_{m \in K_m} \alpha_{ij}^m x_{ij}^m + \sum_{m \in K_m} f_m z_o^m \quad (1)$$

Subject to

$$\sum_{k \in K} x_{0j}^k = 1, \quad \forall j \in V_c \quad (2)$$

$$\sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1, \quad \forall i \in V_c \quad (3)$$

Constraints (2) and (3)) are to ensure that exactly one vehicle regardless their type enters and departs from every customer and from the central depot. and comes back to the depot.

$$\sum_{i \in V} x_{ij}^k - \sum_{i \in V} x_{ji}^k = 1; \quad \forall j \in V_c, \forall k \in K \quad (4)$$

A flow conservation equation is necessarily needed to maintain the continuity of each vehicle route on period of time. This equation is presented in Constraints (4).

$$x_{ij}^m \leq z_o^m, \quad (i, j) \in V_c, \forall m \in K_m \quad (5)$$

Constraint (5) represents that each customer is served only by the available and active vehicle of type m .

$$\sum_{j \in V_c} x_{1j}^k \leq 1; \quad \forall k \in K \quad (6)$$

$$\sum_{i \in V_c, i > 1} x_{i1}^k \leq 1; \quad \forall k \in K \quad (7)$$

Constraints (6) and (7) state the availability of vehicles by bounding the number of route, related to vehicle k for each type, directly leaving from and returning to the central depot, not more than one, respectively.

$$\sum_{k \in K} \sum_{j \in V_c} x_{ij}^k = 1; \quad \forall i \in P \quad (8)$$

$$\sum_{j \in V_c} x_{ij}^k - \sum_{j \in V_c} x_{p+1,j}^k = 0; \quad \forall i \in P, k \in K \quad (9)$$

Constraints (8) and (9) are to make sure that each request from customers are served exactly once and that the pick-up and delivery nodes are visited by the same vehicle.

$$\sum_{i \in V_c} d_i \sum_{j \in V_c} x_{ij}^m \leq Q_m; \quad \forall m \in K_m \quad (10)$$

Constraints (10) ensure that each delivery does not exceed the capacity of each type of vehicle.

$$x_{ij}^m (l_i^m + u_i^m + s_i + t_{ij} - l_j^m) = 0; \quad \forall m \in K_m, (i, j) \in A \quad (11)$$

Constraints (11) establishes the equilibrium among the arrival time, duration of service, service time and travel time at customers in the routes assigned.

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$$l_i^m \leq a_i \sum_{j \in V_c} x_{ij}^m; \quad \forall m \in K_m, i \in V_c \quad (12)$$

$$a_i \sum_{j \in V_c} x_{ij}^m \leq l_i^m + u_i^m \leq b_i \sum_{j \in V_c} x_{ij}^m; \quad \forall m \in K_m, i \in V_c \quad (13)$$

Constraints (12) and (13) present time window as the committed scheduled time-dependent for each customer.

$$\sum_{j \in V_c} x_{oj}^m \leq n_m; \quad \forall m \in K_m \quad (14)$$

Constraint (12) guarantees that the number availability of active vehicle does not exceed the number of vehicle available at the central depot of catering company.

$$W_j^k \geq (W_i^k + q_j)x_{ij}^k; \quad \forall i \in V, \forall j \in V_c, \forall k \in K \quad (15)$$

Constraint (15) represents the consistency of load variable.

It can be observed that the model formulated in expressions (1) – (15) is in the form of large scale Mixed Integer Nonlinear Programming (MINLP).

V. THE ALGORITHM

Stage 0.

Solve the relaxation of model (1) to (15), as a large scale nonlinear programming.

If all variables in the continuous optimal solution is integer valued, the optimal integer solution to the problem (1) to (15) is found. STOP.

Otherwise go to Stage 1.

Stage 1.

Step 1. Get row i^* the smallest integer infeasibility, such that $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i^*}^T \alpha_j$

With j corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\alpha_{ij}} \right| \right\}$$

Calculate the maximum movement of nonbasic j at lower bound and upper bound.

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased from LB or decreased from UB. If none go to next i^* .

Step 4.

Solve $B\alpha_{j^*} = \alpha_{j^*}$ for α_{j^*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \{\emptyset\}$ go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Pass 1 : adjust integer infeasible superbasis by fractional steps to reach complete integer feasibility.

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Pass2 : adjust integer feasible superbasics. The objective of this phase is to conduct a highly localized neighbourhood search to verify local optimality.

VI. CONCLUSION

This paper was intended to present a model and solution for one of the crucial problem in the scope of Logistic and Distribution Systems. The aim of this paper was to develop a model of heterogeneous vehicle routing problem (HVRP) with time windows in which there are nodes for pickup and delivery demand of customers.. This problem has imposed a benefit that could be gained as incentive for offering serviced to customers. The proposed model was a large scale mixed integer nonlinear program The algorithm for solving the model employs nearest neighbor heuristic algorithm using the notion of superbasic variables.

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