On k-Heronian Mean Labeling

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ABSTRACT: In this paper, we introduce k-Heronian mean labeling and investigate k-Heronian mean labeling of Path, \( P_n \circ mK_1; 1 \leq m \leq 4 \) and Ladder.

KEYWORDS: k-Heronian mean labeling, k-Heronian mean graph, Path, \( P_n \circ mK_1; 1 \leq m \leq 4 \), Ladder.

I. INTRODUCTION

In this paper, all graphs are finite, undirected and simple graphs. Terms not defined here are used in the sense of Harary [2]. Graph labeling is first introduced by Rosa in 1967. The concept of mean labeling was introduced and studied by S. Somasundaram and R.Ponraj [5]. The concept of Heronian mean labeling was introduced and studied by S. S. Sandhaya and others [3] [4]. Here we investigate k – Heronian mean labeling of Path, \( P_n \circ nK_1; 1 \leq n \leq 4 \), and Ladder. For brevity, we use k-HML for k-Heronian mean labeling and k is any positive integer.

II. MAIN RESULTS

Definition 2.1:
A graph \( G=(V,E) \) with \( p \) vertices and \( q \) edges is said to be a \textbf{k-Heronian Mean graph} if it is possible to label the vertices \( \forall v \in V \) with distinct labels \( f(v) \) from \( k, k+1, k+2, \ldots, k+q \) in such a way that when each edge \( e=uv \) is labeled with, \( f^*(e) = \frac{f(u) + f(v) + \sqrt{f(u)f(v)}}{3} \) or \( \frac{f(u) + f(v) + \sqrt{f(u)f(v)}}{3} \), then the resulting edge labels are distinct. In this case \( f \) is called a \textbf{k-Heronian Mean labeling} of \( G \).

Theorem 2.2:
Any path \( P_n \) is a k-Heronian mean graph, for all \( n \geq 2 \).

Proof:
Let \( V(P_n) = \{u_i; 1 \leq i \leq n\} \) and \( E(P_n) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \) be the vertices and edges of \( P_n \) respectively.
Define \( f: V(P_n) \to \{k, k+1, k+2, \ldots, k+n-1\} \) by \( f(u_i) = k + i - 1; 1 \leq i \leq n \).
Now the induced edge labels are \( f^*(e_i) = k + i - 1; 1 \leq i \leq n-1 \).
Here \( p = n \) and \( q = n-1 \).
Clearly, \( f \) is k-Heronian mean labeling of \( P_n \).
Hence \( P_n \) is a k-Heronian mean graph, for all \( n \geq 2 \).
Example 2.3:
50-Heronian mean labeling of $P_5$ is given in the figure 2.1:

![50-HML of $P_5$]

Theorem 2.3:
Any comb, $P_n \odot K_1$ is a k-Heronian mean graph, for all $n \geq 2$.

Proof:
Let $V(P_n \odot K_1) = \{u_i, v_i; 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup \{e'_i = (u_i, v_i); 1 \leq i \leq n\}$ be the vertices and edges of $P_n \odot K_1$ respectively.

Define $f: V(P_n \odot K_1) \rightarrow \{k, k + 1, k + 2, ..., k + 2n - 1\}$ by

- $f(u_i) = k + 2i - 2; 1 \leq i \leq n,$
- $f(v_i) = k + 2i - 1; 1 \leq i \leq n.$

Now the induced edge labels are
- $f^*(e_i) = k + 2i - 2; 1 \leq i \leq n,$
- $f^*(e'_i) = k + 2i - 1; 1 \leq i \leq n - 1.$

Here $p = 2n$ and $q = 2n - 1$.
Clearly, $f$ is k-Heronian mean labeling of $P_n \odot K_1$.

Hence $P_n \odot K_1$ is a k-Heronian mean graph, for all $n \geq 2$.

Example 2.4:
75-Heronian mean labeling of $P_5 \odot K_1$ is given in the figure 2.2:

![75-HML of $P_5 \odot K_1$]

Theorem 2.5
The graph $P_n \odot 2K_1$ is a k-Heronian mean graph, for all $n \geq 2$.

Proof:
Let $V(P_n \odot 2K_1) = \{u_i, v_i, w_i; 1 \leq i \leq n\}$ and $E(P_n \odot 2K_1) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup \{e'_i = (u_i, v_i); 1 \leq i \leq n\} \cup \{e''_i = (u_i, w_i); 1 \leq i \leq n\}$ be the vertices and edges of $P_n \odot 2K_1$ respectively.

Define $f: V(P_n \odot 2K_1) \rightarrow \{k, k + 1, k + 2, ..., k + 3n - 1\}$ by

- $f(u_i) = k + 3i - 2; 1 \leq i \leq n,$
- $f(v_i) = k + 3i - 3; 1 \leq i \leq n,$
- $f(w_i) = k + 3i - 1; 1 \leq i \leq n.$

Now the induced edge labels are
- $f^*(e_i) = k + 3i - 2; 1 \leq i \leq n - 1,$
- $f^*(e'_i) = k + 3i - 3; 1 \leq i \leq n,$
- $f^*(e''_i) = k + 3i - 1; 1 \leq i \leq n.$
\[ f^*(e_i') = k + 3i - 2; \quad 1 \leq i \leq n. \]

Here \( p = 3n \) and \( q = 3n-1 \).

Clearly, \( f \) is \( k \)-Heronian mean labeling of \( P_n \odot 2K_1 \).

Hence \( P_n \odot 2K_1 \) is a \( k \)-Heronian mean graph, for all \( n \geq 2 \).

**Example 2.6:**

225-Heronian mean labeling of \( P_3 \odot 2K_1 \) is given in the figure 2.3:

![Figure 2.3: 225-HML of \( P_3 \odot 2K_1 \)](image)

**Theorem 2.7:**

The graph \( P_n \odot 3K_1 \) is a \( k \)-Heronian mean graph, for all \( n \geq 2 \).

**Proof:**

Let \( V(P_n \odot 3K_1) = \{u_i, v_i, w_i, x_i; 1 \leq i \leq n\} \) and \( E(P_n \odot 3K_1) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup \{e_i' = (u_i, v_i); 1 \leq i \leq n\} \cup \{e_i'' = (u_i, w_i); 1 \leq i \leq n\} \cup \{e_i''' = (u_i, x_i); 1 \leq i \leq n\} \) be the vertices and edges of \( P_n \odot 3K_1 \) respectively.

Define \( f: V(P_n \odot 3K_1) \rightarrow \{k, k + 1, k + 2, \ldots, k + 4n - 1\} \) by

\[
\begin{align*}
  f(u_i) &= k + 4i - 3; \quad 1 \leq i \leq n, \\
  f(v_i) &= k + 4i - 4; \quad 1 \leq i \leq n, \\
  f(w_i) &= k + 4i - 3; \quad 1 \leq i \leq n, \\
  f(x_i) &= k + 4i - 1; \quad 1 \leq i \leq n.
\end{align*}
\]

Now the induced edge labels are

\[
\begin{align*}
  f^*(e_i) &= k + 4i - 1; \quad 1 \leq i \leq n - 1, \\
  f^*(e_i') &= k + 4i - 4; \quad 1 \leq i \leq n, \\
  f^*(e_i'') &= k + 4i - 3; \quad 1 \leq i \leq n, \\
  f^*(e_i''') &= k + 4i - 2; \quad 1 \leq i \leq n.
\end{align*}
\]

Here \( p = 4n \) and \( q = 4n-1 \).

Clearly, \( f \) is \( k \)-Heronian mean labeling of \( P_n \odot 3K_1 \).

Hence \( P_n \odot 3K_1 \) is a \( k \)-Heronian mean graph, for all \( n \geq 2 \).

**Example 2.8:**

50-Heronian mean labeling of \( P_3 \odot 3K_1 \) is given in the figure 2.4:

![Figure 2.4: 50-HML of \( P_3 \odot 3K_1 \)](image)
Theorem 2.9: The graph $P_n \odot 4K_1$ is a k-Heranian mean graph, for all $n \geq 2$.

**Proof:** Let $V(P_n \odot 4K_1) = \{u_i, v_i, w_i; 1 \leq i \leq n\}$ and $E(P_n \odot 4K_1) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup \{e'_i = (u_i, v_i); 1 \leq i \leq n\} \cup \{e''_i = (v_i, u_i); 1 \leq i \leq n\} \cup \{e''''_i = (u_i, y_i); 1 \leq i \leq n\}$ be the vertices and edges of $P_n \odot 4K_1$, respectively.

Define $f: V(P_n \odot 4K_1) \rightarrow \{k, k+1, k+2, ..., k+5n-1\}$ by

- $f(u_i) = k + 5i - 4; \ 1 \leq i \leq n$, 
- $f(v_i) = k + 5i - 5; \ 1 \leq i \leq n$, 
- $f(w_i) = k + 5i - 3; \ 1 \leq i \leq n$, 
- $f(x_i) = k + 5i - 2; \ 1 \leq i \leq n$, 
- $f(y_i) = k + 5i - 1; \ 1 \leq i \leq n$.

Now the induced edge labels are

- $f^*(e_i) = k + 5i - 1; \ 1 \leq i \leq n-1$, 
- $f^*(e'_i) = k + 5i - 5; \ 1 \leq i \leq n$, 
- $f^*(e''_i) = k + 5i - 4; \ 1 \leq i \leq n$, 
- $f^*(e'''_i) = k + 5i - 3; \ 1 \leq i \leq n$, 
- $f^*(e''''_i) = k + 5i - 2; \ 1 \leq i \leq n$.

Here $p = 5n$ and $q = 5n-1$.

Clearly, $f$ is k-Heranian mean labeling of $P_n \odot 4K_1$.

Hence $P_n \odot 4K_1$ is a k-Heranian mean graph, for all $n \geq 2$.

**Example 2.10:**

50-Heranian mean labeling of $P_3 \odot 4K_1$ is given in the figure 2.5:

**Theorem 2.5**

Any Ladder, $L_n$, is a k-Heranian mean graph, for all $n \geq 2$.
Proof:
Let \( V(L_n) = \{u_i,v_i; 1 \leq i \leq n\} \) and \( E(L_n) = \{e_i = (u_i,v_i); 1 \leq i \leq n\} \cup \{e_i' = (u_i,u_{i+1}); 1 \leq i \leq n-1\} \cup \{e_i'' = (v_i,v_{i+1}); 1 \leq i \leq n-1\} \) be the vertices and edges of \( L_n \), respectively.
Define \( f: V(L_n) \rightarrow \{k,k+1,k+2,\ldots,k+3n-2\} \) by
\[
f(u_i) = k + 3i - 3; \quad 1 \leq i \leq n,  \]
\[
f(v_i) = k + 3i - 2; \quad 1 \leq i \leq n.  \]
Now the induced edge labels are
\[
f^*(e_i) = k + 3i - 3; \quad 1 \leq i \leq n,  \]
\[
f^*(e_i') = k + 3i - 2; \quad 1 \leq i \leq n - 1,  \]
\[
f^*(e_i'') = k + 3i - 1; \quad 1 \leq i \leq n - 1.  \]
Here \( p = 2n \) and \( q = 3n-2 \).
Clearly, \( f \) is \( k \)-Heronian mean labeling of \( L_n \).
Hence \( L_n \) is a \( k \)-Heronian mean graph, for all \( n \geq 2 \).

Example 2.12:
200-Heronian mean labeling of \( L_5 \) is given in the figure 2.6:

III. CONCLUSION

Graph labeling has its own applications in communication networks and astronomy, so enormous types of labeling have grown. Towards this \( k \)-Heronian mean labeling is also a kind of labeling which an extension of Heronian mean labeling. In this paper we discussed \( k \)-Heronian mean labeling of the graphs, Path, \( P_n \bigcirc m \mathcal{K}_1; 1 \leq m \leq 4 \) and Ladder.

REFERENCES