

Properties of g^*sp -Closed Sets in Topological Spaces

Govindappa Navalagi¹, Sujata Mookanagoudar²

Professor, Department of Mathematics, KIT Tiptur, Karnataka, India

Asst. Professor, Department of Mathematics, G .F. G. College, Haliyal, Karnataka, India

ABSTRACT: The purpose of this paper is to define and study a new class of sets, called the class of g^* -semipreclosed (in brief, g^*sp -closed) sets, which lies between the class of semipreclosed sets and the class of generalized semipreclosed sets. Also, the basic properties of these g^*sp -closed sets, g^*sp -open functions, g^*sp -closed functions and allied g^*sp -Hausdorff spaces are studied in this paper.

KEYWORDS: preopen sets, semipreclosed sets, gsp -closed sets and gp -closed sets g^*p -closed sets.

I. INTRODUCTION

Levine [10] generalized the closed set to generalized closed set (g -closed set) in topology for the first time. Since then it is noticed that some of the weaker forms of closed sets have been generalized. In 1982 and 1986, respectively, A.S.Mashhour et al [11] and D.Andrijevic [1] have defined and studied the concepts of preopen sets and semipreopen sets in topology. In 1995, 1998 and 2002, respectively, Dontchev [7], Noiri et al [14] and M.K.R.S. Veera Kumar [16], have defined and studied the concepts of gsp -closed sets, gp -closed sets and g^*p -closed sets in topological spaces. In the literature, by M.K.R.S. Veera Kumar, see [15 & 16] has defined and studied the concepts of g^* -closed sets and g^*p -closed sets in topological spaces. Latter, many authors have been studied many subsets, separation axioms, continuous functions and irresolute functions via g^* -closed sets and g^*p -closed sets, respectively. In this paper, we define and study a new class of sets, called the class of g^* -semipreclosed (in brief, g^*sp -closed) sets, which lies between the class of semipreclosed sets and the class of generalized semipreclosed sets. Also, the basic properties of these g^*sp -closed sets, g^*sp -open functions, g^*sp -closed functions and allied g^*sp -Hausdorff spaces are studied in this paper. In a successive paper, as related works, we defined and studied the concepts of g^*sp -continuous functions, g^*sp -irresolute functions and various allied g^*sp -continuous functions.

II. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of space X .

We denote the closure of A and the interior of A by $Cl(A)$ and $Int(A)$ respectively.

The following definitions and results are useful in the sequel :

Definition 2.1 : A subset A of a space X is said to be:

- (i) preopen [11] if $A \subset Int Cl(A)$.
- (ii) semiopen [9] if $A \subset Cl Int(A)$.
- (iii) semipreopen [1] if $A \subset Cl Int Cl(A)$.

The complement of a preopen (resp. semiopen, semipreopen) set of a space X is called preclosed [8] (resp. semiclosed [3], semipreclosed [1]).

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 7, Issue 8, August 2018

Definition 2.2 [1]: The union of all semipreopen sets contained in A is called the semipreinterior of A and is denoted by $spInt(A)$.

$pInt(A)$ [12] and $sInt(A)$ [6] can be similarly defined.

Definition 2.3[1]: The intersection of all semipreclosed sets containing A is called the semipreclosure of A and is denoted by $spCl(A)$.

$pCl(A)$ [8] and $sCl(A)$ [5] can be similarly defined.

Definition 2.4 : A subset A of a space X is called :

- (i) generalized closed set (in brief, g -closed) set [10] if $Cl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (ii) generalized semiclosed (in brief, gs -closed) set [2] if $sCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (iii) generalized semipreclosed (in brief, gsp -closed) set [7] if $spCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (iv) generalized preclosed (in brief, gp -closed) set [14] if $pCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (v) g^* -closed set [15] if $Cl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .
- (vi) g^* -preclosed (in brief, g^*p -closed) set [16] if $pCl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .

Definition 2.5 [4]: A space X is said to be g -Hausdroff if whenever x and y are distinct points of X there exist disjoint g -open sets U and V such that $x \in U$ and $y \in V$.

III. PROPERTIES OF G^*SP -CLOSED SETS

We, define the following.

Definition 3.1: A subset A of a space X is said to be g^* -semipreclosed (in brief, g^*sp -closed) set if $spCl(A) \subset U$ whenever $A \subset U$ and U is g -open in X . The complement of a g^*sp -closed set is called g^*sp -open set.

Lemma 3.2 : Let X be a space. Then,

- (i) Every closed set is g^*sp -closed set.
- (ii) Every g -closed set is g^*sp -closed set.
- (iii) Every preclosed set is g^*sp -closed set.
- (iv) Every semipreclosed set is g^*sp -closed set.
- (v) Every g^* -closed set is g^*sp -closed set.
- (vi) Every gs -closed set is g^*sp -closed set.
- (vii) Every g^*sp -closed set is gsp -closed set.
- (viii) Every g^*p -closed set is g^*sp -closed set.
- (ix) Every gp -closed set is g^*sp -closed set.

Lemma 3.3 : Let A be a g^*sp -closed set of space X . Then,

- (i) $spCl(A) - A$ does not contain any nonempty g -closed set.
- (ii) If $A \subset B \subset spCl(A)$, then B is also a g^*sp -closed set of X .

Proof. (i) Let F be a g -closed set contained in $spCl(A) - A$, then $spCl(A) \subset X - F$ since $X - F$ is g -open with $A \subset X - F$ and A is g^*sp -closed set. Then, $F \subset (X - spCl(A)) \cap (spCl(A) - A) \subset (X - spCl(A)) \cap spCl(A) = \emptyset$. Thus, $F = \emptyset$.

(ii) Let G be a g -open set of X such that $B \subset G$. Then, $A \subset G$. Since $A \subset G$ and A is g^*sp -closed set, then $spCl(A) \subset G$. Then, $pCl(B) \subset pCl(spCl(A)) = pCl(A)$ since $B \subset spCl(A)$. Thus, $pCl(B) \subset pCl(A) \subset G$. Hence, B is also a g^*sp -closed set of X .

Next, we introduce the following.

Definition 3.4 : A space X is said to be a :

- (i) T_c^* -space if every g^*sp -closed set in it is closed.
- (ii) T_{pc}^* -space if every g^*sp -closed set in it is preclosed.
- (iii) T_{sp}^* -space if every g^*sp -closed set in it is semipreclosed.

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 7, Issue 8, August 2018

- (iv) T_{gsp}^* -space if every gsp -closed set in it is g^*sp -closed.
- (v) $**T_{gsp}^*$ -space if every g^*sp -closed set in it is g^*p -closed.
- (vi) T_{gp}^{**} -space if every g^*sp -closed set in it is gp -closed .

We, state the following.

Lemma 3.5: A subset A of space X is called g^*sp -open set if $F \subset spInt(A)$ whenever $F \subset A$ and F is g -closed set in X

We , define the following

Definition 3.6 : The union of all g^*sp -open sets which contained in A is called the g^*sp -interior of A and is denoted by $g^*spInt(A)$.

Definition 3.7: The intersection of all g^*sp -closed sets containing set A is called the g^*sp -closure of A and is denoted $g^*spCl(A)$.

Lemma 3.8 : Let $x \in X$, then $x \in g^*spCl(A)$ if and only if $V \cap A \neq \emptyset$ for every g^*sp -open set V containing x .

Remark 3.9 : The g^*sp -closure of a set A is not always g^*sp -closed set.

We , state the following.

Lemma 3.10: Let A and B be subsets of X . Then

- (i) $g^*spCl(\emptyset) = \emptyset$ and $g^*spCl(X) = X$
- (ii) If $A \subset B$, $g^*spCl(A) \subset g^*spCl(B)$.
- (iii) $A \subset g^*spCl(A)$

Lemma 3.11: Let A and B be subsets of X . Then

- (i) $g^*spCl(A) = g^*spCl(g^*spCl(A))$
- (ii) $g^*spCl(A) \cup g^*spCl(B) \subset g^*spCl(A \cup B)$
- (iii) $g^*spCl(A \cap B) \subset g^*spCl(A) \cap g^*spCl(B)$

Theorem 3.12: If $spInt(A) \subset B \subset A$ and A is g^*sp -open, then B is g^*sp -open.

Proof: Let A be g^*sp -open and $spInt(A) \subset B \subset A$. Then $X - A \subset X - B \subset X - spInt(A)$ that implies $X - A \subset X - B \subset spCl(X - A)$. Thus , $X - B$ is g^*sp -closed. This proves that B is g^*sp -open

We, define the following.

Definition 3.13: A space X is said to be g^*sp -Hausdroff if whenever x and y are distinct points of X , there exist disjoint g^*sp -open sets U and V such that $x \in U$ and $y \in V$.

Definition 3.14: A space X is said to be g^*p -Hausdroff if whenever x and y are distinct points of X there exist disjoint g^*p -open sets U and V such that $x \in U$ and $y \in V$.

We, recall the following .

Definition 3.15[13]: A function $f: X \rightarrow Y$ is called g^*sp -continuous if $f^{-1}(V)$ is g^*sp -closed in X for every closed subset V of Y .

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 7, Issue 8, August 2018

Definition 3.16[13]: A function $f: X \rightarrow Y$ is called g^*sp -irresolute if $f^{-1}(V)$ is g^*sp -closed in X for every g^*sp -closed subset V of Y .

Definition 3.17[13]: A function $f: X \rightarrow Y$ is called strongly g^*sp -continuous if the inverse image of each g^*sp -open set of Y is open in X .

Definition 3.18[13]: A function $f: X \rightarrow Y$ is called (g, g^*sp) -continuous if the inverse image of each g -open set of Y is g^*sp -open in X .

Definition 3.19[13]: A function $f: X \rightarrow Y$ is called (g^*p, g^*sp) -continuous if the inverse image of each g^*p -open set of Y is g^*sp -open in X .

Next, we prove the following.

Theorem 3.20: Let X be a space and Y be Hausdorff. If $f: X \rightarrow Y$ be g^*sp -continuous injective, then X is g^*sp -Hausdorff.

Proof: Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is Hausdorff, there exist two disjoint open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is g^*sp -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint g^*sp -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is g^*sp -Hausdorff space.

Theorem 3.21: Let X be a space and Y be g^*sp -Hausdorff. If $f: X \rightarrow Y$ be g^*sp -irresolute injective, then X is g^*sp -Hausdorff.

Proof: Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is g^*sp -Hausdorff, there exist two disjoint g^*sp -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is g^*sp -irresolute and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint g^*sp -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is g^*sp -Hausdorff space.

Theorem 3.22: Let X be a space and Y be g -Hausdorff. If $f: X \rightarrow Y$ be (g, g^*sp) -continuous injective, then X is g^*sp -Hausdorff.

Proof: Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is g -Hausdorff, there exist two disjoint g -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is (g, g^*sp) -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint g^*sp -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is g^*sp -Hausdorff space.

Theorem 3.23: Let X be a space and Y be g^*p -Hausdorff. If $f: X \rightarrow Y$ be (g^*p, g^*sp) -continuous injective, then X is g^*sp -Hausdorff.

Proof: Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is g^*p -Hausdorff, there exist two disjoint g^*p -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is (g^*p, g^*sp) -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint g^*sp -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is g^*sp -Hausdorff space.

Theorem 3.24: Let X be a space and Y be g^*sp -Hausdorff. If $f: X \rightarrow Y$ be strongly g^*sp -continuous injective, then X is Hausdorff.

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 7, Issue 8, August 2018

Proof : Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is g^*sp -Hausdorff, there exist two disjoint g^*sp -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is strongly- g^*sp -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is Hausdorff space.

We define the following.

Definition 3.25: A function $f: X \rightarrow Y$ is said to be g^*sp -open if the image of each open set of X is g^*sp -open in Y .

Definition 3.26: A function $f: X \rightarrow Y$ is said to be always- g^*sp -open if the image of each g^*sp -open set of X is g^*sp -open in Y .

Clearly, every always- g^*sp -open function is g^*sp -open.

Definition 3.27 : A function $f: X \rightarrow Y$ is said to be g^*sp -closed if the image of each closed set of X is g^*sp -closed in Y .

Definition 3.28 : A function $f: X \rightarrow Y$ is said to be always- g^*sp -closed if the image of each g^*sp -closed set of X is g^*sp -closed in Y .

Clearly, every always- g^*sp -closed function is g^*sp -closed.

We, state the following.

Theorem 3.29: Let $f: X \rightarrow Y$ be a bijection. Then the following are equivalent.

- (i) f is always- g^*sp -open.
- (ii) f is always- g^*sp -closed.
- (iii) f^{-1} is g^*sp -irresolute.

Proof: Obvious

We, define the following.

Definition 3.30: A function $f: X \rightarrow Y$ is said to be strongly g^*sp -open if the image of each g^*sp -open set of X is open in Y .

Theorem 3.31: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two functions. Then,

- (i) If f is g^*sp -open and g is always g^*sp -open, then $g \circ f$ is g^*sp -open.
- (ii) If f is strongly g^*sp -open and g is g^*sp -open, then $g \circ f$ is always g^*sp -open.
- (iii) If f is always g^*sp -open and g is strongly g^*sp -open, then $g \circ f$ is strongly g^*sp -open.

Proof : (i) Let U be any open set in X , $f(U)$ is g^*sp -open set in Y , because f is g^*sp -open function. Again, g is always g^*sp -open and $f(U)$ is g^*sp -open set in Y , then $g(f(U)) = g \circ f(U)$ is g^*sp -open set in Z . This shows that $g \circ f$ is g^*sp -open function.

(ii) Let U be any g^*sp -open set in X , $f(U)$ is open set in Y , because f is strongly g^*sp -open function. Again, g is g^*sp -open and $f(U)$ is open set in Y , then $g(f(U)) = g \circ f(U)$ is g^*sp -open set in Z . This shows that $g \circ f$ is always g^*sp -open function.

(iii) Let U be any g^*sp -open set in X , $f(U)$ is g^*sp -open set in Y , because f is always g^*sp -open function. Again, g is strongly g^*sp -open and $f(U)$ is g^*sp -open set in Y , then $g(f(U)) = g \circ f(U)$ is open set in Z . This shows that $g \circ f$ is strongly g^*sp -open function.

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 7, Issue 8, August 2018

We , define the following .

Definition 3.32: A function $f:X \rightarrow Y$ is said to be strongly g^*sp -closed if the image of each g^*sp -closed set of X is closed in Y .

Theorem 3.33: Let $f :X \rightarrow Y$ and $g : Y \rightarrow Z$ be any two functions. Then,

- (i) If f is g^*sp -closed and g is always g^*sp -closed , then $g \circ f$ is g^*sp -closed.
- (ii) If f is strongly g^*sp -closed and g is g^*sp -closed , then $g \circ f$ is always g^*sp -closed.
- (iii) If f is always g^*sp -closed and g is strongly g^*sp -closed ,then $g \circ f$ is strongly g^*sp -closed.

Proof : (i) Let F be any closed set in X , $f(F)$ is g^*sp -closed set in Y , because f is g^*sp -closed function. Again , g is always g^*sp -closed and $f(F)$ is g^*sp -closed set in Y , then $g(f(F))=g \circ f(F)$ is g^*sp -closed set in Z . This shows that $g \circ f$ is g^*sp -closed function.

(ii) Let F be any g^*sp -closed set in X , $f(F)$ is closed set in Y , because f is strongly g^*sp -closed function. Again , g is g^*sp -closed and $f(F)$ is closed set in Y , then $g(f(F))=g \circ f(F)$ is g^*sp -closed set in Z . This shows that $g \circ f$ is always g^*sp -closed function.

(iii) Let F be any g^*sp -closed set in X , $f(F)$ is g^*sp -open set in Y , because f is always g^*sp -closed function. Again , g is strongly g^*sp -closed and $f(F)$ is g^*sp -closed set in Y , then $g(f(F))=g \circ f(F)$ is closed set in Z . This shows that $g \circ f$ is strongly g^*sp -closed function.

Definition 3.34: A bijection $f:X \rightarrow Y$ is called g^*sp -homeomorphism if f is both g^*sp -continuous function and g^*sp - open function .

Theorem 3.35 : Every homeomorphism is an g^*sp - homeomorphism..

We , define the following.

Definition 3.36 : A bijection $f : X \rightarrow Y$ is said to be always g^*sp - homeomorphism if both f and f^{-1} are g^*sp - irresolute functions.

Theorem 3.37: Every always g^*sp - homeomorphism is an g^*sp - homeomorphism.

Proof: Obvious.

IV.CONCLUSION

We , introduced and studied a new class of sets in this paper , namely, g^*sp -closed sets in topological spaces . As applications of these sets , we defined six low-dimensional spaces in Definition -3.4 above . We , give their brief implications in the following.:

Theorem 4.1 : Let X be a space .Then,

- (i) If X is T_c^* - space ,then it is a T_{pc}^* -space (and hence it is a T_{sp}^* -space).
- (ii) If X is T_{gsp}^* -space , then it is a T_c^* -space (and hence it is a T_{pc}^* -space , it is a T_{sp}^* -space.
- (iii) If X is T_{gsp}^{**} -space , then it is a T_{sp}^* -space.
- (iv) If X is T_{gp}^{**} -space , then it is a T_{sp}^* -space.

Also,these spaces are utilized in our another paper, entitled, Applications of g^*sp -closed sets in topological spaces.

*** We thank reviewer, for his valuable suggestions , which improve the quality of the paper.

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 7, Issue 8, August 2018

REFERENCES

- [1] D.Andrijevic , Semipreopen sets, Mat.Vesnik ,38 (1886),24-32.
- [2] S. P. Arya and T.M.Nour,Characterizations of s -Normal spaces, Indian J.Pure & Appl. Math. 21(1990), N0.8,717-719.
- [3] N.Biswas, "On characterization of semi-continuous functions", Rendiconti, Accademia Nazionale Dei Lincei, April 1970.
- [4] Chaalit Boonpok , Preservation Theorems Concerning g -Hausdorff and rg -Hausdorff spaces ,Naresuan University Journal , 11(3),(2003),75-77.
- [5] S.G.Crossley and S.K.Hildebrand, "Semi-Closure", Texas J. Soci., 22,No.2-3(1970), 99-112.
- [6] P.Das "Note on some applications of semi-open sets", Progre. Math. Soc.(Allahabad), 7(1973), 3-44.
- [7] Julian. Dontchev, "On generalized semi-preopen sets, Fac. Sci. Kochi Univ. Ser. A, Math., 16(1995), 35-48.
- [8] S.N.El – Deeb, I. A. Hasanein, A. S. Mashhour and T. Noiri, " On p -regular Spaces", Bull. Math. Soc.Sci. Math. R.S.Roumanie (N.S), 27 (75), (1983), 311-315.
- [9] N.Levine, "Semi-open and semi-continuity in topological spaces", Amer. Math. Monthly, 70(1963), 36-41.
- [10] N.Levine, Generalized Closed Sets in Topology. Rend. Circ. Mat. Palermo. 19(2) (1970), 89-96.
- [11] A. S. Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb, On Precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, 53(1982) 47-53.
- [12] A. S. Mashhour, M. E. Abd El-Monsef and I.A.Hasanein "On pretopological Spaces", Bull. Math. Soc. Sci. Math. R.S. Roumanie, 28(76) (1984), 39-45.
- [13] Govindappa Navalagi and Sujata Mookanagoudar , Applications of g^*sp -closed sets topology , IJREI (accepted)
- [14] T.Noiri, H. Maki and J.Umehara, 'Generalized preclosed Functions' Mem. Fac. Sci. Kochi. Univ. (Math.) 19(1998), 13-20.
- [15] M.K.R.S.Veera Kumar , Between closed sets and g -closed sets, Mem.Fac.Sci.Kochi Univ.(Math.),21(2000),1-19.
- [16] M.K.R.S.Veera Kumar , g^* -preclosed sets , Acta Ciencia Indica ,Vol.XXVIII M .No.1, (2002), 51-60.