

Fuzzy Multiple Split Domination Number

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ABSTRACT: In this paper we introduced the concepts of fuzzy multiple split domination number. We determine the number $\gamma_{ms}(G)$ of a fuzzy multiple split domination number and obtain the definitions and theorems.

KEYWORD: Graph, Fuzzy Graph, Domination set, Multiple domination set, Split dominating set, Split domination number.

I. INTRODUCTION

The study of dominating sets in graphs was begun by Ore and Berge, the domination number is introduced by Cockayne and Hedetniemi, Rosenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycle and connectedness. A. Somasundaram and S. Somasundaram discussed domination in fuzzy graph. A. Nagoor Gani and V. T. Chandrasekaran discussed domination in fuzzy graph. We also discussed the domination number of the fuzzy digraph, fuzzy k-dominating set, fuzzy k-domination number, fuzzy multiple domination. Then the concept of split domination was introduced by V. R. Kulli and B. Janakiram discussed about the split domination number by using this finally about the fuzzy multiple split domination number $\gamma_{ms}(G)$.

II. PRELIMINARIES

Definition 2.1: A fuzzy graph $G = (\sigma, \mu)$ is a set with two functions $\sigma: V \rightarrow [0,1]$ and $\mu: E \rightarrow [0,1]$ such that $\mu(\{x, y\}) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. Then we write $\mu(xy)$ for $\mu(\{x, y\})$.

Definition 2.2: Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(x) = \sigma(x)$ for all $x \in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(x, y) = \mu(xy)$ for all $x, y \in V_1$. Then (σ_1, μ_1) is called the fuzzy sub graph of G induced by V_1 and is denoted by $\langle V_1 \rangle$.

Definition 2.3: Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $S \subseteq V$. Then the fuzzy cardinality of S is defined to be $\sum_{v \in S} \sigma(v)$.

Definition 2.4: A subset D of V is said to be a **dominating set** of G if every vertex in $V - D$ is adjacent to a vertex in D .

Definition 2.5: The **domination number** $\gamma(G)$ of G is the minimum cardinality of a dominating set.

Definition 2.6: Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Then $D \subseteq V$ is said to be a **fuzzy dominating set** of G if for every $v \in V - D$ there exists u in D such that $\mu(v, u) = \sigma(u) \wedge \sigma(v)$.

Definition 2.7: The minimum fuzzy cardinality of a fuzzy dominating set is called the **fuzzy domination number** of G and is denoted by $\gamma_f(G)$.

Definition 2.8: A fuzzy dominating set D of a fuzzy graph G is called a **multiple dominating set** of G if for each vertex in $V - D$ be dominated by multiple vertices in D .

Definition 2.9: A split dominating set D of graph G is called a **split dominating set**, if the induced sub graph $\langle V - D \rangle$ is disconnected.

Definition 2.10: The **split domination number** $\gamma_s(G)$ of G is the minimum cardinality of the split dominating set.

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III. FUZZY MULTIPLE SPLIT DOMINATION NUMBER

Definition 3.1: A fuzzy dominating set D of a fuzzy graph G is called *multiple split domination number* of G if for each vertex in $V - D$ be dominated by multiple vertices in D and $\gamma_{ms}(G)$ is the minimum cardinality of the split dominating set.

Example: The fuzzy multiple split domination number of given fuzzy graph (Fig.4) is $D = \{v_2, v_4, v_5\}$, $V - D = \{v_1, v_3\}$

The cardinality of a fuzzy multiple split domination number is 3.

The fuzzy multiple split domination number $\gamma_{ms}(G) = 3$ and $\{v_2, v_4, v_5\}$ is a set of cut vertices.

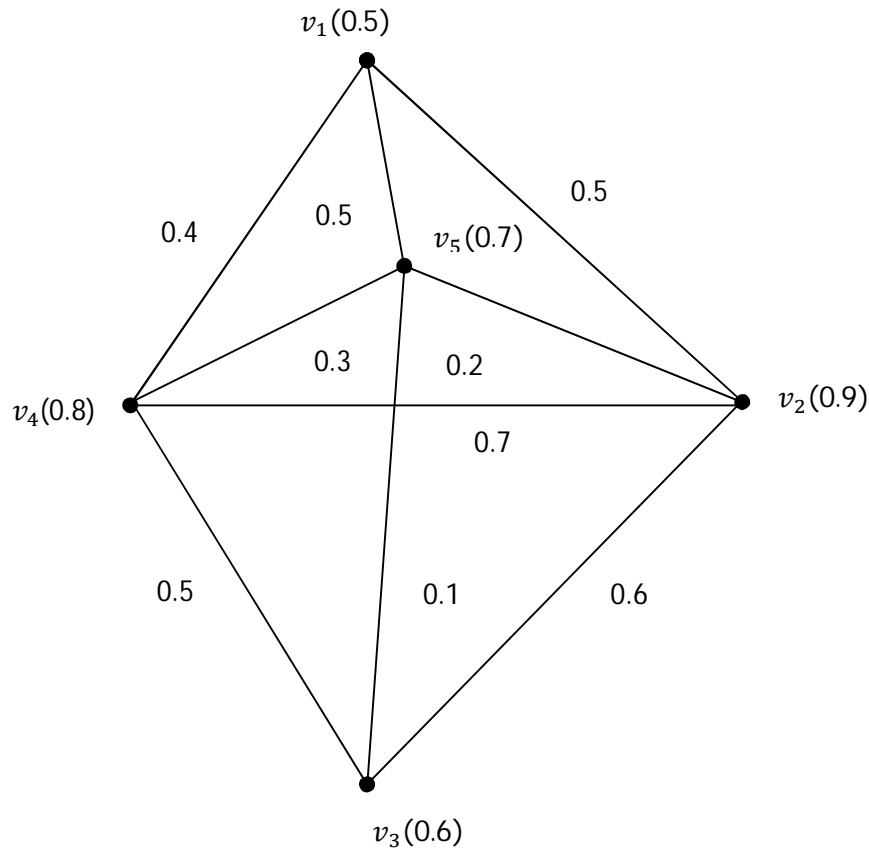


Fig. 1

Theorem 3.1: If D is a fuzzy γ_{ms} - set of a fuzzy graph G , then at least one vertex in $V - D$ is not dominated by multiple vertices in D and is not a set of cut vertices.

Proof: Let D be a fuzzy multiple split dominating set in G . Assume that, every vertex in $V - D$ is dominated by multiple vertices in D and is a set of cut vertices. Let $u \in V - D$ and let v and w be two cut vertices in D with multiple vertices which dominates u . By our assumption, every vertex in $V - D$ is dominated by at least one cut vertices in $D - \{v, w\}$. The set $D^1 = D - \{v, w\} \cup \{u\}$ is fuzzy multiple split domination number $\gamma_{ms}(G)$. But, $|D^1| \leq |D|$ contradict the assumption that D is a fuzzy multiple split domination number.

Corollary 3.1: If the fuzzy graph G has no cut vertices and no multiple vertices in D , then $\gamma_{ms}(G) \geq 2$.

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Remark 3.1: Converse of **corollary 3.1** need not be true as one can see from the fuzzy graph G given in **Fig. 2**. Here $\gamma_{ms}(G) = 4$ and $\{v_5, v_6, v_7, v_8\}$ is a set of cut vertices and has multiple vertices in D .

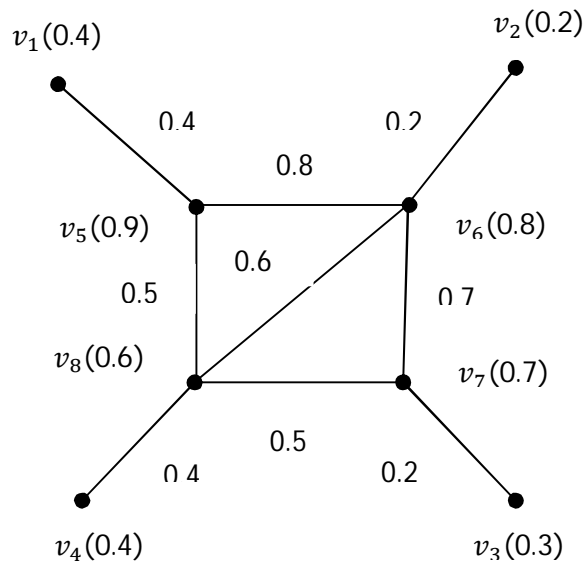


Fig. 2

Theorem 3.2: Let v be a vertex in a fuzzy graph G with degree k and $\langle N(v) \rangle$ disconnected. Then $\gamma_{ms}(G) \leq p - k$.

Proof: Since v is of degree k and $\langle N(v) \rangle$ disconnected. $V - N(v)$ is a fuzzy multiple split domination number $\gamma_{ms}(G)$. Therefore $|V - N(v)| \geq \gamma_{ms}(G)$.

Theorem 3.3: Let S be a γ_{ms} - set of a fuzzy graph G . If $V - S$ is a fuzzy multiple split domination number of $\gamma_{ms}(G)$, then $\gamma_{ms}(G) \leq p/2$.

Proof: Since $V - S$ is a multiple split domination number in a fuzzy graph G . $|V - S| \geq \gamma_{ms}(G)$.

Remark 3.2: The converse of the above **Theorem 3.3** need not be true and the same can be seen from the fuzzy graph given in **Fig. 3** In this fuzzy graph, $S = \{v_1, v_4\}$ is a γ_{ms} - set with $\gamma_{ms} < p/2$. But, $V - S = \{v_2, v_3, v_5\}$ is not a fuzzy multiple split domination number $\gamma_{ms}(G)$.

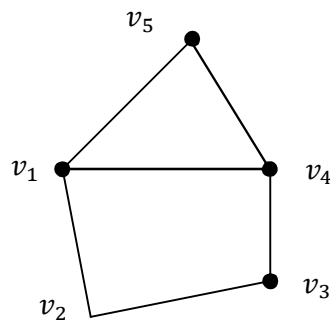


Fig. 3

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Theorem 3.4: If a dominating set S of fuzzy graph is also a fuzzy multiple split domination number, then there exists two vertices v_1, v_2 in $V - S$ such that $d(v_1, v_2) \geq 2$.

Proof: If not, assume that for any two vertices v_1, v_2 in $V - S$, $d(v_1, v_2) = 1$. Then $\langle V - S \rangle$ is connected which is a contradiction to S is a fuzzy multiple split domination number $\gamma_{ms}(G)$.

IV. CONCLUSION

The concept of domination in graphs is very rich both in the theoretical developments and applications. More than thirty domination parameters have been investigated by different authors, and in this paper we have introduced, the concepts of fuzzy multiple split domination number by using fuzzy multiple domination and Split domination number in graphs.

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