

Connected Edge Perfect Domination in Fuzzy Graphs

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ABSTRACT: In this paper we introduce the concept of connected edge perfect domination and connected edge domination in fuzzy graph. We determine the connected edge perfect domination number for standard theorems and some examples. Finally the connected edge perfect domination set and number for a connected fuzzy graph is obtained.

KEYWORDS: Fuzzy graph, Dominating set, Edge domination, Edge domination number, Effective edge, Connected edge domination, Perfect dominating set, Connected edge perfect domination, Connected edge perfect domination number.

I. INTRODUCTION

An interesting application in voting situation using the concept of domination. The study of dominating sets in graphs was begun by Ore and Berge, the domination number, independent domination number are introduced by Cockayne and Hedetniemi. A. Somasundram and S. Somasundram discussed domination in fuzzy graphs. V.R. Kulli and D.K. Patwari discussed the total edge domination number of graph. Nagoor Gani and Chandrasekaran discussed domination in fuzzy graph using strong arcs. We also discuss domination, independent domination and perfect domination in fuzzy graph using strong arcs. In this paper we introduce the connected edge perfect domination in fuzzy graph, the connected edge perfect domination number, and give some important results.

II. PRELIMINARIES

Definition 2.1: A *fuzzy graph* $G = (\sigma, \mu)$ is a set with two functions, $\sigma: V \rightarrow [0,1]$ and $\mu: E \rightarrow [0,1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y) \forall x, y \in V$. Let $\sigma: V \rightarrow [0,1]$ be a fuzzy subset of V then the *complete fuzzy graph* on σ is defined on $G = (\sigma, \mu)$ where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$ and is denoted by K_σ . The *complement of a fuzzy graph* G denoted by $\bar{G} = (\sigma, \bar{\mu})$ where $\bar{\mu}(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$.

Definition 2.2: Let $G = (\sigma, \mu)$ be a fuzzy graph on V . Let $x, y \in V$. We say that x dominated y in G if $\mu(xy) = \sigma(x) \wedge \sigma(y)$. A subset S of V is called a *dominating set* in G if for every $v \notin S$, there exists $u \in S$ such that u dominates v . The minimum fuzzy cardinality of a dominating set in G is called the *domination number* of G and is denoted by $\gamma(G)$ or γ .

Definition 2.3: Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $S \subseteq V$. Then the *fuzzy cardinality* of S is defined to be $\sum_{v \in S} \sigma(v)$. Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $D \subseteq E$. Then the *fuzzy edge cardinality* of D is defined to be $\sum_{e \in D} \mu(e)$.

Definition 2.4: Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, X) . A subset S of X is said to be an *edge domination set* in G if for every $x \in X - S$ is adjacent to atleast one effective edge in S . The minimum fuzzy cardinality of an edge dominating

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set in G is called the **edge domination number** of G and is denoted by $\gamma'(G)$ or γ' . An edge $e = xy$ of a fuzzy graph is called an **effective edge** if $\mu(xy) = \sigma(x) \wedge \sigma(y)$.

Definition 2.5: Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, X) , an edge dominating set F of a fuzzy graph G is **connected edge dominating set** with $\langle F \rangle$ is connected. The **connected edge domination number** $\gamma_{ce'}(G)$ or $\gamma_{ce'}$ is the minimum fuzzy cardinality of connected edge dominating set.

Definition 2.6: Let G be a fuzzy graph without isolated edges. A subset D of E is said to be a **total edge dominating set** if every edge in E is dominated by an edge in D . The minimum fuzzy cardinality of a total edge dominating set is called the **total edge domination number** of G and is denoted by $\gamma_t'(G)$.

Definition 2.7: An edge dominating set X of a fuzzy graph G is said to be a **perfect edge dominating set** if every edge of $E - X$ is adjacent to exactly one edge in X . The perfect edge dominating set X of a fuzzy graph G is said to be a **minimal perfect edge dominating set** if for each $uv \in X, X - \{u, v\}$ is not a perfect edge dominating set. The cardinality of a minimum perfect edge dominating set is called as **perfect edge domination number** and is denoted by $\gamma_{pf}'(G)$.

III. CONNECTED EDGE PERFECT DOMINATION IN FUZZY GRAPH

Definition 3.1: Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, X) , an edge dominating set F of a fuzzy graph G is **connected edge perfect dominating set** with $\langle F \rangle$ is connected.

The **connected edge perfect domination number** $\gamma_{cep'}(G)$ or $\gamma_{cep'}$ is the minimum fuzzy cardinality of connected edge perfect dominating set.

Example 3.1:

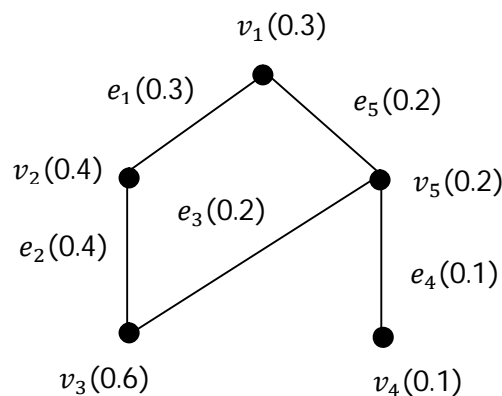


Fig. 1

$\{e_1, e_5\}$ is a connected edge perfect dominating set.
Connected edge perfect domination number $\gamma_{cep'}(G) = 0.5$

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Theorem 3.1: [4] For any fuzzy graph $\frac{q}{\Delta_{cep}'(G)+1} \geq \gamma_{cep}'(G)$

Proof: Let S be connected edge perfect edge domination set of G .

Since $|S|\Delta_{cep}'(G) \leq \sum_{e \in S} d_E(e)$

$$= \sum_{e \in S} |N(e)|$$

$$\leq |\cup_{e \in S} N(e)|$$

$$\leq |E - S|$$

$$\leq q - |S|$$

$$|S|\Delta_{cep}'(G) + |S| \leq q$$

$$\text{Thus } \frac{q}{\Delta_{cep}'(G)+1} \geq \gamma_{cep}'(G).$$

Theorem 3.2: A connected edge perfect dominating set D is minimal if and only if for each edge $e \in D$, one of the following two conditions holds.

I. $N(e) \cap D = \emptyset$

II. There exists an edge $q \in E - D$ such that $N(q) \cap D = \{e\}$ and q is an effective edge.

Proof: Let D be a minimal connected edge perfect dominating set and $e \in D$. Then $D_e = D - \{e\}$ is not an connected edge perfect dominating set. Hence there exists $q \in E - D_e$ such that q is not dominated by any element of D_e . If $q = e$ we get (I) and if $q \neq e$ we get (II).

Conversely,

Let I. $N(e) \cap D = \emptyset$ II. There exists an edge $q \in E - D$ such that $N(q) \cap D = \{e\}$ and q is an effective edge. To prove that D is a minimal connected edge perfect dominating set. Suppose that $e \in E - D_e$, then q is not dominated by any element of D_e . Now $D_e = D - \{e\}$ is not a connected edge perfect dominating set. Hence D is a minimal connected edge perfect dominating set.

Theorem 3.3: [4] Let G be a connected fuzzy graph and let S be a minimal connected edge perfect dominating set of G . Then $E - S$ is also a minimal connected edge perfect dominating set of G .

Proof: Let S be a minimal connected edge perfect dominating set of G . Suppose that $E - S$ is not a connected edge perfect dominating set. Then there exists a edge $E - S$ such that e is not a dominated by any edge in $E - S$. Since G is connected, e is a strong neighbor of atleast one edge in $S - \{e\}$. Then $S - \{e\}$ is a dominating set which is a contradiction. Thus every edge in S is a strong neighbor of atleast one edge in $E - S$. Hence $E - S$ is also a connected edge perfect dominating set.

Result 3.3: [4] For any fuzzy graph G without isolated edges $\gamma_{cep}' \leq \frac{E}{2}$.

Given that G be any fuzzy graph without isolated edges. To prove that $\gamma_{cep}' \leq \frac{E}{2}$. If any graph without isolated edges has two disjoint connected edge perfect dominating set. Then $\gamma_{cep}' \leq \frac{E}{2}$.

For the connected edge perfect domination in fuzzy graph given in fig. 2

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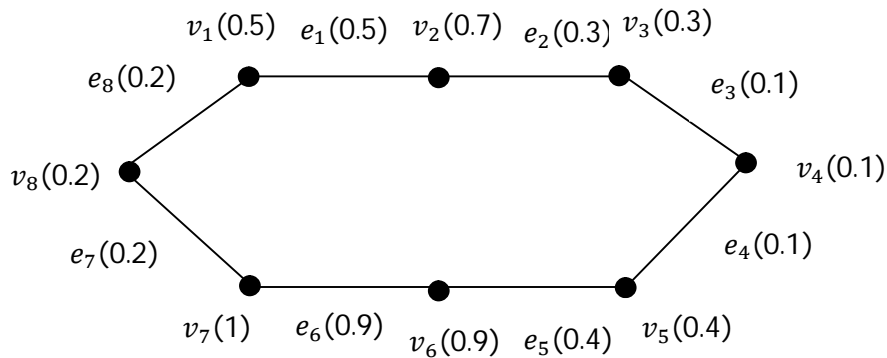


Fig. 2

The connected edge perfect domination number $\gamma_{cep}' = 0.6, E = 2.7$ and $\frac{E}{2} = 1.35$.

$$\text{Hence } \gamma_{cep}' \leq \frac{E}{2}.$$

Theorem 3.4: For any fuzzy graph $\frac{X}{\Delta_{cep}'(G)} \geq \gamma_{cep}'(G)$.

Proof: Let S be a connected edge perfect dominating set with minimum number of fuzzy edge cardinality. Then every edge in S is adjacent to atleast $\Delta_{cep}'(G)$ fuzzy edge cardinality. Therefore $|S|\Delta_{cep}'(G) \leq X$. Hence $\frac{X}{\Delta_{cep}'(G)} \geq \gamma_{cep}'(G)$.

Theorem 3.5: [3] Every complete fuzzy graph G is a connected edge perfect dominating set with $n = 3$.

Proof: Let G be a complete fuzzy graph. Therefore every arc in G is strong arc. (ie) every edge $e, E - S$ is incident to exactly one edge of the dominating set S in G . Hence S is a connected edge perfect dominating set with $n = 3$. Therefore, every complete fuzzy graph G is a connected edge perfect dominating set with $n = 3$.

Theorem 3.6: [4] If a fuzzy graph G has no isolated edges then $\gamma_{cep}'(G) \leq \gamma_{tcep}'$.

Proof: Let G be any fuzzy graph without isolated edges. To prove that $\gamma_{cep}'(G) \leq \gamma_{tcep}'$. By the definition, clearly every minimal connected edge perfect dominating set which is less than or equal to minimal total connected edge perfect dominating set. Hence $\gamma_{cep}'(G) \leq \gamma_{tcep}'$.

Theorem 3.7: [6] If G be any fuzzy graph without isolated edges then $\delta_{cep}(G) \leq \frac{2q}{p} \leq \Delta_{cep}(G)$

Proof: Let G be connected edge perfect domination in fuzzy graph. Let $E(G) = \{e_1, e_2, e_3, e_4\}$. We have $\delta_{cep}(G) \leq degE_i(G) \leq \Delta_{cep}(G)$

$$\text{Hence } p\delta_{cep}(G) \leq \sum_{i=1}^4 degE_i(G) \leq p\Delta_{cep}(G).$$

$$p\delta_{cep}(G) \leq 2q \leq p\Delta_{cep}(G).$$

$$\delta_{cep}(G) \leq \frac{2q}{p}.$$

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IV. CONCLUSION

The concept of domination in graphs is very rich both in the theoretical developments and applications. More than thirty domination parameters have been investigated by different authors, and in this paper we have introduced, the concepts of connected edge perfect domination and minimal connected edge perfect domination in fuzzy graph by using connected edge domination in fuzzy graph.

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