

Excellent Domination in Square Fuzzy Graphs

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ABSTRACT: In this chapter we introduce the concept of excellent domination in square fuzzy graph and also we discuss about some related theorems and examples of excellent domination in square fuzzy graph.

KEYWORDS: Fuzzy domination, square fuzzy graph, excellent fuzzy graph, strong fuzzy graph, complete fuzzy graph, excellent square fuzzy graph.

I. INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. The study of dominating sets in graph was begun by Orge and Berge. A. Somasundaram, S. Somasundaram [12] presented the concepts of Domination in fuzzy graph. An Excellent domination in fuzzy graph was presented by K.M Dharmalingam and P. Nithya [2]. Some results in square fuzzy graph were presented by G. Sanjeevi [7]. Here we introduce the concept of excellent domination in square fuzzy graph and their related concepts.

II. PRELIMINARIES

Definition 2.1: A *fuzzy graph* $G = (\sigma, \mu)$ is a pair of functions $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ Where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u), \sigma(v)$.

Definition 2.2: The *underlying crisp graph* of $G: (\sigma, \mu)$ is denoted by $G^*(V, E)$ Where $E \subseteq V \times V$.

Definition 2.3: A subset D of V is called a *fuzzy dominating set* if for every $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$. The minimum cardinality of such a dominating set is denoted by γ^f and is called the *fuzzy domination number* of G .

Definition 2.4: A fuzzy graph G is said to be *fuzzy excellent* if for every vertex of G belongs to γ^f - sets of G . A vertex which belongs to γ^f - set is called *Fuzzy good*. (i.e) A Fuzzy graph G is said to be Fuzzy excellent if for every vertex of G is Fuzzy good.

Definition 2.5: A fuzzy graph $G: (\sigma, \mu)$ is a *strong fuzzy graph* if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $(u, v) \in E$ and is a *complete fuzzy graph* if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $(u, v) \in V$

Definition 2.6: A fuzzy graph $G: (\sigma, \mu)$ is said to be *regular fuzzy graph* if all the vertices have same degree.

Definition 2.7: Let $G: (\sigma, \mu)$ be a fuzzy graph with underlying crisp graph $G^*(V, E)$. Then the *Square fuzzy graph* of G is denoted by $G^2(\sigma^2, \mu^2)$ and is defined as

$$\sigma^2(\mu) = \sigma(\mu), \forall u \in V \text{ and } \mu^2(u, v) = \begin{cases} \mu(u, v), & \text{if } (u, v) \in E \\ \sigma(u) \wedge \sigma(v), & \text{if } (u, v) \notin E \end{cases}$$

If u and v are joined by a path of length is less than or equal to two in $G^*(V, E)$.

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Vol. 7, Issue 2, February 2018

Definition 2.8: A *homomorphism* of fuzzy graph $G: (\sigma, \mu)$ and $G': (\sigma', \mu')$ with underlying crisp graphs $G^*(V, E)$ and $G'^*(V', E')$ respectively is a bijective map $h: V \rightarrow V'$ which satisfies $\sigma(x) = \sigma'(h(x)), \forall x \in V$ and $\mu(x, y) \leq \mu'(h(x), h(y)), \forall x, y \in V$.

Definition 2.9: A *weak isomorphism* $h: G \rightarrow G'$ is a homomorphism such that the map $h: V \rightarrow V'$ is bijective and satisfies $\sigma(x) = \sigma'(h(x)) \forall x \in V$.

Definition 2.10: If $\gamma^{uf}(G, u) = \gamma^f(G)$ then u is *fuzzy level vertex* of G

Definition 2.11: An edge (u, v) of fuzzy graph $G: (\sigma, \mu)$ is called an *effective edge* [5] if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$.

Definition 2.12: A dominating set S of a fuzzy graph $G: (\sigma, \mu)$ is said to be a *minimal dominating set* if no proper subset of S is a dominating set of G .

III. EXCELLENT DOMINATION IN SQUARE FUZZY GRAPH

Definition 3.1: A square fuzzy graph $G^2(\sigma^2, \mu^2)$ is said to be an *Excellent square fuzzy graph* if for every vertex of G^2 belongs to γ^f sets of G^2 , and is denoted by $G_E^2(\sigma^2, \mu^2)$.

Example 3.1:

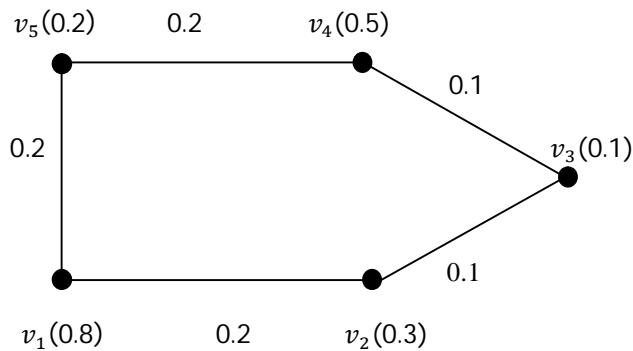


Fig 1: Fuzzy Graph $G(\sigma, \mu)$

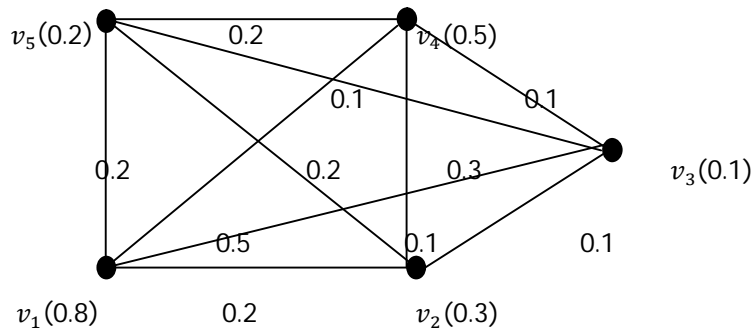


Fig 2: Square fuzzy graph $G^2(\sigma^2, \mu^2)$

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Vol. 7, Issue 2, February 2018

γ^f Sets of G^2 are $\{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}$

$\therefore G^2(\sigma^2, \mu^2)$ is a $G_E^2(\sigma^2, \mu^2)$

Theorem 3.1: Let $G(\sigma, \mu)$ be a strong fuzzy graph and $G^*(V, E)$ is cycle of length $n \leq 5$. Then the square fuzzy graph $G^2(\sigma^2, \mu^2)$ is a complete and is an excellent square fuzzy graph.

Proof: Given $G(\sigma, \mu)$ be a strong fuzzy graph and $G^*(V, E)$ is cycle of length $n \leq 5$. Since G is a strong fuzzy graph if $(u, v) \in V$.

Then $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ in G^2 also.

Since $G^*(V, E)$ is cycle of length $n \leq 5$ if u and v are non adjacent than there is a path of length 2.

$\therefore \mu(u, v) = \sigma(u) \wedge \sigma(v)$ in G^2 , if $(u, v) \notin E$.

\therefore Any two points are joined by an effective edge. $\therefore G^2(\sigma^2, \mu^2)$ is a complete fuzzy graph. Since G^2 is a complete, then every vertex of G^2 belongs to γ^f sets of G^2 . $\therefore G^2$ is an excellent square fuzzy graph.

Theorem 3.2: If $G(\sigma, \mu)$ is a fuzzy graph with underlying crisp graph $G^*(V, E)$ being complete then G and G^2 are same also G and G^2 are excellent square fuzzy graph.

Proof: $G^*(V, E)$ being complete every pair of vertices in V are adjacent in G^* . i.e) For every u, v in $V, (u, v)$ is in $E \Rightarrow \mu(u, v) > 0 \quad \forall u, v$ in V .

$\therefore \mu^2(u, v) = \mu(u, v) \quad \forall u, v$ in V .

$\therefore G^2(\sigma^2, \mu^2) = G(\sigma, \mu)$.

$\therefore G$ and G^2 are same. Since G and G^2 are complete. Then each vertex of G and G^2 are a minimum dominating set. Hence G and G^2 are excellent square fuzzy graph.

Theorem 3.3: Let G_E^2 be an excellent square fuzzy graph and a dominating set D of G_E^2 is a minimal dominating set if and only if for each $d \in D$ one of the following two conditions holds.

1. $N(d) \cap D = \emptyset$.

2. There is a vertex $c \in V \setminus D$ such that $N(c) \cap D = \{d\}$.

Proof: Given D be a minimal dominating set in G_E^2 , and $d \in D$. Then $D_d - D\{d\}$ is not a dominating set in G_E^2 , hence there exists $x \in V \setminus D_d$ such that x is not dominated by any element of D_d . If $x = d$ we get $N(d) \cap D = \emptyset$ and if $x \neq d$ we get $N(c) \cap D = \{d\}$. The converse is obvious.

Theorem 3.4: Let $G(\sigma, \mu)$ be a fuzzy graph and $G_E^2(\sigma^2, \mu^2)$ be its excellent square fuzzy graph. Then

(i) $O(G) = O(G_E^2)$ [i.e. Order of G = Order of G_E^2]

(ii) Size of $G \leq$ Size of G_E^2

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Vol. 7, Issue 2, February 2018

Proof: Given $G(\sigma, \mu)$ be a fuzzy graph and $G_E^2(\sigma^2, \mu^2)$ be its excellent square fuzzy graph. (i) Since G and G_E^2 has same point set with their respective weights.

$$\therefore \text{Order of } G = \sum \sigma(u),$$

$$\text{Order of } G_E^2 = \sum \sigma^2(u) = \sum \sigma(u),$$

$$\therefore O(G) = O(G_E^2).$$

(ii) If (u, v) be an edge in $G(\sigma, \mu)$ then (u, v) is also be an edge in $G_E^2(\sigma^2, \mu^2)$. If u and v are at a distance 2 in $G^*(V, E)$ then they are made adjacent in $G_E^2(\sigma^2, \mu^2)$. \therefore The number of edges is more in $G_E^2(\sigma^2, \mu^2)$.

Hence Size of $G \leq$ Size of G_E^2 .

Results 3.4.1: 1. A fuzzy graph G is weak isomorphic to the excellent square fuzzy graph G_E^2 .

2. [9] For any excellent square fuzzy graph $G_E^2(\sigma^2, \mu^2)$. $\gamma^f(G_E^2) \leq \gamma^f(G)$.

3. An excellent square fuzzy graph of a regular fuzzy graph needs not be regular.

Theorem 3. 5: If $G_E^2(\sigma^2, \mu^2)$ is complete then any vertex of G_E^2 is a fuzzy level vertex.

Proof: Let $G_E^2(\sigma^2, \mu^2)$ be a complete excellent square fuzzy graph. Since G_E^2 is complete then every pair of vertices are adjacent. Since each vertex of G_E^2 are minimum dominating set

$$\therefore \gamma^f(G_E^2) = 1.$$

Now remove any vertex $\{u\}$ of G_E^2 and their incident edges. We get $G_E^2 - \{u\}$ is complete. Since $G_E^2 - \{u\}$ complete then each vertex of $G_E^2 - \{u\}$ is a minimum dominating set.

$$\therefore \gamma^{uf}(G_E^2, u) = 1,$$

$$\therefore \gamma^f(G_E^2) = \gamma^{uf}(G_E^2, u)$$

$\therefore \{u\}$ is a fuzzy level vertex.

Since any vertex of G_E^2 is a fuzzy level vertex.

IV. CONCLUSION

In this paper we have define the concept of excellent domination in square fuzzy graph and derive the related theorems to this concept.

ACKOWLEGEMENT

Authors would like to thank referees for their helpful comments.

REFERENCES

- [1]. Bhattacharya, P. 1987. Some remarks on fuzzy graphs. Pattern Recognition Letters , pp. 297-302.
- [2]. Dharmalingam, K.M and Nithya, P. Excellent domination in fuzzy graph, vol. 7(2017), pp. 257-266.
- [3]. Haynes, T.W., Hedetniemi, S.T. and Slater, P.J. Fundamentals of domination in Graphs, Marcel Dekker, New York, 1998.

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(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Vol. 7, Issue 2, February 2018

- [4]. Nagoor Gani, A. and BasheerAhamed, M. Order and Size in fuzzy graphs, Bulletin of Pure and Applied Sciences, Vol. 22E (No. 1), 2003, pp. 145-148.
- [5]. Nagoorgani, A. and chandrasekaran, V.T A first look at Fuzzy graph Theory, allied publishers pvt.It, pp.21-24.
- [6]. Nagoor Gani, A. and Radha, K. On regular Fuzzy Graphs , Journal of Physical Sciences, Vol. 12, 2008, pp. 33-40
- [7]. Sanjeevi, G. Some Results on Square Fuzzy Graph, international journal of mathematical archive-8(2), 2017, pp. 124-128
- [8]. Sampathkumar, E. and Pushpalatha, L. Set domination in Graphs, J. Graph Theory, 18(5)(1994), 489-495.
- [9]. Sarala, N. and Kavitha, T. Connected domination numbers of Fuzzy Square graph, vol. 10, Issue 6 Ver.III (Nov-Dec. 2014), pp 13-14.
- [10]. Sridharan, N. and Yamuna, M. A note on Excellent graphs, Ars Combinatorial, 78(2006), pp.267-276
- [11]. Sridharan, N. and Yamuna, M. Excellent-Just Excellent-Very Excellent graphs, J. Math. Phy. Sci., 14(5) (1980), pp. 471-475.
- [12]. Somasundaram, A. and Somasundaram, S. Domination in Fuzzy Graphs-I, Pattern Recognition Letters, 19(9) (1998), pp. 787-789.