

# Results on n-Edge Magic Labeling

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**ABSTRACT:** In this paper is to define and study some fundamental graphs related to n – Edge Magic Labeling. The results of n – Edge magic labeling were implemented and generalized through Dyck graph, Petersen graph, Complete k – ary tree, Double quadrilateral snake graph and Firecracker graph.

**KEYWORDS:** Magic Labeling, n – Edge magic labeling, Petersen graph, Double quadrilateral snake, Firecracker graph.

## I. INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. In 1963 [10] Sedlack introduced labelings that simplify the thought of a magic labeling. In 1967 [9], Rosa introduced the origin of this labeling. Jayapriya.J, Thirusangu.k, P. Nedumaran [3], [4], [5], and [6] introduced 0-edge magic labeling of ladder, wheel, and windmill graph. Neelam Kumari, Seema Mehra introduced 1- edge magic labeling of path, cycle, double star [8] in 2013. Neelam kumari, Seema mehra introduced n - edge magic labeling for path and cycle [7] in 2013. In [11] S.Vimala and N.Nandhini have extended n – edge magic labeling on Ladder graph, Flower graph, Armed Crown graph,  $G = P_{t+1} \odot K_{1,t}$ , Splitting graph, Prism Graph, Web graph satisfied. In this paper, we extended to find the order and size of the graphs namely Dyck graph, Petersen graph, Complete k – ary tree, Double quadrilateral snake and Fire cracker graph through n – edge magic labeling.

## II. PRELIMINARIES

### 2.1 Magic labeling [9]

A finite graph  $G = [V(G), E(G)]$  without loops, multiple edges or isolated vertices. If there exists a mapping  $f$  from the set of edges  $E(G)$  into positive real numbers such that

(i)  $f(e_i) \neq f(e_j)$  for all  $e_i \neq e_j : e_i, e_j \in E(G)$ ,

(ii)  $\sum_{e \in E(G)} \eta(v, e)f(e) = r$  for all  $v \in E(G)$  where  $\eta(v, e)$  is 1 when vertex  $v$  and edge  $e$  are incident and 0 in the opposite case.

Then the graph  $G$  is called magic. The mapping  $f$  is called a labeling of  $G$ .

### 2.2 Vertex magic labeling [9]

A one-to-one map  $\lambda$  from  $E(G) \cup V(G)$  onto the integers  $1, 2, \dots, n + m$  is a vertex magic labeling on a graph  $G$  if there is a constant  $h$  so that for every vertex  $x$ ,  $w(x) = \lambda(x) + \sum_{x-y} \lambda(xy) = h$ .  $h$  is called magic sum. As above, any graph with a vertex-magic labeling is called vertex-magic.

### 2.3 Edge magic labeling [8]

An edge magic labeling of a  $(p; q)$ -graph  $G$  is a bijective function  $f : V(G) \cup E(G) \rightarrow \{1; 2; \dots; p + q\}$  such that  $f(u) + f(v) + f(uv) = k$  is a constant for any edge  $uv$  of  $G$ . In such a case,  $G$  is said to be edge-magic and  $k$  is called the valence of  $f$ .

### 2.4 Zero - edge magic labeling [3]

Let  $G = (V, E)$  be a graph where  $V = \{v_i, 1 \leq i \leq n\}$ , and  $E = \{v_i v_{i+1}, 1 \leq i \leq n\}$ . Let  $f : V \rightarrow \{-1, 1\}$ , and  $f^* : E \rightarrow \{0\}$ , such that all  $uv \in E$ ,  $f^*(uv) = f(u) + f(v) = 0$  then the labeling is said to be 0- edge magic labeling.

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## 2.5 One -edge magic labeling [8]

Let  $G=(V, E)$  be a graph where  $V= \{v_i, 1 \leq i \leq t\}$  and  $E= \{v_i v_{i+1}, 1 \leq i \leq t-1\}$ . Let  $f: V \rightarrow \{-1, 2\}$  and  $f^*: E \rightarrow \{1\}$  such that for all  $uv \in E$ ,  $f^*(u v) = f(u) + f(v) = 1$  then the labeling is said to be 1-edge magic labeling.

## 2.6 n -edge magic labeling [7]

Let  $G=(V, E)$  be a graph where  $V= \{v_i, 1 \leq i \leq t\}$  and  $E= \{v_i v_{i+1}, 1 \leq i \leq t-1\}$ . Let  $f: V \rightarrow \{-1, n+1\}$  and  $f^*: E \rightarrow \{n\}$  such that for all  $uv \in E$ ,  $f^*(u v) = f(u) + f(v) = n$  then the labeling is said to be n -edge magic labeling.

## 2.7 Dyck graph [2]

The Dyck graph is unique cubic symmetric graph (regular of order 3). The Dyck graph is a 3-regular graph with 16 vertices and 24 edges.

## 2.8 Petersen graph [2]

The Petersen graph is an undirected graph with 10 vertices and 15 edges.

## 2.9 Complete k-ary tree

In a complete k – ary tree of n vertices,  $k \geq 2$  all non-leaf vertices have exactly k children and all leaves have the same depth.

## 2.10 Double quadrilateral snake

The Double quadrilateral snake  $D(Q_n)$  consists of two quadrilateral snakes that have a common path.

## 2.11 Firecrackers

Let  $G_1, G_2 \dots G_n$  be a family of disjoint stars. Let  $v_1$  be a pendant vertex of  $G_1$   $1 \leq i \leq n$ . The fire which contains all the n stars and path joining  $v_1, v_2, \dots v_n$  is called fire crackers.

**2.1.1 Theorem:**  $P_t$  admits n – edge magic labeling for all t. [7]

**2.1.2 Theorem:**  $C_p$  admits n – edge magic labeling for all p is even. [7]

### III. MAIN RESULTS

**Theorem 3.1:** A Dyck graph admits n-Edge Magic Labeling.

**Proof:**

Let  $G = (V, E)$  be a graph.

Where  $V= \{v_i, 1 \leq i \leq p\}$  and  $E= \{v_i v_{i+1} \ 1 \leq i \leq p-1\}$

Let  $f: V \rightarrow \{-1, (n+1)\}$

Such that

$$f(v_i) = \begin{cases} -1 & \text{if } i \text{ is odd} \\ n+1 & \text{if } i \text{ is even} \end{cases} \quad \text{for } 1 \leq i \leq 16$$

We have,

$$f^*(v_i v_{i+1}) = \begin{cases} (-1) + (n+1) = n & \text{if } i \text{ is odd,} \\ (n+1) + (-1) = n & \text{if } i \text{ is even.} \end{cases}$$

$$f^*(v_i v_{i+5}) = (-1) + (n+1) = n \text{ if } i= 5, 7$$

$$f^*(v_i v_{i+7}) = (-1) + (n+1) = n \text{ if } i=1, 9$$

$$f^*(v_i v_{i+9}) = (n+1) + (-1) = n \text{ if } i=2, 4, 6$$

$$f^*(v_i v_{i+13}) = (-1) + (n+1) = n \text{ if } i=1, 3$$

Hence the proof

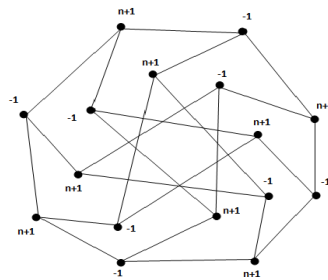
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**Example :**  
Dyck Graph (16,24)



**Theorem 3.2:** Let  $G$  be a Petersen graph  $Y_p$ .  $n$  – edge magic labeling of Petersen graph have  $2p$  vertices and  $3p$  edges when  $p$  is even (where  $p=4,6,\dots,n$ ).

**Proof:**

Given by method of induction.

Let  $G=(V, E)$  be a graph.

Where  $V= \{v_i, 1 \leq i \leq p\}$  and  $E= \{v_i v_{i+1}, 1 \leq i \leq p-1\}$

Let  $f: V \rightarrow \{-1, n+1\}$

Such that

$$f(v_i) = \begin{cases} -1 & \text{if } i \text{ is odd,} \\ n+1 & \text{if } i \text{ is even.} \end{cases}$$

$$f(u_i) = \begin{cases} -1 & \text{if } i \text{ is even,} \\ n+1 & \text{if } i \text{ is odd.} \end{cases} \quad \text{For } 1 \leq i \leq p.$$

We have

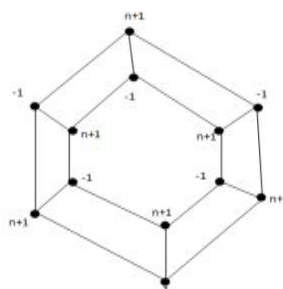
$$f^*(v_i v_{i+1}) = \begin{cases} (-1) + (n+1) = n & \text{if } i \text{ is odd,} \\ (n+1) + (-1) = n & \text{if } i \text{ is even} \end{cases}$$

$$f^*(v_i u_i) = \begin{cases} (-1) + (n+1) = n & \text{if } i \text{ is even,} \\ (n+1) + (-1) = n & \text{if } i \text{ is odd.} \end{cases}$$

$$f^*(u_i u_{i+1}) = \begin{cases} (-1) + (n+1) = n & \text{if } i \text{ is even,} \\ (n+1) + (-1) = n & \text{if } i \text{ is odd.} \end{cases}$$

**Example**

When  $p=6, v=12, e=18$



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**Theorem 3.3:** Let  $G$  be a complete  $k$ -ary tree  $f_{m,p}$   $m$  - odd vectors and  $p$  – components admits  $n$  – edge magic labeling have  $4n+1$  vertices and  $4n$  edges (where  $n=1,2,3\dots n$ ).

**Proof:**

Let  $G=(V, E)$  be a complete  $k$ -ary tree  $f_{m,p}$  and  $w_1$  is a common vertex in  $f_{m,p}$ . Let  $v_i$ 's and  $u_i$ 's vertices in  $f_{m,p}$ .

Let  $f: V \rightarrow \{-1, n+1\}$

Such that

$$\begin{aligned} f(w_1) &= -1, \\ f(v_i) &= n+1, \quad \text{for } 1 \leq i \leq m \\ f(u_i) &= -1, \quad \text{for } 1 \leq i \leq p. \end{aligned}$$

We have

$$\begin{aligned} f^*(w_1 v_i) &= (-1) + (n+1) = n \text{ for } 1 \leq i \leq m, \\ f^*(v_i u_i) &= (n+1) + (-1) = n \text{ for } 1 \leq i \leq m, 1 \leq i \leq p. \end{aligned}$$

Complete  $k$ -ary tree for  $n$ -Edge magic labeling exists only when  $m > 4$  for all  $m$  is odd

$$(i.e.) f_{m,p} = \begin{cases} m > 4 & \text{for all } m \text{ is odd,} \\ P & \text{for all } p=1, 2 \dots n. \end{cases}$$

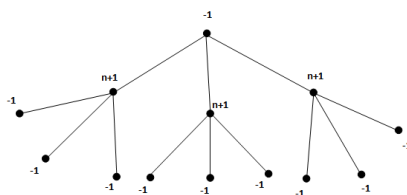
Let us fix  $m=5$

Complete  $k$  – ary tree have  $4n+1$  vertices and  $4n$  edges.

Hence the proof.

**Example:**

When  $m = 5, p = 3, v = 13, e = 12$



**Theorem 3.4:** Let  $G$  be a double quadrilateral snake  $D(Q_p)$   $n$ -edge magic labeling of double quadrilateral snake graph have  $5p+1$  vertices and  $6p$  edges (where  $p=1,2,\dots$ ).

**Proof:**

Let  $G=(V, E)$  be a graph.

Where  $V = \{v_i \mid 1 \leq i \leq p\}$  and  $E = \{v_i v_{i+1} \mid 1 \leq i \leq p-1\}$

Let  $f: V \rightarrow \{-1, n+1\}$

Such that

$$f(v_i) = \begin{cases} -1 & \text{if } i \text{ is odd for } 1 \leq i \leq p, \\ n+1 & \text{if } i \text{ is even for } 1 \leq i \leq p. \end{cases}$$

We have

$$\begin{aligned} f^*(v_i v_{i+1}) &= \begin{cases} (-1) + (n+1) = n & \text{if } i \text{ is odd,} \\ (n+1) + (-1) = n & \text{if } i \text{ is even.} \end{cases} \\ f^*(v_i u_i) &= (-1) + (n+1) = n \text{ for } 1 \leq i \leq p. \end{aligned}$$

$$f^*(u_i u_{i+1}) = \begin{cases} (-1) + (n+1) = n & \text{if } i \text{ is even,} \\ (n+1) + (-1) = n & \text{if } i \text{ is odd.} \end{cases}$$

Hence  $D(Q_p)$  is  $n$ -edge magic labeling.

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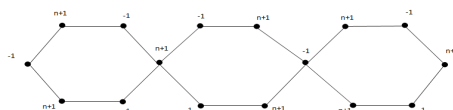
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**Example:**

When  $p=3, v=16, e=18$



**Theorem 3.5:** The fire crackers  $(fc)_p$  admits  $n$ -edge magic labeling of fire cracker graph have  $p$  vertices  $p-1$  edges.(where  $p=4,6,8,\dots$ )

**Proof:**

Let  $G=(V, E)$  be a graph.

Where  $V= \{v_i, 1 \leq i \leq p\}$  and  $E= \{v_i v_{i+1}, 1 \leq i \leq p-1\}$

Let  $f: V \rightarrow \{-1, n+1\}$

Such that

$$f(v_i) = \begin{cases} (-1) & \text{if } i \text{ is odd for } 1 \leq i \leq p, \\ (n+1) & \text{if } i \text{ is even for } 1 \leq i \leq p. \end{cases}$$

We have

$$f^*(v_i v_{i+1}) = \begin{cases} (-1) + (n+1) = n & \text{if } i \text{ is odd,} \\ (n+1) + (-1) = n & \text{if } i \text{ is even.} \end{cases}$$

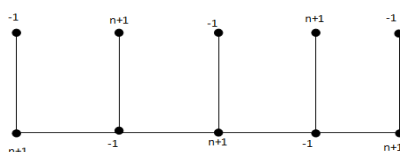
$$f^*(u_i u_{i+1}) = \begin{cases} (-1) + (n+1) = n & \text{if } i \text{ is even,} \\ (n+1) + (-1) = n & \text{if } i \text{ is odd.} \end{cases}$$

$$f^*(v_i u_i) = (-1) + (n+1) = n \quad \text{for } 1 \leq i \leq p.$$

Hence  $G$  admits  $n$ -edge magic labeling.

**Example:**

When  $p=4, v=10, e=9$ .



## IV. CONCLUSION

The  $n$  – Edge Magic Labeling was extended by the Dyck graph, Petersen graph, Complete  $k$ -ary tree graph, Double quadrilateral snake graph and Firecracker graph. Further it is extended for some other families of graphs.

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