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SD Closed set, SD Open set and its Properties

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ABSTRACT: In this paper, we introduce and investigate the notation of SD closed set and SD open sets by utilizing generalized closed sets and regular b-closed set. We obtain fundamental properties of SD closed set and SD closed set and discuss the relationships between SD closed set and SD open set and other related sets.

KEY WORDS: SD closed set, SD open set.

I. INTRODUCTION

The notion of regular b-closed (briefly rb-closed) sets is introduced and studied recently by Nagaveni and Narmadha [1],[2]. Regular open sets and b-open sets have been introduced and investigated by Stone [3] and Andrijevi'c [4], respectively. Levine [5] (resp. Bhattacharya and Lahiri [6], Palaniappan and Rao [7], Arya and Nour [8], Maki et al [9], Maki et al [10],[11]) introduced and investigated generalized closed sets (resp. semi-generalized closed sets, regular generalized closed sets, generalized semi-closed sets, generalized preclosed sets, generalized α -closed sets and α -generalized closed sets). Al-Omari and Noorani [12] investigated the class of generalized b-closed sets and obtained some of its fundamental properties. We introduce and study the concepts of rb-open sets and rb-closed spaces. Throughout this paper, a space means a topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $Cl(A)$ and $Int(A)$ denote the closure of A and interior of A, respectively.

$X - A$ or A^c denotes the complement of A in X . The concept of generalized closed sets plays a significant role in topology. There are many research papers which deal with different types of generalized closed sets. Bhattacharya and Lahiri [6] introduced sg-closed sets in topological spaces. Arya and Nour [8] introduced gs-closed sets in topological spaces. Sheik John [14] introduced w-closed sets in topological spaces. Ravi and Ganesan [15] introduced \tilde{g} -closed sets in topological spaces. Quite Recently, Ravi et. al. [16] have introduced $\approx g$ -closed sets in topological spaces. In this paper we introduce a new class of sets namely sd-closed sets in topological spaces and study their basic properties.

II. PRELIMINARIES

Definition: 2.1 A subset A of a space (X, τ) is called

- (i) Semi-open set [18] if $A \subseteq Cl(Int(A))$
- (ii) preopen set [19] if $A \subseteq Int(Cl(A))$
- (iii) α -open set [20] if $A \subseteq Int(Cl(Int(A)))$
- (iv) semi-preopen set [21] if $A \subseteq Cl(Int(Cl(A)))$

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Definition: 2.2 A subset A of a space (X, τ) is called

- (i) a generalized semi-preclosed (briefly gsp-closed) set [17] if $spCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) . The complement of gsp-closed set is called gsp-open set;
- (ii) \ddot{g} -closed set [15] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X, τ) . The complement of \ddot{g} -closed set is called \ddot{g} -open set;
- (iii) a A -closed set [16] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is \ddot{g} -open in (X, τ) . The complement of A -closed set is called A -open set;
- (iv) a B -closed set [16] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is A -open in (X, τ) . The complement of B -closed set is called B -open set;
- (v) a $\approx g$ -closed set [16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is B -open in (X, τ) . The complement of $\approx g$ -closed set is called $\approx g$ -open set.

Definition 2.3: A subset A of a space X is said to be regular closed [3] if $A = Cl(Int(A))$. The complement of regular closed set is called regular open set;

Definition 2.4: A subset A of a space X is said to be b -closed [4] if $Int(Cl(A)) \cap Cl(Int(A)) \subseteq A$. The complement of b -closed set is called b -open set;

Definition 2.5: A subset A of a space X is said to be

1. generalized semi-closed (briefly gs-closed) [8] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X , The complement of gs-closed set is called gs-open set;
2. semi-generalized closed (briefly sg-closed) [6] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X , The complement of sg-closed set is called sg-open set;
3. regular-generalized closed (briefly rg-closed) [7] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open in X , The complement of rg-closed set is called rg-open set;
4. generalized pre-closed (briefly gp-closed) [9] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X , The complement of gp-closed set is called gp-open set;
5. generalized α -closed (briefly $g\alpha$ -closed) [10] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in X , The complement of $g\alpha$ -closed set is called $g\alpha$ -open set;
6. α -generalized closed (briefly αg -closed) [11] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X , The complement of αg -closed set is called αg -open set;

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7. generalized b-closed (briefly gb-closed) [12] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X , The complement of gb -closed set is called gb- open set;

8. δ -generalized closed (briefly δg -closed) [13] if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X . The complement of δg -closed set is called δg -open set;

Definition 2.6: A generalized closed (briefly g-closed) set [5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) . The complement of g-closed set is called g-open set.

Definition 2.7: A subset A of a space X is said to be regular b-closed (briefly rb-closed) [2] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is b-open in X . The complement of regular b-closed set is called regular b-open set.

3. SD Closed set

Definition 3.1: A Set A is said to be SD closed set if

(i) A is generalized closed set[22] and (ii) A is regular b-closed[23]

Example:3.1

Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{b\}\}$, $\tau^c = \{\phi, X, \{a,c\}\}$ and $U=X$.

SD closed set = $\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \phi, X\}$.

Proposition:3.1 Let (X, τ) be a topological space. Prove that if A is regular closed set then A is SD closed set in X .

Converse is need not be true. ($U=X$).

Proof: Let A be a regular closed then $A=Cl(Int(A))$.

$\Rightarrow A$ is regular b-closed set. ... (1)

Since every regular closed set is regular b-closed and also Every regular closed set is regular open set.

Now Every regular open is open. Therefore $Int(A) = A$.

$A = Cl(Int(A)) = Cl(A)$.

$\Rightarrow Cl(A) = A$.

$\Rightarrow Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

Then A is generalized closed (g-closed). ... (2)

From (1) and (2) A is SD closed set.

Converse :

Example:

Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{b\}\}$, $\tau^c = \{\phi, X, \{a,c\}\}$ and $U = X$

(i) Regular closed set = $\{\{a\}, \{b,c\}, \phi, X\}$

(ii) SD closed set = $\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \phi, X\}$

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Here the element $\{c, a\}$ is in SD closed set but not in regular closed set.

Lemma: Let (X, τ) be a topological space. Prove that regular closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.2 Let (X, τ) be a topological space. Prove that b-closed set is SD closed set in X. Converse is need not be true. (U=X)

Proof: Let A be b-closed set then $Int(Cl(A)) \cap Cl(Int(A)) \subset A$.

Since Every b-closed set is a subset of regular b-closed set, A is regular b-closed set. ... (1)

we know that $Int(A) \subseteq A \subseteq Cl(A)$

$\Rightarrow Int(A) \subseteq Cl(A)$

$\Rightarrow Cl(Int(A)) \subseteq Cl(Cl(A)) \subseteq Cl(A) \subseteq A$

$\Rightarrow Cl(A) \subseteq A$

$\Rightarrow Cl(A) \subseteq U$ since $A \subset U$

$\Rightarrow Cl(A) \subseteq U$ whenever $A \subset U$ and U is open in X.

Therefore A is g-closed set. ... (2)

From (1) and (2) A is SD closed set.

Converse:

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{a, b\} \}$, $\tau^c = \{ \phi, X, \{c\} \}$ and $U = X$

(i) b - closed set = $\{ \{a, c\}, \{b, c\}, \{a\}, \{c\}, \{b\}, \phi, X \}$

(ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{a, b\}$ is in SD closed set but not in b-closed set.

Lemma: Let (X, τ) be a topological space. Prove that b-closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.3 Let (X, τ) be a topological space. Prove that generalized semi closed set (gs-closed) is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{b\}, \{b, c\} \}$, $\tau^c = \{ \phi, X, \{a\}, \{a, c\} \}$ and $U = X$

(i) gs - closed set = $\{ \{a, c\}, \{a\}, \phi, X \}$

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(ii) SD closed set = $\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \phi, X\}$

Here the element $\{b\}$ is in SD closed set but not in gs-closed set.

Lemma: Let (X, τ) be a topological space. Prove that gs-closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.4 Let (X, τ) be a topological space. Prove that generalized semi open set (gs open) is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{b\}, \{b,c\}, \{a,b\}\}$, $\tau^c = \{\phi, X, \{a\}, \{c\}, \{a,c\}\}$ and $U = X$

(i) gs - open set = $\{\{a,b\}, \{b\}, \{b,c\}, \phi, X\}$

(ii) SD closed set = $\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \phi, X\}$

Here the element $\{b\}$ is in both SD closed set and gs-open set. The element $\{c\}$ is in SD closed set but not in gs-open set.

Lemma: Let (X, τ) be a topological space. Prove that gs-open set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.5 Let (X, τ) be a topological space. Prove that semi open set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{b\}, \{a,c\}\}$, $\tau^c = \{\phi, X, \{b\}, \{a,c\}\}$ and $U = X$

(i) semi open set = $\{\{b\}, \{a,c\}, \phi, X\}$

(ii) SD closed set = $\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \phi, X\}$

Here the element $\{b\}$ is in both SD closed set and semi open set. The element $\{c\}$ is in SD closed set but not in semi open set.

Lemma: Let (X, τ) be a topological space. Prove that semi open set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.6 Let (X, τ) be a topological space. Prove that semi closed set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$, $\tau^c = \{\phi, X, \{c\}, \{a,c\}, \{b,c\}\}$ and $U = X$

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- (i) semi closed set = $\{ \{c\}, \{a, c\}, \{b, c\}, \phi, X \}$
(ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{c\}$ is in both SD closed set and semi closed set. The element $\{a\}$ is in SD closed set but not in semi closed set.

Lemma: Let (X, τ) be a topological space. Prove that semi closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.7 Let (X, τ) be a topological space. Prove that semi generalized closed (sg-closed) set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\} \}$, $\tau^c = \{ \phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\} \}$ and $U = X$

- (i) sg-closed set = $\{ \{b\}, \{c\}, \{a, b\}, \{b, c\}, \phi, X \}$
(ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{c\}$ is in both SD closed set and sg-closed set. The element $\{a, c\}$ is in SD closed set but not in sg-closed set.

Lemma: Let (X, τ) be a topological space. Prove that sg-closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.8 Let (X, τ) be a topological space. Prove that semi generalized open (sg-open) set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{b\}, \{c\}, \{a, c\}, \{b, c\} \}$, $\tau^c = \{ \phi, X, \{a\}, \{b\}, \{a, c\}, \{a, b\} \}$ and $U = X$

- (i) sg-open set = $\{ \{b\}, \{c\}, \{a, c\}, \phi, X \}$
(ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{c\}$ is in both SD closed set and sg-open set. The element $\{a\}$ is in SD closed set but not in sg-open set.

Lemma: Let (X, τ) be a topological space. Prove that sg-open set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.9 Let (X, τ) be a topological space. Prove that generalized pre closed (gp-closed) set is SD closed set in X. Converse is need not be true. (U=X)

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Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{b\}, \{c\}, \{b, c\}, \{a, c\} \}$, $\tau' = \{ \phi, X, \{a\}, \{b\}, \{a, c\}, \{a, b\} \}$ and $U = X$

- (i) gp-closed set = $\{ \{b\}, \{a, c\}, \{a, b\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{b\}$ is in both SD closed set and gp-closed set. The element $\{b, c\}$ is in SD closed set but not in gp-closed set.

Lemma: Let (X, τ) be a topological space. Prove that gp-closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.10 Let (X, τ) be a topological space. Prove that generalized pre open (gp-open) set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{b\}, \{c\}, \{b, c\}, \{a, c\} \}$, $\tau' = \{ \phi, X, \{a\}, \{b\}, \{a, c\}, \{a, b\} \}$ and $U = X$

- (i) gp-open set = $\{ \{b\}, \{c\}, \{a, b\}, \{b, c\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{a, b\}$ is in both SD closed set and gp-open set. The element $\{a\}$ is in SD closed set but not in gp-open set.

Lemma: Let (X, τ) be a topological space. Prove that gp-open set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.11 Let (X, τ) be a topological space. Prove that pre open set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{b\} \}$, $\tau' = \{ \phi, X, \{a, c\} \}$ and $U = X$

- (i) Pre open set = $\{ \{a\}, \{b\}, \{a, b\}, \{b, c\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{a, b\}$ is in both SD closed set and pre open set. The element $\{c\}$ is in SD closed set but not in pre open set.

Lemma: Let (X, τ) be a topological space. Prove that pre open set is SD open set in X. Converse is need not be true. (U=X)

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Proposition: 3.12 Let (X, τ) be a topological space. Prove that pre closed set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{b\}, \{b, c\} \}$, $\tau' = \{ \phi, X, \{a\}, \{a, c\} \}$ and $U = X$

- (i) pre closed set = $\{ \{a\}, \{c\}, \{a, c\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{a\}$ is in both SD closed set and pre closed set. The element $\{b, c\}$ is in SD closed set but not in pre closed set.

Lemma: Let (X, τ) be a topological space. Prove that pre closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.13 Let (X, τ) be a topological space. Prove that generalized α -closed ($g\alpha$ -closed) set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{a, b\} \}$, $\tau' = \{ \phi, X, \{c\} \}$ and $U = X$

- (i) $g\alpha$ -closed set = $\{ \{c\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{c\}$ is in both SD closed set and $g\alpha$ -closed set. The element $\{b, c\}$ is in SD closed set but not in $g\alpha$ -closed set.

Lemma: Let (X, τ) be a topological space. Prove that $g\alpha$ -closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.14 Let (X, τ) be a topological space. Prove that α -open set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{b\}, \{b, c\} \}$, $\tau' = \{ \phi, X, \{a\}, \{a, c\} \}$ and $U = X$

- (i) α -open set = $\{ \{b\}, \{a, b\}, \{b, c\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{b\}$ is in both SD closed set and α -open set. The element $\{b, c\}$ is in SD closed set but not in α -open set.

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Lemma: Let (X, τ) be a topological space. Prove that α -open set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.15 Let (X, τ) be a topological space. Prove that α -closed set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{b\}, \{a, b\}, \{b, c\} \}$, $\tau' = \{ \phi, X, \{a\}, \{c\}, \{a, c\} \}$ and $U = X$

- (i) α -closed set = $\{ \{a\}, \{c\}, \{a, c\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{c\}$ is in both SD closed set and α -closed set. The element $\{a, b\}$ is in SD closed set but not in α -closed set.

Lemma: Let (X, τ) be a topological space. Prove that α -closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.16 Let (X, τ) be a topological space. Prove that α -generalized closed (αg -closed) set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{a\}, \{b\}, \{a, b\} \}$, $\tau' = \{ \phi, X, \{b, c\}, \{a, c\}, \{c\} \}$ and $U = X$

- (i) αg -closed set = $\{ \{c\}, \{b, c\}, \{a, c\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{c\}$ is in both SD closed set and αg -closed set. The element $\{a, b\}$ is in SD closed set but not in αg -closed set.

Lemma: Let (X, τ) be a topological space. Prove that αg -closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.17 Let (X, τ) be a topological space. Prove that α -generalized open (αg -open) set is SD closed set in X. Converse is need not be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\} \}$, $\tau' = \{ \phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\} \}$ and $U = X$

- (i) αg -open set = $\{ \{a\}, \{c\}, \{a, b\}, \{a, c\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

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Here the element $\{a\}$ is in both SD closed set and αg -open set. The element $\{b\}$ is in SD closed set but not in αg -open set.

Lemma: Let (X, τ) be a topological space. Prove that αg -open set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.18 Let (X, τ) be a topological space. Prove that generalized b-closed (gb closed) set is SD closed set in X. Converse is need not be true. (U=X)

Proof: Let A be a gb closed set then $bcl(A) \subset U$ whenever $A \subset U$ and U is open in X.

$$\Rightarrow bcl(A) \subset U$$

$$\Rightarrow cl(A) \subset U$$

$$\Rightarrow A \text{ is g-closed set. ... (1)}$$

Since every regular closed set is b-closed set.

$$\Rightarrow rcl(A) \subset bcl(A) \subset U$$

$$\Rightarrow rcl(A) \subset U \text{ whenever } A \subset U \text{ and } U \text{ is open in X.}$$

$$\Rightarrow A \text{ is regular b-closed set. ... (2)}$$

From (1) and (2) A is SD closed set.

Converse:

Example:

Let $X = \{a,b,c\}$, $\tau = \{ \phi, X, \{b\}, \{c\}, \{a,c\}, \{b,c\} \}$, $\tau^c = \{ \phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\} \}$ and $U = X$

(i) gb-closed set = $\{ \{a\}, \{c\}, \{a,c\}, \phi, X \}$

(ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \phi, X \}$

Here the element $\{c\}$ is in both SD closed set and gb-closed set. The element $\{a,b\}$ is in SD closed set but not in gb-closed set.

Lemma: Let (X, τ) be a topological space. Prove that gb-closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.19 Let (X, τ) be the topological space. Prove that semi pre open (sp open) set is a SD closed set in X. converse is need not to be true. (U=X)

Example:

Let $X = \{a,b,c\}$, $\tau = \{ \phi, X, \{b\}, \{b,c\} \}$, $\tau^c = \{ \phi, X, \{a\}, \{a,c\} \}$ and $U = X$

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- (i) sp open set = $\{\{b\}, \{a,b\}, \{b,c\}, \phi, X\}$
(ii) SD closed set = $\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \phi, X\}$

Here the element $\{b\}$ is in both SD closed set and sp open set. The element $\{a\}$ is in SD closed set but not in sp open set.

Lemma: Let (X, τ) be a topological space. Prove that semi pre open set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.20 Let (X, τ) be the topological space. Prove that semi pre closed (sp closed) set is a SD closed set in X. converse is need not to be true. (U=X)

Proof: Let A be a semi pre closed set.

Since every semi pre closed set is generalized semi pre closed set (gsp closed)

\Rightarrow A is gsp closed set

Since every gsp closed set is b-closed set then by proposition 3.2.

Hence A is SD closed set in X.

Converse:

Example:

Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{b\}, \{c\}, \{a,b\}, \{b,c\}\}$, $\tau^f = \{\phi, X, \{a\}, \{c\}, \{a,b\}, \{a,c\}\}$ and $U = X$

- (i) sp closed set = $\{\{a\}, \{c\}, \{a,b\}, \{a,c\}, \phi, X\}$
(ii) SD closed set = $\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \phi, X\}$

Here the element $\{b\}$ is in SD closed set but not in sp closed set.

Lemma: Let (X, τ) be a topological space. Prove that semi pre closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.21 Let (X, τ) be a topological space. Prove that generalized semi pre closed (gsp-closed) set is a SD closed in X. Converse is need not to be true (U=X)

Example:

Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{b\}\}$, $\tau^f = \{\phi, X, \{a,c\}\}$ and $U=X$

- (i) gsp-closed set - $\{\{a\}, \{c\}, \{a,c\}, \{b,c\}, \phi, X\}$
(ii) SD closed set = $\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \phi, X\}$

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Here the element $\{a\}$ is in both SD closed set and gsp-closed set. The element $\{a, b\}$ is in SD closed set but not in gsp-closed set.

Lemma: Let (X, τ) be a topological space. Prove that gsp-closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.22 Let (X, τ) be a topological space. Prove that generalized semi pre open (gsp-open) set is a SD closed in X. Converse is need not to be true (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{a, b\} \}$, $\tau^c = \{ \phi, X, \{c\} \}$ and $U=X$

- (i) gsp-open set - $\{ \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{a\}$ is in both SD closed set and gsp-open set. The element $\{c\}$ is in SD closed set but not in gsp-open set.

Lemma: Let (X, τ) be a topological space. Prove that gsp-open set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.23 Let (X, τ) be a topological space. Prove that B-closed set is a SD closed in X. Converse is need not to be true. (U=X)

Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{a, b\} \}$, $\tau^c = \{ \phi, X, \{c\} \}$ and $U=X$

- (i) B-closed set - $\{ \{c\}, \phi, X \}$
- (ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{c\}$ is in both SD closed set and B-closed set. The element $\{a, b\}$ is in SD closed set but not in B-closed set.

Lemma: Let (X, τ) be a topological space. Prove that B-closed set is SD open set in X. Converse is need not be true. (U=X)

Proposition: 3.24 Let (X, τ) be a topological space. Prove that B-open set is a SD closed in X. Converse is need not to be true. (U=X)

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Example:

Let $X = \{a, b, c\}$, $\tau = \{ \phi, X, \{b\}, \{b, c\} \}$, $\tau^c = \{ \phi, X, \{a\}, \{a, c\} \}$ and $U=X$

(i) B-open set - $\{ \{b\}, \{b, c\}, \phi, X \}$

(ii) SD closed set = $\{ \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \phi, X \}$

Here the element $\{b, c\}$ is in both SD closed set and B-open set. The element $\{a\}$ is in SD closed set but not in B-open set.

Lemma: Let (X, τ) be a topological space. Prove that B-open set is SD open set in X. Converse is need not be true. ($U=X$)

IV. PROPERTIES OF SD CLOSED SETS

Let (X, τ) be a topological space. Let A and B be the SD closed set,

1. $\text{Int}(\text{SD closed set}) \neq \text{Cl}(\text{SD closed set})$
2. $\text{Int}(\text{SD closed set}) \subseteq \text{Cl}(\text{SD closed set})$
3. $A \cap B = \rho(A)$
4. $A \cup B = \rho(A)$
5. $A \cup B = A \cap B$
6. $(A \cap B)^c = \phi$
7. $(A \cup B)^c = \phi$
8. $(A \cap B)^c = (A \cup B)^c$
9. $(A \cap B)^c = A^c \cup B^c$
10. $(A \cup B)^c = A^c \cap B^c$

Remark: The above properties are true for SD open sets.

V. CONCLUSION

The following sets are subset of SD closed set and SD open set

1. regular closed set, 2. b closed set, 3. gs closed set, 4. gs open, 5. semi open set, 6. semi closed set, 7. sg closed set, 8. sg open set, 9. gp closed set, 10. gp open set, 11. pre open set, 12. pre closed set, 13. α closed set, 14. α -open set, 15. α -closed set, 16. ag closed set, 17. ag open set, 18. gb closed set, 19. sp open set, 20. sp closed set, 21. gsp closed set, 22. gsp open set, 23. B-closed set, 24. B-open set.

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