δmk-Closed Set and δmk-Open Set in Topological Spaces

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ABSTRACT: In this paper, we introduce and investigate the notation of δmk-closed set and δmk-open set. We obtain fundamental properties of δmk-closed set, δmk-open set and discuss the relation between δmk-closed set, δmk-open set and other related sets.

KEYWORDS: δmk-closed set, δmk-open set.

I. INTRODUCTION

The concept of generalized closed sets introduced by Levine [1] plays a significant role in general topology. After the introduction of generalized closed sets many research papers were published which deal with different types of generalized closed sets. Y. Gnanambal [2] introduced the concept of gpr-closed set and investigated its basic properties. H. Maki et al. [3] defined the concept of gp-closed set in topological spaces and established results related to it. These concepts motivated us to define a new class of sets called the delta generalized pre-closed sets. Throughout this paper, (X, τ) represents topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), we denote the closure of A, the interior of A and complement of A as Cl(A), int(A) and A' respectively. The modified forms of generalized closed sets and δα-closed sets were studied by K. Balachandran, P. Sundaram and H. Maki [4]. In 2008, S. Jafari, T. Noiri, N. Rajesh and M.L. Thivagar [5] introduced the concept of δβ-closed sets and their properties. In this paper, we introduce new classes of sets called δβ-closed sets for topological spaces.

II. PRELIMINARIES

2.1. A subset A of a topological space X is called,
pre-open [6] if \( A \subseteq \text{int}(c\text{l}(A)) \) and Pre-closed if \( \text{c\text{l}}(\text{int}(A)) \subseteq \text{c\text{l}}(\text{int}(A)). \)

2.2. A subset A of a topological space X is called,
\( \alpha - \text{open} \) [7] if \( A \subseteq \text{int}(\text{c\text{l}}(A)) \) and \( \alpha - \text{closed} \) if \( \text{c\text{l}}(\text{int}(A)) \subseteq A. \)

2.3. A subset A of a topological space X is called,
\( b - \text{open} \) [8] if \( A \subseteq \text{c\text{l}}(\text{int}(A)) \cap \text{int}(\text{c\text{l}}(A)) \) and \( b - \text{closed} \) if \( \text{c\text{l}}(\text{int}(A)) \cap \text{int}(\text{c\text{l}}(A)) \subseteq A. \)

2.4. A subset A of a topological space X is called,
\( \text{regular - open} \) [9] if \( A = \text{c\text{l}}(\text{int}(A)) \) and \( \text{regular - closed} \) if \( A = \text{c\text{l}}(\text{int}(A)). \)

2.5. A subset A of a topological space X is called,
\( \text{semi - open} \) [10] if \( A \subseteq \text{cl}(\text{int}(A)) \) and \( \text{semi - closed} \) if \( \text{int}(\text{c\text{l}}(A)) \subseteq A. \)
2.6. A subset A of a topological space X is called,
   Semi preopen [11] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and semi preclosed if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.

2.7. A subset A of a topological space X is called,
   $\delta$-closed [12] if $A = cl_\delta(A)$ where $cl_\delta(A) = \{x \in X : \text{cl}(\text{int}(U)) \cap A \neq \emptyset, \forall U \in \tau \text{ and } x \in U\}$. The Complement of $\delta$-closed is $\delta$-open.

2.8. Let $(X, \tau)$ be a topological space and $A \subseteq X$.
   The pre-interior of A, denoted by $\text{pint}(A)$, is the union of all preopen subsets of A.
   The pre-closure of A, denoted by $\text{pcl}(A)$, is the intersection of all preclosed sets containing A.

2.9. A subset A of a topological space X is called,
   Generalized closed (briefly, g-closed) [1] if $\text{cl}(A) \subseteq G$ whenever $A \subseteq G$ and G is open in X. The complement of g-closed is g-open.

2.10. A subset A of a topological space X is called,
   Generalized $\alpha$-closed (briefly, $g\alpha$-closed ) [113] if $\text{acl}(A) \subseteq G$ whenever $A \subseteq G$ and G is $\alpha$-open in X. The complement of g-$\alpha$-closed is g-open.

2.11. A subset A of a topological space X is called,
   $\alpha$-generalized closed (briefly, $g\alpha$-closed ) [14] if $\text{acl}(A) \subseteq G$ whenever $A \subseteq G$ and G is Open in X. The complement of g-$\alpha$-closed is g-open.

2.12. A subset A of a topological space X is called,
   Generalized pre-closed (briefly, gp-closed ) [15] if $\text{pcl}(A) \subseteq G$ whenever $A \subseteq G$ and G is Open in X. The complement of g-pc-closed is gp-open.

2.13. A subset A of a topological space X is called,
   Generalized prerregular closed (briefly, gpr-closed ) [2] if $\text{pcl}(A) \subseteq G$ whenever $A \subseteq G$ and G is regular Open in X. The complement of gpr-closed is gpr-open.

2.14. A subset A of a topological space X is called,
   $\delta$-generalized closed (briefly, $g\delta$-closed ) [16] if $\text{cl}(A) \subseteq G$ whenever $A \subseteq G$ and G is Open in X. The complement of g-$\delta$-closed is g-$\delta$-open.

2.15. A subset A of a topological space X is called,
   Pregeneralized closed (briefly, pg-closed ) [3] if $\text{pcl}(A) \subseteq G$ whenever $A \subseteq G$ and G is preopen in X. The complement of pg-closed is pg-open.

2.16. A subset A of a topological space X is called,
   Generalized semi-preclosed (briefly, gsp-closed ) [17] if $\text{spcl}(A) \subseteq G$ whenever $A \subseteq G$ and G is Open in X. The complement of gsp-closed is gsp-open.

Example: 2.1

Let $X =\{a,b,c\}, \tau =\{\emptyset,x\{a\}\{b\}\{a,b\}\} \tau^c =\{\emptyset,x,\{b\}\{ac\}\{bc\}\} \text{ then } \delta\text{GPC}(X) =\{\emptyset,x\{c\}\{a,c\}\{b,c\}\}.$

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2.19. Let \((X, \tau)\) be a topological space. A subset \(A \subseteq X\) is said to be \(\tilde{g}\)-closed set[20] if \(cl(A) \subseteq G\) whenever \(A \subseteq G\) and \(G\) is semi o-open in \(X\). The complement of \(\tilde{g}\)-closed is \(\tilde{g}\)-open.

**Example:** Let \(X=\{a,b,c\}, \tau=\{\emptyset,\{a\},\{a,b\},\{a,c\}\}\) and \(\tau^c=\{\{b\},\{b,c\}\}\) then \(\tilde{g}=\{\emptyset,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}\).

### III. MK-CLOSED SET

**Definition:** Let \((X, \tau)\) be a topological space. A subset \(A \subseteq X\) is said to be MK-closed set if

(i) \(\delta\)-gp-closed,

(ii) \(\tilde{g}\)-closed.

The complement of MK-closed is MK-open.

**Example:** Let \(X=\{a,b,c\}, \tau=\{\emptyset,\{a\},\{a,b\},\{a,c\}\}\), \(\tilde{g}^c=\{\emptyset,\{a\},\{a,b\},\{a,c\}\}\) then \(MK\)-closed = \(\{\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{a,c\}\}\).

**Theorem:** Let \((x, \tau)\) be a Topological Space. Prove that Every closed set is \(MK\)-closed set in \(X\).

But not conversely. (U=X)

**Proof:**

Let A be a Closed set.

then \(Cl(A)=A\),

Take \(A \subseteq U\) and U is semiopen in \(X\).

Then \(Cl(A) \subseteq U\)

\(\therefore\) A is \(\tilde{g}\)-closed. Every Semi open is \(\delta\)-open in \(X\). Therefore U is \(\delta\)-open in \(X\).

\(Cl(A) \subseteq Pcl(A)\) and A is \(\tilde{g}\)-closed.

Therefore A is Preclosed in \(X\).

Therefore A is MK-closed set.

**Example:** Let \(X=\{a,b,c\}, \tau=\{\emptyset,\{a\},\{a,b\}\}\) and \(\tau^c=\{\emptyset,\{a\},\{b\},\{ac\}\}\), \(\delta MK\)-closed set = \(\{\emptyset,\{a\},\{b\},\{ac\}\}\), Preclosed = \(\{\emptyset,\{a\},\{c\}\}\).

\(\tilde{g}\)-closed = \(\{\emptyset,\{a\},\{b\},\{ac\}\}\), \(\tilde{g}\)-closed and \(\delta MK\)-closed.

Therefore closed set is \(\delta MK\)-closed set.

The converse of the above theorem is not true.
Example: 3.3

Let $X = \{a, b, c\}, \tau = \{\{c\}, \{a, b\}, \emptyset, X\}$ and $\tau^c = \{\{c\}, \{a, b\}, \emptyset, X\}$.

$\delta\text{mk-closed set} = \{\emptyset, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$\text{closed set} = \{\emptyset, \{c\}, \{a, c\}\}$. The element $\{a\}$ is in $\delta\text{mk-closed}$ but not in $\text{closed set}$

Theorem 3.2

Let $(x, \tau)$ be a Topological Space. Prove that Every open set is $\delta\text{mk-open set}$ in $X$ $(U = X)$. But not conversely.

Example: 3.4

Let $X = \{a, b, c\}, \tau = \{\emptyset, \{c\}, \{b, c\}\}$, $\tau^c = \{\emptyset, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}$

$\delta\text{mk-open set} = \{\emptyset, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}$

The element $\{a, b\} \in \delta\text{mk-open set}$ but not in open set.

Theorem 3.3

Let $(x, \tau)$ be a Topological Space. Prove that Regular Closed Set is $\delta\text{mk-closed}$ set in $X$. But not conversely. ($U = X$)

Proof:

Let $A$ be a Regular Closed set in $X$.

Then $A = \text{Cl}(\text{Int}(A))$.

And also $A$ is Regular open in $X$.

Therefore $\text{Int}(A) = A$

$\Rightarrow A = \text{Cl}(\text{Int}(A)) = \text{Cl}(A)$

$\Rightarrow A = \text{Cl}(A)$

Take $A \subseteq U$ and $U$ is semiopen in $X$.

Then $\text{Cl}(A) \subseteq U$

$\therefore A$ is $\delta\text{-closed}$.

Every Semi open is $\delta$-open in $X$. Therefore $U$ is $\delta$-open in $X$.

$\text{Cl}(A) \subseteq \text{Pcl}(A) \subseteq A \subseteq U$

$A$ is $\delta\text{-closed}$.

Therefore $A$ is $\delta\text{-generalized preclosed}$ in $X$.

Therefore $A$ is $\delta\text{mk-closed}$ set in $X$.

Example: 3.5

Let $X = \{a, b, c\}, \tau = \{\emptyset, \{c\}, \{b, c\}, \{a, c\}\}$ and $\tau^c = \{\emptyset, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}$.

Regular closed $= \{\emptyset, \{a, c\}, \{b\}\}$, Regular open $= \{\emptyset, \{a, c\}, \{b\}\}$ and $\delta\text{mk-closed set} = \{\emptyset, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}$.

Here Regular closed set = Regular open set
The converse of the above theorem need not to be true.

In the above example, element \{a\} in δmk-closed set but not in Regular Closed Set.

**Theorem 3.4**

Let \((x, \tau)\) be a Topological Space. Prove that Regular Open Set is δmk-open set in \(X(U=X)\). But not conversely.

**Example: 3.6**

Let \(X=\{a,b,c\}\), \(\tau=\{\emptyset, x, \{b\}, \{c\}, \{b,c\}\}\) and \(\tau^c=\{\emptyset, x, \{a,c\}, \{a,b\}, \{a\}, \{c\}\}\).

Regular open= \{\emptyset, x, \{a\}, \{b\}, \{c\}\} and δmk-Open set= \{\emptyset, x, \{a\}, \{b\}, \{c\}, \{b\}\}.

The element \{b\} is in δmk-Open set but not in Regular open.

**Theorem: 3.5**

Let \((x, \tau)\) be a Topological Space. Prove that δmk-closed Set is Semi closed set in \(X\). But not conversely. (U=X)

**Proof:**

Let \(A\) be a δmk- Closed set in \(X\).

Then \(A\) is \(\tilde{g}\) closed and \(\delta\)-generalized preclosed.

\[ \Rightarrow Cl(A) \subseteq U \text{ and } Pcl(A) \subseteq U \text{ whenever } A \subseteq U \text{ and } U \text{ is Semiopen and } \delta\text{-open in } X. \]

\[ \Rightarrow Cl(A) \subseteq Pcl(A) \subseteq U \]

\[ \Rightarrow Cl(A) \subseteq U \]

Every \(\delta\)-open set is open set in \(X\).

\[ \text{Int(cl(A))} \subseteq \text{int } U = U \]

\[ \Rightarrow \text{Int(Cl(A))} \subseteq U \]

\[ \Rightarrow \text{Int(Cl(A))} \subseteq A. \text{ since } A \subseteq U \]

Therefore \(A\) is Semi closed in \(X\).

The converse of the above theorem need not to be true.

**Example: 3.7**

Let \(X=\{a,b,c\}, \quad \tau=\{\emptyset, x, \{b\}, \{c\}, \{b,c\}\}\) and \(\tau^c=\{\emptyset, x, \{a,c\}, \{a,b\}\}\), Semi closed= \{\emptyset, x, \{a\}, \{b\}, \{c\}, \{a\}, \{c\}\} and δmk-closed set= \{\emptyset, x, \{a\}, \{a,b\}\}.

The elements \{b\}, \{c\} in Semi closed set but not in δmk- closed set.

**Theorem: 3.6**

Let \((x, \tau)\) be a Topological Space. Prove that δmk-open Set is Semi open set in \(X\).

But not conversely. (U=X)
Example: 3.8

Let $X = \{a, b, c\}$, $\tau = \{\phi, x, \{b\}, \{c\}, \{b, c\}\}$ and $\tau^c = \{\phi, x, \{a\}, \{a, c\}, \{a, b\}\}$. Semi Open = $\{\phi, x, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $\delta$mk-Open set = $\{\phi, x, \{b\}, \{c\}, \{b, c\}\}$.

The elements $\{a, c\}$ in Semi open set but not in $\delta$mk-open set.

Theorem: 3.7

Let $(x, \tau)$ be a Topological Space. Prove that Every $\delta$mk-closed set is generalized closed set. But not conversely. ($U = X$)

Proof:

Let $A$ be a $\delta$mk-closed set in $X$.

Then $A$ is $\bar{g}$ closed and $\delta$-generalized preclosed.

$\Rightarrow \text{Cl}(A) \subseteq U$ and $\text{Pcl}(A) \subseteq U$ Whenever $A \subseteq U$ and $U$ is semiopen and $\delta$-open in $X$.

$\Rightarrow \text{Cl}(A) \subseteq \text{Pcl}(A) \subseteq U$

Every $\delta$-open set is open set in $X$.

Then $A \subseteq U$ and $U$ is open in $X$.

Therefore $\text{Cl}(A) \subseteq U$ Whenever $A \subseteq U$ and $U$ is open in $X$.

Hence $A$ is generalized closed.

The converse of the above theorem need not to be true.

Example: 3.9

Let $X = \{a, b, c\}$, $\tau = \{\phi, x, \{b\}, \{bc\}\}$ and $\tau^c = \{\phi, x, \{a\}, \{ac\}\}$, generalized closed $= \{\phi, x, \{a\}, \{b\}, \{bc\}\}$ and $\delta$mk-closed set $= \{\phi, x, \{a\}, \{c\}, \{ac\}\}$

The element $\{b, c\}$ is in generalized closed set but not in $\delta$mk-closed set.

Theorem: 3.8

Let $(x, \tau)$ be a Topological Space. Prove that Every $\delta$mk-open set is generalized open set. But not conversely. ($U = X$).

Example: 3.10

Let $X = \{a, b, c\}$, $\tau = \{\phi, x, \{a\}, \{b\}, \{bc\}\}$ and $\tau^c = \{\phi, x, \{a\}, \{c\}, \{ac\}, \{b, c\}\}$, generalized Open $= \{\phi, x, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{bc\}\}$ and $\delta$mk-Open set $= \{\phi, x, \{a\}, \{b\}, \{b, c\}, \{a, b\}\}$

The element $\{c\}$ is in generalized open set but not in $\delta$mk-open set.

Properties:

If $A$ and $B$ are $\delta$mk-closed sets, then

1. $A \cap B \subseteq A \cup B$
2. \((A \cup B)^c = A^c \cup B^c\)

3. \(A^c \cap B^c = A \cap B^c\)

The above properties are true for \(\delta mk\)-Open set.

IV. CONCLUSION

1. The following sets are subset of \(\delta mk\)-closed set
   (i) \(\alpha\)-closed, (ii) Regular closed, (iii) \(g\alpha\)-closed, (iv) \(\alpha g\)-closed

2. The following sets are subset of \(\delta mk\)-open set
   (i) \(\alpha\)-open, (ii) Regular open, (iii) \(g\alpha\)-open, (iv) \(\alpha g\)-open.

3. \(\delta mk\)-closed set is subset of the following sets
   (i) Semi closed, (ii) Semi preclosed, (iii) \(g\)-closed, (iv) \(gsp\)-closed, (v) pre semiclosed.

4. \(\delta mk\)-open set is subset of the following sets
   (i) Semi open, (ii) Semi preopen, (iii) \(g\)-open, (iv) \(gsp\)-open, (v) pre semiopen.

REFERENCES