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Integer Triples in Arithmetic Progression through Diophantine Equation

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ABSTRACT. We search for two non-zero distinct integers a and d such that the triple $(a-d, a, a+d)$ forms an Arithmetic progression where in section 1 each of the expression $2a-d$ and $2a+d$ is a perfect square and $2a$ is (k^2+1) times a cube. And in section 2 each of the expression $2a-d$ and $2a+d$ is a perfect square and $2a = (k^2+1)$ times a quintic

KEYWORDS. Arithmetic Progression, Perfect square, Cube 2010 Mathematics subject Classification: 11D09, 11D25,

I. INTRODUCTION

Number theory, called the Queen of Mathematics, is a broad and diverse part of Mathematics that developed from the study of the integers. The foundations for Number theory as a discipline were laid by the Greek Mathematician Pythagoras and his disciples (known as Pythagoreans). One of the oldest branches in mathematics itself, is the Diophantine equations since its origins can be found in texts of the ancient Babylonians, Chinese, Egyptians, Greeks and so on [7-8]. Diophantine problems were first introduced by Diophantus of Alexandria who studied this topic in the algebra. The theory of Diophantine equations is a treasure house in which the search for many hidden relations and properties among numbers form a treasure hunt. In fact Diophantine problems dominated most of the celebrated unsolved mathematical problems. Certain Diophantine problems are neither trivial nor difficult to analyse. In this communication we have two sections. In section 1 we search for two non-zero distinct integers a and d such that $2a-d = \alpha^2$, $2a+d = \beta^2$, and $2a = (k^2+1)\gamma^3$ such that the triple $(a-d, a, a+d)$ forms an Arithmetic Progression. And in section 2 we search for two non-zero distinct integers a and d such that $2a-d = \alpha^2$, $2a+d = \beta^2$, and $2a = (k^2+1)\gamma^5$ such that the triple $(a-d, a, a+d)$ forms an Arithmetic Progression.

II. SECTION 1

METHOD OF ANALYSIS

The problem under consideration is equivalent to finding the values for a and d such that the following systems of equations is solvable.

$$2a-d = \alpha^2, \tag{1}$$

$$2a+d = \beta^2 \tag{2}$$

$$2a = (k^2+1)\gamma^3 \tag{3}$$

Eliminating a, d from (1), (2), (3)

$$\alpha^2 + \beta^2 = 2(k^2+1)\gamma^3 \tag{4}$$

APPROACH: 1

Assume $\gamma = p^2 + q^2$ (5)

Write 2 as $2 = \frac{(7+i)(7-i)}{25}$ (6)

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Substitute (5), (6) in (4) and use method of factorization

$$(\alpha+i\beta)(\alpha-i\beta) = \frac{(7+i)(7-i)}{25}(k+i)(k-i)(p+iq)^3(p-iq)^3$$

$$= \frac{1}{25} \{ [(7k-1)(p^3-3pq^2)-(k+7)(3p^2q-q^3)] + i[(7k-1)(3p^2q-q^3) + (k+7)(p^3-3pq^2)] \}$$

$$[(7k-1)(p^3-3pq^2)-(k+7)(3p^2q-q^3)] - i[(7k-1)(3p^2q-q^3) + (k+7)(p^3-3pq^2)] \}$$

Equating real and imaginary parts on both the sides

$$\alpha = \frac{1}{5} [(7k-1)(p^3-3pq^2)-(k+7)(3p^2q-q^3)]$$

$$\beta = \frac{1}{5} [(7k-1)(3p^2q-q^3) + (k+7)(p^3-3pq^2)]$$

Since we should find integer solution so substitute $p=5P, q=5Q$

$$\alpha = 25 [(7k-1)(P^3-3PQ^2)-(k+7)(3P^2Q-Q^3)]$$

$$\beta = 25 [(7k-1)(3P^2Q-Q^3) + (k+7)(P^3-3PQ^2)]$$

$$\gamma = 25(P^2+Q^2)$$

Substitute γ in (3) $2a = (k^2+1) 25^3(P^2+Q^2)^3$

$$2a = (k^2+1) 25^3(P^2+Q^2)^3 \tag{7}$$

From (2) $d = 25[(7k-1)(3P^2Q-Q^3) + (k+7)(P^3-3PQ^2)]^2 - (k^2+1) 25^3(P^2+Q^2)^3$ (8)

$$2a-d = 2(k^2+1) 25^3(P^2+Q^2)^3 - 25[(7k-1)(3P^2Q-Q^3) + (k+7)(P^3-3PQ^2)]^2$$

$$= 625 \{ ((k+7)^2 + (7k-1)^2) (P^2+Q^2)^3 - (7k-1)^2 (3P^2Q-Q^3)^2 - (k+7)^2 (P^3-3PQ^2)^2 - 2(7k-1)(3P^2Q-Q^3)(k+7)(P^3-3PQ^2) \}$$

$$= 625 \{ (k+7)^2 (3P^2Q-Q^3)^2 + (7k-1)^2 (P^3-3PQ^2)^2 - 2(7k-1)(3P^2Q-Q^3)(k+7)(P^3-3PQ^2) \}$$

$$= \alpha^2 \tag{9}$$

$$2a+d = 625[(7k-1)(3P^2Q-Q^3) + (k+7)(P^3-3PQ^2)]^2 = \beta^2 \tag{10}$$

$$2a = (k^2+1) 25^3(P^2+Q^2)^3 = (k^2+1) \gamma^3 \tag{11}$$

Therefore the set of triples forming A.P is

$$\left\{ \frac{3}{2}(k^2+1)25^3(P^2+Q^2)^3 - (25(7k-1)(3P^2Q-Q^3) + (k+7)(P^3-3PQ^2))^2, \frac{(k^2+1)}{2}25^3(P^2+Q^2)^3, \frac{1}{2}(k^2+1)25^3(P^2+Q^2)^3 + (25(7k-1)(3P^2Q-Q^3) + (k+7)(P^3-3PQ^2))^2 \right\}$$

From (9),(10),(11) It is seen that the required Triple in A.P Satisfying required condition

	k = 1, P = 3, Q = 5	k = 2, P = 1, Q = 3	k = 3, P = 4, Q = 6
a	614,125,000	39,062,500	625,000,000
d	223,360,000	58,765,000	1,054,000,000
α	31,700	-4400	-14,000
β	-38,100	-11,700	48,000
γ	850	250	500
2a-d	1,004,890,000	19,360,000	1,96,000,000
2a+d	1,451,610,000	136,890,000	2,304,000,000
2a	1,228,250,000	78,125,000	1,250,000,000

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Remark:

Instead of (6) we may have the following representation for 2

If $2=(1+i)(1-i)$ then proceeding as in approach 1 we get

$$\alpha = [(k-1)(p^3-3pq^2)-(k+1)(3p^2q-q^3)]$$

$$\beta = [(k+1)(p^3-3pq^2)+(k-1)(3p^2q-q^3)]$$

$$\gamma = 25(p^2+q^2)$$

$$2a = (k^2+1)25^3 (p^2+q^2)^3$$

$$d = \beta^2 - 2a$$

Here we have some Numerical Example

	k = 2 , p = 1 , q = 3	k = 3, p = 4 , q = 6	k = 1, p = 3 , q = 5
a	2500	703,040	39,104
d	4216	357,504	78,208
α	28	-1024	-20
β	-96	-1328	-396
γ	10	52	34
2a-d	784	1,048,576	400
2a+d	9216	1,763,584	156,816
2a	5000	140,6080	78,208

If $2 = \frac{(7+17i)(7-17i)}{13^2}$ then proceeding as in approach 1 we get

$$\alpha = 169 [(7k-17)(P^3-3PQ^2)-(17k+7)(3P^2Q-Q^3)]$$

$$\beta = 169[(17k+7)(P^3-3PQ^2)+(7k-17)(3P^2Q-Q^3)]$$

$$\gamma = 169(P^2+Q^2)$$

$$2a = (k^2+1)169^3 (P^2+Q^2)^3$$

$$d = \beta^2 - 2a$$

Here we have some Numerical Example

	k = 1 , P = 2 , Q = 3	k = 1, P = 3 , Q = 5	k = 2, P = 1 , Q = 3
A	$1.060449375 \times 10^{10}$	1.8971×10^{11}	$1.20670225 \times 10^{10}$
d	$1.950859105 \times 10^{10}$	2.9296×10^{11}	5116531784
α	41236	294060	137904
β	201786	-819988	-171028
γ	2197	5746	1690
2a-d	1700407700	8.647×10^{10}	$1.901751322 \times 10^{10}$
2a+d	4.0717×10^{10}	6.7238×10^{11}	2.925057×10^{10}
2a	$2.120899875 \times 10^{10}$	3.7942×10^{11}	2.4134045×10^{10}

If $2 = \frac{(17+7i)(17-7i)}{13^2}$ then proceeding as in approach 1 we get

$$\alpha = 169 [(17k-7)(P^3-3PQ^2)-(7k+17)(3P^2Q-Q^3)]$$

$$\beta = 169[(7k+17)(P^3-3PQ^2)+(17k-7)(3P^2Q-Q^3)]$$

$$\gamma = 169(P^2+Q^2)$$

$$2a = (k^2+1)169^3 (P^2+Q^2)^3$$

$$d = \beta^2 - 2a$$

Here we have some Numerical Example

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	k =1 , P =2 ,Q =3	k =2, P =1 ,Q =3	k = 3,P =1 ,Q =2
a	1.0604495×10^{10}	$1.20670225 \times 10^{10}$	3016755625
d	8157315956	$2.35418041 \times 10^{10}$	1279132946
α	-114244	-24336	-68952
β	-171366	-218348	85514
γ	2197	1690	845
2a-d	$1.305167404 \times 10^{10}$	592240900	4754378304
2a+d	$2.936630596 \times 10^{10}$	$4.76758491 \times 10^{10}$	7312644196
2a	2.120899×10^{10}	2.4134045×10^{10}	6033511250

If $2 = \frac{(1+7i)(1-7i)}{25}$ then proceeding as in approach 1 we get

$$\alpha = 25[(k-7)(P^3-3PQ^2)-(7k+1)(3P^2Q-Q^3)]$$

$$\beta = 25[(7k+1)(3P^2Q-Q^3)+(k-7)(P^3-3PQ^2)]$$

$$\gamma = 25(P^2+Q^2)$$

$$2a = (k^2+1)\gamma^3$$

$$d = \beta^2 - 2a$$

Here we have some Numerical Example

	k = 1,P =2 ,Q = 3	k =2, ,P =1 ,Q =3	k =3 ,P =2 ,Q = 4
a	34328125	39062500	625000000
d	42646250	-21875000	940240000
α	5100	10000	17600
β	10550	-7500	-46800
γ	325	250	500
2a-d	26010000	100000000	309760000
2a+d	111302500	56250000	2190240000
2a	68656250	78125000	1250000000

If $2 = \frac{(7+23i)(7-23i)}{17^2}$, then proceeding as in approach 1 we get

$$\alpha = 289 [(7k-23)(P^3-3PQ^2)-(23k+7)(3P^2Q-Q^3)]$$

$$\beta = 289[(23k+7)(P^3-3PQ^2)+(7k-23)(3P^2Q-Q^3)]$$

$$\gamma = 289(P^2+Q^2)$$

$$2a = (k^2+1)289^3 (P^2+Q^2)^3$$

$$d = \beta^2 - 2a$$

Here we have some Numerical Example

	k = 1,P =3 ,Q = 5	k =2, ,P =4 ,Q =5	k =1,P =2,Q =3
a	$9.48703012 \times 10^{11}$	$4.158963483 \times 10^{12}$	$9.699193505 \times 10^{10}$
d	$1.210410386 \times 10^{12}$	$7.000897595 \times 10^{12}$	$8.79233919 \times 10^{10}$
α	828852	1147619	134674
β	-1762900	3913927	440436
γ	9826	11849	3757
2a-d	6.8699563×10^{11}	$1.317029377 \times 10^{12}$	$1.81370863 \times 10^{10}$
2a+d	$3.10781641 \times 10^{12}$	$1.531882456 \times 10^{13}$	$1.939838701 \times 10^{11}$
2a	$1.897406024 \times 10^{12}$	$8.317926965 \times 10^{12}$	$1.060604782 \times 10^{11}$

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If $2 = \frac{(23+7i)(23-7i)}{17^2}$, then proceeding as in approach 1 we get

$$\begin{aligned} \alpha &= 289 [(23k-7)(P^3-3PQ^2)-(7k+23)(3P^2Q-Q^3)] \\ \beta &= 289[(7k+23)(P^3-3PQ^2)+ (23k-7) (3P^2Q-Q^3)] \\ \gamma &= 289(P^2+Q^2) \\ 2a &= (k^2+1)289^3 (P^2+Q^2)^3 \\ d &= \beta^2-2a \end{aligned}$$

	k = 1, P = 2, Q = 3	k = 2, P = 4, Q = 5	k = 3, P = 1, Q = 2
a	$9.699193505 \times 10^{10}$	4.1589634×10^{12}	$1.508598063 \times 10^{10}$
d	$2.153421942 \times 10^{10}$	$-6.811457936 \times 10^{12}$	702745694
α	-290734	-3.889651	171666
β	-357204	-1227383	-175712
γ	3757	11849	1445
2a-d	$8.452625878 \times 10^{10}$	$1.51293849 \times 10^{13}$	$2.946921556 \times 10^{10}$
2a+d	1.2759469×10^{10}	$1.506469029 \times 10^{12}$	$3.087470694 \times 10^{10}$
2a	$1.060604782 \times 10^{11}$	$8.317926965 \times 10^{12}$	$3.017196125 \times 10^{10}$

APPROACH:2

Introducing the Linear Transformation $\alpha = u+v$, $\beta = u-v$ (12)

substitute (5),(12) in (4) $(u^2+v^2)=(k^2+1)(p^2+q^2)^3$

Using method of factorization

$$\begin{aligned} (u+iv)(u-iv) &= (k+i)(k-i)(p+iq)^3(p-iq)^3 \\ &= k[(p^3-3pq^2)-(3p^2q-q^3)] + i[(p^3-3pq^2)+k(3p^2q-q^3)] [k(p^3-3pq^2)-(3p^2q-q^3)] - \\ & i[(p^3-3pq^2)+k(3p^2q-q^3)] \end{aligned}$$

Equating real and imaginary parts on both the sides

$$u = k(p^3-3pq^2)-(3p^2q-q^3), \quad v = (p^3-3pq^2)+k(3p^2q-q^3) \quad (13)$$

$$\alpha = (p^3-3pq^2)(k+1) + (3p^2q-q^3)(k-1)$$

$$\beta = (p^3-3pq^2)(k-1) - (3p^2q-q^3)(k+1)$$

From (3) $2a = (k^2+1)(p^2+q^2)^3$

From (2) $d = ((p^3-3pq^2)(k-1) - (3p^2q-q^3)(k+1))^2 - (k^2+1)(p^2+q^2)^3$

$$\begin{aligned} \text{Now } 2a-d &= 2(k^2+1)(p^2+q^2)^3 - [(p^3-3pq^2)(k-1) - (3p^2q-q^3)(k+1)]^2 \\ &= ((k+1)^2+(k-1)^2) ((3p^2q-q^3)^2 + (p^3-3pq^2)^2) - ((p^3-3pq^2)(k-1) - (3p^2q-q^3)(k+1))^2 + 2(p^3-3pq^2)(k-1) (3p^2q-q^3)(k+1) \\ &= ((p^3-3pq^2)(k+1) + (3p^2q-q^3)(k-1))^2 = \alpha^2 \end{aligned} \quad (14)$$

$$2a+d = ((p^3-3pq^2)(k-1) - (3p^2q-q^3)(k+1))^2 = \beta^2 \quad (15)$$

$$2a = (k^2+1) \gamma^3 \quad (16)$$

The set of triples forms A.P

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$$\left\{ \frac{3}{2}(k^2+1)(p^2+q^2)^3 - [(p^3-3pq^2)(k-1) - (3p^2q-q^3)(k+1)]^2, \frac{1}{2}(k^2+1)(p^2+q^2)^3, -\frac{1}{2}(k^2+1)(p^2+q^2)^3 + [(p^3-3pq^2)(k-1) - (3p^2q-q^3)(k+1)]^2 \right\}$$

From the equations (14),(15),(16) it is seen that the required triple in arithmetic progression is satisfying the required condition.

	k = 2, p = 3, q = 1	k = 1, p = 3, q = 5	k = 2, p = 4, q = 6
a	2,500	39,304	3,51,520
d	-1400	-78,208	-361,984
α	80	-396	-1032
β	-60	20	-584
γ	10	10	52
2a-d	6400	156,816	1,065,024
2a+d	3600	400	341,056
2a	5000	78,208	703040

APPROACH : 3

Let $\alpha = (k^2+1)^2x, \beta = (k^2+1)^2y, \gamma = (k^2+1)z$

Apply α, β, γ in (4)

$$x^2 + y^2 = 2z^3$$

Choose $z = A^2 + B^2$ and $2 = \frac{(1+7i)(1-7i)}{25}$

$$x^2 + y^2 = 2(A^2 + B^2)^3$$

$$x^2 + y^2 = \frac{(1+7i)(1-7i)}{25} (A+iB)^3 (A-iB)^3$$

$$(x+iy)(x-iy) = \frac{(1+7i)(1-7i)}{25} ((A^3-3AB^2) + i(3A^2B-B^3)) ((A^3-3AB^2) - i(3A^2B-B^3))$$

$$= \frac{1}{25} \{ [(A^3-3AB^2) - 7(3A^2B-B^3)] + i[7(A^3-3AB^2) + (3A^2B-B^3)] \} [(A^3-3AB^2) - 7(3A^2B-B^3)] - i[7(A^3-3AB^2) + (3A^2B-B^3)] \}$$

Equating real & imaginary parts on both the sides

$$x = \frac{1}{5} [(A^3-3AB^2) - 7(3A^2B-B^3)]$$

$$y = \frac{1}{5} [7(A^3-3AB^2) + (3A^2B-B^3)]$$

Let $A = 5A, B = 5B$

$$x = 25 [(A^3-3AB^2) - 7(3A^2B-B^3)]$$

$$y = 25 [7(A^3-3AB^2) + (3A^2B-B^3)]$$

Substitute x, y in α, β, γ

$$\alpha = (k^2+1)^2 25 [(A^3-3AB^2) - 7(3A^2B-B^3)]$$

$$\beta = (k^2+1)^2 25 [7(A^3-3AB^2) + (3A^2B-B^3)]$$

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$$\gamma = (k^2+1) 25 (A^2 + B^2)$$

$$\text{From (3) } a = \frac{25^3}{2}(k^2+1)^4 (A^2+B^2)^3 \tag{17}$$

$$\text{From (4) } d = (k^2+1)^4 25^2 [7(A^3-3AB^2) + (3A^2B-B^3)]^2 - (k^2+1)^4 25^3 (A^2+B^2)^3 \tag{18}$$

$$\begin{aligned} \text{Now } 2a-d &= 2(k^2+1)^4 25^3 (A^2+B^2)^3 - (k^2+1)^4 25^2 [7(A^3-3AB^2) + (3A^2B-B^3)]^2 \\ &= (k^2+1)^4 25^2 [50A^6 + 150A^4B^2 + 150A^2B^4 + 50B^6 - 49A^6 + 49 \times 2 \times A^3 \times 3AB^2 - 9A^2B^4 \times 49 - 2 \times 7 \times \\ &\quad (A^3 - 3AB^2) (3A^2B - B^3) - 9A^4B^2 + 2 \times 3A^2B \times B^3 - B^6] \\ &= (k^2+1)^4 25^2 [(A^6 - 6A^4B^2 + 9A^2B^4) + 49 (9A^4B^2 - 2 \times 3A^2B \times B^3 + B^6) - 2 \times 7(A^3-3AB^2) (3A^2B - B^3)] \\ &= (k^2+1)^4 25^2 [(A^3 - 3AB^2) - 7(3A^2B - B^3)]^2 = \alpha^2 \tag{19} \end{aligned}$$

$$\begin{aligned} \text{Now } 2a+d &= (k^2+1)^4 25^3 (A^2+B^2)^3 + (k^2+1)^4 25^2 [7(A^3-3AB^2) + (3A^2B - B^3)]^2 - \\ &\quad (k^2+1)^4 25^3 (A^2+B^2)^3 = \beta^2 \tag{20} \end{aligned}$$

$$\begin{aligned} \text{Now } 2a &= (k^2+1)^4 25^3 (A^2+B^2)^3 \\ &= (k^2+1) (k^2+1)^3 25^3 (A^2+B^2)^3 \\ &= (k^2+1) \gamma^3 \tag{21} \end{aligned}$$

The set of triples forming A.P is

$$\left\{ \frac{46875}{2}(k^2+1)^4 (A^2+B^2)^3 - [(k^2+1)^4 25^2 (7(A^3-3AB^2) + (3A^2B - B^3))]^2, \right. \\ \left. \frac{25^3}{2}(k^2+1)^4 (A^2+B^2)^3, \frac{-15625}{2}(k^2+1)^4 (A^2+B^2)^3 - ((k^2+1)^4 25^2 (7(A^3-3AB^2) + (3A^2B - B^3)))^2 \right\}$$

From the equations (19),(20),(21) it is seen that the required triple in arithmetic progression is satisfying the required condition.

	k = 1, A = 2, B = 4	k = 2, A = 1, B = 3	k = 1, A = 3, B = 5	k = 2, A = 4, B = 6
a	1000,000,000	4,882,812,500	4913000000	
d	1,994,240,000	5,859,375,000	9,107,760,000	1.076×10 ¹²
α	2400	62500	-26800	545000
β	63200	-12500	-137600	-1,565,000
γ	1000	12500	3400	6500
2a-d	5,76,0000	3,906,250,000	718,240,000	2.97×10 ¹¹
2a+d	3,994,240,000	1.5625×10 ¹⁰	1.8933×10 ¹⁰	2.449×10 ¹²
2a	2,000,000,000	9,765,625,000	9,826,000,000	1.373125×10 ¹²

APPROACH :4

Assume $\gamma = p^2+q^2$

$$\text{choose } 2 = \frac{(17+7i)(17-7i)}{13^2} \tag{22}$$

Substitute (5),(6) in (4) and use method of factorization

$$(\alpha+i\beta)(\alpha-i\beta) = \frac{(17+7i)(17-7i)}{13^2} (1+ki)(1-ki)(p+iq)^3(p-iq)^3$$

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$$= \frac{1}{13^2} \{ [(17-7k) + i(7+17k)] [(p^3-3pq^2) + i(3p^2q-q^3)] [(17-7k) - i(7+17k)] [(p^3-3pq^2) - i(3p^2q-q^3)] \}$$

$$= \frac{1}{13^2} \{ [(17-7k) (p^3-3pq^2) - (7+17k) (3p^2q-q^3)] + i[(3p^2q-q^3)(17-7k) + (7+17k)(p^3-3pq^2)] [(17-7k) (p^3-3pq^2) - (7+17k) (3p^2q-q^3)] - i [(3p^2q-q^3) (17-7k) + (7+17k) (p^3-3pq^2)] \}$$

Equating real & Imaginary parts on both the sides

$$\alpha = \frac{1}{13} [(17-7k) (p^3-3pq^2) - (7+17k) (3p^2q-q^3)]$$

$$\beta = \frac{1}{13} [(3p^2q-q^3) (17-7k) + (7+17k) (p^3-3pq^2)]$$

Let $P = 13P$ $Q = 13Q$

$$\alpha = 169 [(17-7k) (P^3-3PQ^2) - (7+17k) (3P^2Q-Q^3)]$$

$$\beta = 169 [(3P^2Q-Q^3) (17-7k) + (7+17k) (P^3-3PQ^2)]$$

$$\gamma = 169 (P^2+Q^2)$$

$$\text{From (3) } 2a = (k^2+1) 169^3 (P^2+Q^2)^3$$

$$a = \frac{(k^2+1)}{2} (169)^3 (P^2+Q^2)^3 \tag{23}$$

$$d = \beta^2 - 2a$$

$$= 169^2 [(3P^2Q-Q^3) (17-7k) + (7+17k) (P^3-3P^2Q)]^2 - (k^2+1) (169 (P^2+Q^2))^3 \tag{24}$$

$$\text{Now } 2a-d = 169^2 [169 \times 2 (k^2+1) (P^2+Q^2)^3 - (3P^2Q-Q^3)^2 (17-7k) - 2(3P^2Q-Q^3) (17-7k) (7+17k) (P^3-3PQ^2) - (7+17k)^2 (P^3-3PQ^2)^2]$$

$$= 169^2 [(17-7k) (P^3-3PQ^2) - (7+17k) (3P^2Q-Q^3)]^2 = \alpha^2 \tag{25}$$

$$\text{Now } 2a+d = (k^2+1) 169^3 (P^2+Q^2)^3 + 169^2 [(3P^2Q-Q^3)(17-7k) + (7+17k) (P^3-3PQ^2)]^2 - (k^2+1) 169^3 (P^2+Q^2)^3 = \beta^2 \tag{26}$$

$$\text{Now } 2a = (k^2+1) 169^3 (P^2+Q^2)^3 = (k^2+1) \gamma^3 \tag{27}$$

The set of triples forming A.P is

$$(a-d, a, a+d) = \left\{ \frac{14480427}{2} (k^2+1) (P^2+Q^2)^3 - 28561 [(3P^2Q-Q^3) (17-7k) + (7+17k) (P^3-3P^2Q)]^2, \frac{14480427}{2} (k^2+1) (P^2+Q^2)^3, \frac{4826809}{2} (k^2+1) (P^2+Q^2)^3 + 28561 [(3P^2Q-Q^3) (17-7k) + (7+17k) (P^3-3P^2Q)]^2 \right\}$$

From the equations (25), (26), (27) it is seen that the required triple in arithmetic progress is satisfying the required condition.

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	k = 1, P=2,Q = 3	k =1 ,P =3 ,Q =5	k =2 ,P = 4,Q = 6
a	1.06044×10^{10}	1.8971×10^{11}	1.6967×10^{12}
d	8,166,000,000	2.3867×10^{11}	2.9236×10^{12}
α	-114,244	-375180	685,464
β	-171,366	-786188	-2,513,368
γ	2197	5746	8788
2a-d	1.305×10^{10}	1.4076×10^{11}	4.698×10^{11}
2a+d	2.9366×10^{10}	6.8193×10^{12}	6.3170×10^{12}
2a	2.120×10^{10}	3.7942×10^{11}	3.3934×10^{12}

III. SECTION 2

METHOD OF ANALYSIS

The problem under consideration is equivalent to finding the values for a and d such that the following system of equations is solvable.

$$2a-d=\alpha^2, \tag{28}$$

$$2a+d=\beta^2 \tag{29}$$

$$2a = (k^2+1)\gamma^5 \tag{30}$$

Eliminating a, d from (28),(29) ,(30)

$$\alpha^2 + \beta^2 = 2(k^2+1)\gamma^5 \tag{31}$$

APPROACH I

Assume $\gamma = p^2+q^2$ (32)

Write 2 as $2 = \frac{(7+i)(7-i)}{25}$ (33)

Doing the same procedure as in Section 1 Approach 1 We get

$$\alpha = 625 \{ (7k-1) [(P^3-3PQ^2) (P^2-Q^2) - (3P^2Q-Q^3) 2PQ] - (k+7) [(3P^2Q-Q^3) (P^2-Q^2) + (P^3-3PQ^2) 2PQ] \}$$

$$\beta = 625 \{ (k+7) [(P^3-PQ^2) (P^2-Q^2) - (3P^2Q-Q^3) 2PQ] + (7k-1) [(3P^2Q-Q^3) (P^2-Q^2) + (P^3-3PQ^2) 2PQ] \}$$

$$\gamma = 25 (P^2+Q^2)$$

$$2a = 25^5 (k^2+1) (P^2+Q^2)^5 \tag{34}$$

$$d = [625 \{ (k+7) [(P^3-3PQ^2) (P^2-Q^2) - (3P^2Q-Q^3) 2PQ] + (7k-1) [(3P^2Q-Q^3) (P^2-Q^2) + (P^3-3PQ^2) 2PQ] \}]^2 - (k^2+1) 25^5 (P^2+Q^2)^5 \tag{35}$$

Now,

$$2a-d = [625 \{ (7k-1) [(P^3-3PQ^2) (P^2-Q^2) - (3P^2Q-Q^3) 2PQ] - (k+7) [(3P^2Q-Q^3) (P^2-Q^2) + (P^3-3PQ^2) 2PQ] \}]^2 = \alpha^2 \tag{36}$$

Similarly we can prove $2a+d = \beta^2$ (37)

$$2a = (k^2+1) \gamma^5 \tag{38}$$

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Therefore the set of triples forming A.P is

$$\left\{ \frac{3 \times 25^3}{2} (k^2+1) (P^2+Q^2)^5 - [625\{(k+7) [(P^3-3PQ^2) (P^2-Q^2) - (3P^2Q-Q^3) 2PQ] + (7k-1) [(3P^2Q-Q^3) (P^2-Q^2) + (P^3-3PQ^2) 2PQ]\}]^2, \frac{25^3}{2} (k^2+1) (P^2+Q^2)^5, -\frac{25^3}{2} (k^2+1) (P^2+Q^2)^5 + [625\{(k+7) [(P^3-PQ^2) (P^2-Q^2) - (3P^2Q-Q^3) 2PQ] + (7k-1) [(3P^2Q-Q^3) (P^2-Q^2) + (P^3-3PQ^2) 2PQ]\}]^2 \right\}$$

From (36), (37),(38) It is seen that the required triple in A.P satisfying required condition.

	k = 1, P=2,Q = 3	k =1 ,P =3 ,Q =5	k =2 ,P = 3,Q = 1
a	3.6259 x 10 ¹²	4.437053125 x 10 ¹⁴	2.44140625 x 10 ¹²
d	- 4.5998 x 10 ¹²	-8.14564625 x 10 ¹⁴	1.3671875 x 10 ¹²
α	3,442,500	41255000	1875000
β	-1,628,750	-8535000	2500000
γ	325	850	250
2a-d	1.18508x10 ¹³	1.701975 x 10 ¹⁵	3.515625 x 10 ¹²
2a+d	2652 x 10 ¹²	7.284225 x 10 ¹³	6.25 x 10 ¹²
2a	7.2518164062 x 10 ¹²	8.87410625 x 10 ¹⁴	4.8828125 x 10 ¹²

APPROACH II

Introducing the linear transformation $\alpha = u+v$, $\beta = u-v$, (39)

Substitute (32), (39) in (31) using the same procedure as in Section 1. Approach 2 We get

$$\alpha = (k+1) [(p^3-3pq^2) (p^2-q^2) - (3p^2q - q^3) 2pq] + (k-1) [(3p^2q-q^3) (p^2-q^2) + (p^3-3pq^2) 2pq]$$

$$\beta = (k-1) [(p^3-3pq^2) (p^2-q^2) - (3p^2q-q^3) 2pq] - (k+1) [(3p^2q-q^3) (p^2-q^2) + (p^3-3pq^2) 2pq]$$

$$\gamma = p^2+q^2$$

$$2a = (k^2+1) (p^2+q^2)^5 \tag{40}$$

$$d = \{(k-1)[(p^3-3pq^2)(p^2-q^2)-(3p^2q-q^3)2pq]-(k+1)[(3p^2q-q^3)(p^2-q^2)+(p^3-3pq^2)2pq]\}^2 - (k^2+1) (p^2+q^2)^5 \tag{41}$$

Now doing the same procedure as in section 1 Approach 1 We get

$$2a-d = \alpha^2, \tag{42}$$

$$2a+d = \beta^2, \tag{43}$$

$$2a = (k^2+1) \gamma^5 \tag{44}$$

The set of triples from A.P.

$$\left\{ \frac{3}{2} (k^2+1) (p^2+q^2)^5 - \{(k-1)[(p^3-3pq^2)(p^2-q^2)-(3p^2q-q^3)2pq] - (k+1)[(3p^2q-q^3) (p^2-q^2) + (p^3-3pq^2) (2pq)]\}^2, \frac{(k^2+1)}{2} (p^2+q^2)^5, -\frac{1}{2} (k^2+1) (p^2+q^2)^5 + \{(k-1) [(p^3-3pq^2) (p^2-q^2) - (3p^2q-q^3) 2pq] - (k+1)[(3p^2q-q^3) (p^2-q^2) + (p^3-3pq^2) (2pq)]\}^2 \right\}$$

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	k=2, p = 4, q = 6	k=1, p = 2, q = 3	k=1, p = 3, q = 5
a	950,510,080	371,293	45,435,424
d	1,846,378,496	683,050	57969152
α	-7392	244	5736
β	61216	-1194	12200
γ	380204032	13	34
2a-d	54,641,664	59,536	32,901,696
2a+d	37,47,398,656	1,425,636	148,840,000
2a	1,901,020,160	742,586	90,870,848

APPROACH III

Let $\alpha = (k^2+1)^3x$; $\beta = (k^2+1)^3y$; $\gamma = (k^2+1)z$,

apply α, β, γ in (31) we get $x^2 + y^2 = 2z^5$

Choose $z = A^2+B^2$, and $2 = \frac{(7+23i)(7-23i)}{17^2}$

Doing the same procedure as in Section 1 Approach : 3

$$\alpha = (k^2+1)^3 83521 \{ 7[(A^3-3AB^2)(A^2-B^2) - (3A^2B-B^3)2AB] - 23 [(3A^2B-B^3)(A^2-B^2) + (A^3-3AB^2)2AB] \} \quad (45)$$

$$\beta = (k^2+1)^3 83521 \{ 23[(A^3-3AB^2)(A^2-B^2) - (3A^2B-B^3)2AB] + 7 [(3A^2B-B^3)(A^2-B^2) + (A^3-3AB^2)2AB] \} \quad (46)$$

$$\gamma = (k^2+1)17^2 (A^2+B^2) \quad (47)$$

$$2a = 17^{10} (k^2+1)^6 (A^2+B^2)^5$$

$$d = \{ 83521 (k^2+1)^3 \{ 23 [(A^3-3AB^2)(A^2-B^2) - (3A^2B-B^3)2AB] + 7[(A^2-B^2)(3A^2B-B^3) + (A^3-3AB^2)2AB] \} \}^2 - 17^{10} (k^2+1)^6 (A^2+B^2)^5 \quad (48)$$

Doing the same procedure as in Approach :3 in section 1 we get

$$2a-d = \alpha^2 \quad (49)$$

$$2a+d = \beta^2 \quad (50)$$

$$2a = (k^2+1) \gamma^5 \quad (51)$$

From the equations (49), (50), (51) it is seen that the required triple in A.P is satisfying the required condition.

	k=1, A = 2, B = 4	k=2, A = 1, B = 3	k=3, A = 2, B = 1
a	2.6643×10^{20}	1.574×10^{21}	3.14999×10^{21}
d	-2.03344×10^{20}	2.4753×10^{21}	$-3.8963503 \times 10^{21}$
α	2.4823×10^{10}	2.597×10^{10}	$1.00976889 \times 10^{11}$
β	1.4475×10^{10}	7.50018×10^{10}	$-4.9026827 \times 10^{10}$
γ	11560	14450	14450
2a-d	6.1618×10^{20}	6.746×10^{20}	$1.01963303 \times 10^{22}$
2a+d	2.095×10^{20}	5.625×10^{21}	2.4036297×10^{21}
2a	4.1287×10^{20}	3.1499×10^{21}	3.14999×10^{21}

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IV. CONCLUSION

In this paper, we have presented many triples (a, b, c) forming Arithmetic progression. Such that in each triple in section 1, the expressions $a+b, b+c$ represent perfect squares and $a+c$ is (k^2+1) times a cube and in section 2 the expressions $a+b, b+c$ represent perfect squares and $a+c$ is (k^2+1) times a quintic.

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