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Applications of Ordinary Differential Equations in Engineering Field-A Review

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ABSTRACT: The paper deals with the analysis of L-R circuit by using linear differential equation of first order. A circuit containing an inductance L or a capacitor C and resistor R with current and voltage variable given by differential equation. The general solution of differential equation represent the complete response of network . The general solution of differential equation have two parts complementary function (C.F) and particular integral (P.I) in which C.F. represent transient response and P.I. represent steady response. ODE also help to find time t and current i In Newtons law of cooling.

KEYWORDS: First order differential equation, L-R circuit, Newtons law of cooling

I. INTRODUCTION

In “real-world,” there are many physical quantities that can be represented by functions involving only one of the four variables e.g., (x, y, z, t)
Equations involving highest order derivatives of order one = 1st order differential equations
Examples:
Function $\sigma(x)$ = the stress in a uni-axial stretched tapered metal rod , or
Function $v(x)$ = the velocity of fluid flowing a straight channel with varying cross-section
Mathematical modeling using differential equations involving these functions are classified as First Order Differential Equations.

II. APPLICATION ORDINARY DIFFERENTIAL EQUATION TO ELECTRICAL ENGINEERING FIELD

Using Differential equation we find current and charge at any time t.
First order circuits include only one capacitor or inductor (after using series/parallel reduction).
The solution of these circuits results in a “first-order” linear differential equation with constant coefficients. It requires one initial condition.

First-order RL Circuits-

The switch s is closed at time t=0. Find the current i(t) through the voltage across the resistor and inductor.
the voltage across resistance= Ri(t)

Voltage drop across inductance= $L \frac{di(t)}{dt}$

From Kirchhoff's law $L \frac{di(t)}{dt} + R i(t) = V$

$\Rightarrow \frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V}{L}$

Which is first order linear differential equation

$$I.F = e^{\int \frac{R}{L} dt} = e^{\frac{Rt}{L}}$$

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General solution is $i(t) e^{\frac{Rt}{L}} = \int e^{\frac{Rt}{L}} \cdot \frac{V}{L} dt + k = \frac{V}{L} \frac{e^{\frac{Rt}{L}}}{R/L} + k = \frac{V}{R} e^{\frac{Rt}{L}} + k$

$$\therefore i(t) = \frac{V}{R} + k e^{-\frac{Rt}{L}}$$

Since the inductor behaves as a open circuit.

$$\therefore i(0+) = 0$$

$$\text{From (1)} \therefore k = -\frac{V}{R}$$

$$\therefore i(t) = \frac{V}{R} - \frac{V}{R} e^{-\frac{Rt}{L}} = \frac{V}{R} (1 - e^{-\frac{Rt}{L}})$$

The voltage across the resistor and inductor are given as

$$V_R(t) = i(t) \cdot R = V (1 - e^{-\frac{Rt}{L}})$$

$$V_L(t) = L \frac{di(t)}{dt} = L \cdot \frac{V}{R} [0 - (-\frac{R}{L}) e^{-\frac{Rt}{L}}]$$

$$V_L(t) = V e^{-\frac{Rt}{L}}$$

$$\therefore V_L(t) = V, V_R(t) = 0$$

$$\text{At } t=0, i(t)=0 \text{ and At } t= \infty, i(t) = \frac{V}{R} \therefore V_L(t) = 0, V_R(t) = V$$

$$\text{At } t = \frac{L}{R} = \tau, i(t) = \frac{V}{R} (1 - e^{-1}) = 0.632 \frac{V}{R}$$

$$\text{And } V_L(t) = V e^{-1} = 0.368 V, V_R(t) = 0.632 V$$

$\frac{L}{R} = \tau$ is known as the time constant of the circuit and is defined as the interval after which current or voltage changes 63.2 percent of its total change.

III. APPLICATION OF ORDINARY DIFFERENTIAL EQUATION IN NEWTONS LAW OF COOLING

In Newtons law of cooling we find temperature at any time t also find time at any temperature.

We see this with example

1. Determine the time required to cool down a solid object initially at 80 °C to 8 °C. It is placed in a refrigerator with its interior air maintained at 5 °C. If the coefficient $\alpha = 0.002/\text{m}^2\text{-s}$ and the contact area between the solid and the cool air in the refrigerator is $A = 0.2 \text{ m}^2$.

Solution:

We have $T_0 = 80 \text{ }^\circ\text{C}$, $T_f = 5 \text{ }^\circ\text{C}$, $\alpha = 0.002/\text{m}^2\text{-s}$ and $A = 0.2 \text{ m}^2$

Substituting the above into Equation will lead to the following

1st order differential equation:

$$\frac{dT(t)}{dt} = -\alpha A [T(t) - t_f] = - (0.002)(0.2)[T(t) - 5] = - 0.0004 [T(t) - 5]$$

$$\text{with the condition: } T(0) = 80 \text{ }^\circ\text{C}, \frac{dT(t)}{T(t)-5} = - 0.0004 dt$$

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Integrating both sides of Equation $\int \frac{dT(t)}{T(t)-5} = \int -0.0004 dt + c$
 $T(t) - 5 = e^{-0.0004t+c} = c_1 e^{-0.0004t}$

The integration constant c_1 in Equation can be obtained by the condition $T(0) = 80^\circ\text{C}$ in Equation with $c_1 = 75$.

consequently, the solution $T(t)$ is: $T(t) = 5 + 75e^{-0.0004t}$ (f)

If t_e = required time for the solid to drop its temperature from 80°C to 8°C , we should have:

$$T(t_e) = 8 = 5 + 75e^{-0.0004t_e}$$

Solve the equation for $t_e = 8047\text{s}$ or 2.24h

We also use ordinary differential equation to find orthogonal trajectories of various curves.

We solve here one of the example here.

Find the orthogonal trajectories of the family of straight lines $y=Cx$, where C is a parameter.

1) First, we construct the differential equation for the family of straight lines $y=Cx$. By differentiating the last equation with respect to x , we get:

$$y' = C = \text{const.}$$

Eliminate the constant C from the system of equations:

$$\begin{aligned} y &= Cx \\ y' &= C, \quad \Rightarrow y' = \frac{y}{x}. \end{aligned}$$

We obtain the differential equation of the initial set of straight lines.

2) Replace y' with $(-1/y')$. This gives the differential equation of the orthogonal trajectories:

$$-1/y' = \frac{y}{x'} \Rightarrow y' = -\frac{x}{y}.$$

3) Now we solve the last differential equation to find the algebraic equation of the family of orthogonal trajectories:

$$\begin{aligned} y' &= -\frac{x}{y} \Rightarrow \frac{dy}{dx} = -\frac{x}{y} \\ \Rightarrow y \, dy &= -x \, dx \end{aligned}$$

$$\Rightarrow \int y \, dy = -\int x \, dx$$

$$\Rightarrow \frac{y^2}{2} = -\frac{x^2}{2} + C$$

$$\Rightarrow x^2 + y^2 = 2C = r^2, \text{ where } 2C = r^2$$

we see that the orthogonal trajectories for the family of straight lines are concentric circles.

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We have to see one more example

A family of hyperbolic curves is given by the equation $y = \frac{C}{x}$. Find the orthogonal trajectories for these curves.

Solution –

1) Determine the differential equation for the given family of hyperbolas. Differentiating the equation with respect to x gives:

$$y' = -\frac{C}{x^2}$$

Now we eliminate the parameter C from the system of two equations:

$$y = \frac{C}{x}$$

$$y' = -\frac{C}{x^2}$$

It follows from the first equation that $C=xy$. Substituting into the second equation yields

$$y' = -\frac{xy}{x^2} = -\frac{y}{x}$$

2) Replace y' with $(-1/y')$:

$$\frac{-1}{y'} = -\frac{y}{x}$$

$$\Rightarrow y^2 = x/y$$

3) Now we integrate the differential equation of the orthogonal trajectories

$$y' = \frac{x}{y}$$

$$\frac{dy}{dx} = \frac{x}{y} \Rightarrow y dy = x dx$$

$$\Rightarrow \int y dy = \int x dx$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\Rightarrow x^2 - y^2 = 2C = C', \text{ where } 2C = C'$$

Thus, we have obtained the equation of the family of orthogonal trajectories. As it can be seen, these orthogonal trajectories are also hyperbolas.

Ordinary differential equation is also used in electronics engineering in AC power analysis, AC circuit analysis, E&M, transmission lines, control systems, signal conditioning/processing

It is used in fluid dynamics - Design of containers and funnels , in heat conduction analysis- Design of heat spreaders in microelectronics, in combined heat conduction and convection- Design of heating and cooling chambers and in rigid-body dynamic analysis.

IV. CONCLUSION

By using first order ordinary differential equation in L-R and C-R circuit we can find the current(i) and voltage (v) in the circuit when inductance (L) or capacitance(C) and resistance (R) are given. In Newton's law of cooling we use Ordinary differential equation to find time and temperature. It is used to find orthogonal trajectories of curves.



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