

$\psi\alpha$ -Totally Continuous Functions in Topological Spaces

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ABSTRACT: The aim of this paper is to introduce a new type of totally-continuous functions called totally $\psi\alpha$ -continuous functions and $\psi\alpha$ -totally continuous functions in topological spaces. Also we analyze the association of these continuous functions with various kinds of continuous functions.

KEYWORDS: Totally continuous function, $\psi\alpha$ -continuous function, $\psi\alpha$ -closed set and $\psi\alpha$ -open set.

I. INTRODUCTION

Many different forms of continuous functions have been introduced over the years. Levine (5) introduced the idea of continuous functions in topological spaces. Jain (3) introduced totally continuous functions in topological spaces. Mashhour (6) introduced and studied α -continuous functions in topological spaces. Balachandran (1) introduced generalized continuous functions in topological spaces. Veera Kumar (8) introduced and studied ψ -continuous functions in topological spaces. Shakila and Balamani (9) introduced $\psi\alpha$ -closed sets and $\psi\alpha$ -continuous functions in topological spaces.

The purpose of this paper is to study a new type of totally-continuous functions namely totally $\psi\alpha$ -continuous functions and $\psi\alpha$ -totally continuous functions in topological spaces. Also we obtain the interrelations between these continuous functions.

II. PRELIMINARIES

Definition 2.1 Let (X, τ) be a topological space. A Subset A of a topological space (X, τ) is called

- 1) α -open (7) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$.
- 2) Semi-open (4) if $A \subseteq \text{cl}(\text{int}(A))$.
- 3) Semi generalized closed (2) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ) .
- 4) ψ -closed (8) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X, τ) .
- 5) $\psi\alpha$ -closed (9) if $\psi\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X, τ) .
- 6) $\psi\alpha$ -clopen (9) if it is both $\psi\alpha$ -open and $\psi\alpha$ -closed in (X, τ) .

Results 2.2

- Every closed (open) subset in (X, τ) is $\psi\alpha$ -closed ($\psi\alpha$ -open).
- Every clopen subset in (X, τ) is $\psi\alpha$ -clopen.

Definition 2.3 Let (X, τ) and (Y, σ) be two topological spaces. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- 1) Continuous (5) if $f^{-1}(V)$ is closed in (X, τ) for every closed set V of (Y, σ) .
- 2) α -Continuous (6) if $f^{-1}(V)$ is α -closed in (X, τ) for every closed set V of (Y, σ) .

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- 3) **Totally Continuous** (3) if $f^{-1}(V)$ is clopen in (X, τ) for every open set V of (Y, σ) .
 4) **ψ -Continuous** (8) if $f^{-1}(V)$ is ψ -closed in (X, τ) for every closed set V of (Y, σ) .
 5) **$\psi\alpha$ -Continuous** (9) if $f^{-1}(V)$ is $\psi\alpha$ -closed in (X, τ) for every closed set V of (Y, σ) .

III. TOTALLY $\psi\alpha$ -CONTINUOUS FUNCTIONS

Definition 3.1 Let (X, τ) and (Y, σ) be two topological spaces. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be **totally $\psi\alpha$ -continuous** if $f^{-1}(V)$ is $\psi\alpha$ -clopen subset of (X, τ) for every open set V of (Y, σ) .

Example 3.2 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is totally $\psi\alpha$ -continuous.

Theorem 3.3 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is totally $\psi\alpha$ -continuous if and only if the inverse image of every closed subset of (Y, σ) is a $\psi\alpha$ -clopen subset of (X, τ) .

Proof: (Necessity) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a totally $\psi\alpha$ -continuous function. Let V be any closed set in (Y, σ) . Then $Y - V$ is open in (Y, σ) . Since f is totally $\psi\alpha$ -continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) .

(Sufficiency) Assume that U is any open set in (Y, σ) . Then $Y - U$ is closed in (Y, σ) . By assumption, $f^{-1}(Y - U) = X - f^{-1}(U)$ is $\psi\alpha$ -clopen in (X, τ) which implies that $f^{-1}(U)$ is $\psi\alpha$ -clopen in (X, τ) . Hence f is a totally $\psi\alpha$ -continuous function.

Proposition 3.4 Every totally continuous function is a totally $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any open set in (Y, σ) . Since f is totally continuous, $f^{-1}(V)$ is clopen in (X, τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) . Hence f is a totally $\psi\alpha$ -continuous function.

Example 3.5 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is a totally $\psi\alpha$ -continuous function but not a totally continuous function, since for the open set $\{a\}$ in (Y, σ) , $f^{-1}(\{a\}) = \{b\}$ is $\psi\alpha$ -clopen in (X, τ) but not clopen in (X, τ) .

Proposition 3.6 Every totally $\psi\alpha$ -continuous function is a $\psi\alpha$ -continuous function but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a totally $\psi\alpha$ -continuous function. Let V be any open set in (Y, σ) . Since f is totally $\psi\alpha$ -continuous, $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) . Hence f is a $\psi\alpha$ -continuous function.

Example 3.7 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is $\psi\alpha$ -continuous but not totally $\psi\alpha$ -continuous, since for the open set $\{a\}$ in (Y, σ) , $f^{-1}(\{a\}) = \{a\}$ is $\psi\alpha$ -open but not $\psi\alpha$ -closed in (X, τ) .

Proposition 3.8 Every continuous function is independent from totally $\psi\alpha$ -continuous function.

Example 3.9 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is continuous but not totally $\psi\alpha$ -continuous, since for the open set $\{a, b\}$ in (Y, σ) , $f^{-1}(\{a, b\}) = \{a, b\}$ is open but not $\psi\alpha$ -clopen in (X, τ) .

Example 3.10 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by the identity function. Then f is totally $\psi\alpha$ -continuous but not continuous, since for the open set $\{a, b\}$ in (Y, σ) , $f^{-1}(\{a, b\}) = \{a, b\}$ is $\psi\alpha$ -clopen but not open in (X, τ) .

Proposition 3.11 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a totally $\psi\alpha$ -continuous function and X is $\psi\alpha$ -connected, then Y is an indiscrete space.

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Proof: Suppose that Y is not an indiscrete space. Let V be a non-empty open subset of Y . Since f is totally $\psi\alpha$ -continuous, $f^{-1}(V)$ is non-empty $\psi\alpha$ -clopen subset of X . Then $X = f^{-1}(V) \cup (f^{-1}(V))^c$. Thus X is union of two non-empty disjoint $\psi\alpha$ -open sets which is a contradiction to the fact that X is $\psi\alpha$ -connected. Therefore Y must be indiscrete space.

Theorem 3.12 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a continuous function from discrete space (X, τ) into a topological space (Y, σ) if and only if f is a totally $\psi\alpha$ -continuous function.

Proof: (Necessity) Assume that f is a totally $\psi\alpha$ -continuous function. Let V be any open set in (Y, σ) . Since f is a totally $\psi\alpha$ -continuous function, $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) . Since (X, τ) is discrete space, every subset of (X, τ) is open and closed subset in (X, τ) which implies that $f^{-1}(V)$ is open in (X, τ) . Therefore f is a continuous function.

(Sufficiency) Assume that f is a continuous function. Let V be any open set in (Y, σ) . Then $f^{-1}(V)$ is open in (X, τ) . Since (X, τ) is discrete space, every subset of (X, τ) is clopen in (X, τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) . Therefore f is totally $\psi\alpha$ -continuous.

Theorem 3.13 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\psi\alpha$ -continuous function from a discrete space (X, τ) into a topological space (Y, σ) if and only if f is a totally $\psi\alpha$ -continuous function.

Proof: (Necessity) Assume that f is a totally $\psi\alpha$ -continuous function. Let V be any open set in (Y, σ) . Then $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) . Since (X, τ) is discrete space, every subset of (X, τ) is open and closed subset in (X, τ) which implies that $f^{-1}(V)$ is clopen in (X, τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) . Therefore f is $\psi\alpha$ -continuous.

(Sufficiency) Assume that f is $\psi\alpha$ -continuous. Let V be any open set in (Y, σ) . Then $f^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) . Since (X, τ) is discrete space, $f^{-1}(V)$ is clopen in (X, τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) . Therefore f is totally $\psi\alpha$ -continuous.

Proposition 3.14 The composition of two totally $\psi\alpha$ -continuous functions need not be a totally $\psi\alpha$ -continuous function as seen from the following example.

Example 3.15 Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$ and $\eta = \{\emptyset, \{a, b\}, Z\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be the identity function. Then f and g are totally $\psi\alpha$ -continuous but their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is not totally $\psi\alpha$ -continuous, since $\{a, b\}$ is open in (Z, η) , whereas $(g \circ f)^{-1}(\{a, b\}) = \{a, b\}$ is not $\psi\alpha$ -clopen in (X, τ) .

Proposition 3.16 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a totally $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a continuous function. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a totally $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . Since g is a continuous function, $g^{-1}(V)$ is open in (Y, σ) . Since f is totally $\psi\alpha$ -continuous $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -clopen in (X, τ) . Hence $g \circ f$ is a totally $\psi\alpha$ -continuous function.

Proposition 3.17 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a totally $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a totally continuous function. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a totally $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) which implies that $g^{-1}(V)$ is open in (Y, σ) . Since f is totally $\psi\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -clopen in (X, τ) . Hence $g \circ f$ is a totally $\psi\alpha$ -continuous function.

Proposition 3.18 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a totally continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a totally continuous function. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a totally $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) which implies that $g^{-1}(V)$ is open in (Y, σ) . Since f is totally continuous, $f^{-1}(g^{-1}(V))$ is clopen in (X, τ) , By result 2.2, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -clopen in (X, τ) . Hence $g \circ f$ is totally $\psi\alpha$ -continuous.

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IV. $\psi\alpha$ -TOTALLY CONTINUOUS FUNCTIONS

Definition 4.1 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called $\psi\alpha$ -totally continuous if $f^{-1}(V)$ is clopen subset of (X, τ) for every $\psi\alpha$ -open set V of (Y, σ) .

Theorem 4.2 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is $\psi\alpha$ -totally continuous if and only if the inverse image of every $\psi\alpha$ -closed subset of (Y, σ) is a clopen subset of (X, τ) .

Proof: (Necessity) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\psi\alpha$ -totally continuous function. Let V be any $\psi\alpha$ -closed set in (Y, σ) . Then $Y-V$ is $\psi\alpha$ -open in (Y, σ) . Since f is a $\psi\alpha$ -totally continuous function, $f^{-1}(Y-V) = X - f^{-1}(V)$ is clopen in (X, τ) which implies that $f^{-1}(V)$ is clopen in (X, τ) .

(Sufficiency) Assume that U is any $\psi\alpha$ -open set in (Y, σ) . Then $Y-U$ is $\psi\alpha$ -closed in (Y, σ) . By assumption, $f^{-1}(Y-U) = X - f^{-1}(U)$ is clopen in (X, τ) which implies that $f^{-1}(U)$ is clopen in (X, τ) . Hence f is $\psi\alpha$ -totally continuous.

Proposition 4.3 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a $\psi\alpha$ -totally continuous function, then f is a totally continuous function.

Proof: Let V be any open set in (Y, σ) . By result 2.2, V is a $\psi\alpha$ -open set in (Y, σ) . Since f is a $\psi\alpha$ -totally continuous function, $f^{-1}(V)$ is clopen in (X, τ) . Therefore f is totally continuous.

The converse of proposition 4.3 need not be true as seen from the following example

Example 4.4 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is totally continuous, but f is not $\psi\alpha$ -totally continuous, since for the $\psi\alpha$ -open set $\{b\}$ in (Y, σ) , $f^{-1}(\{b\}) = \{a\}$ is not clopen subset in (X, τ) .

Proposition 4.5 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a $\psi\alpha$ -totally continuous function, then f is a totally $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Y, σ) . By result 2.2, V is a $\psi\alpha$ -open set in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, $f^{-1}(V)$ is clopen in (X, τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) . Therefore f is totally $\psi\alpha$ -continuous.

The converse of proposition 4.5 need not be true as seen from the following example.

Example 4.6 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is totally $\psi\alpha$ -continuous, but f is not $\psi\alpha$ -totally continuous, since for the $\psi\alpha$ -open set $\{a, b\}$ in (Y, σ) , $f^{-1}(\{a, b\}) = \{a, b\}$ is not clopen in (X, τ) .

Proposition 4.7 Every $\psi\alpha$ -totally continuous function is continuous but not conversely.

Proof: Let V be any open set in (Y, σ) . By result 2.2, V is a $\psi\alpha$ -open set in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, $f^{-1}(V)$ is clopen in (X, τ) which implies that $f^{-1}(V)$ is open in (X, τ) . Therefore f is continuous.

Example 4.8 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is continuous, but not $\psi\alpha$ -totally continuous, since for the $\psi\alpha$ -open set $\{b\}$ in (Y, σ) , $f^{-1}(\{b\}) = \{b\}$ is not clopen in (X, τ) .

Proposition 4.9 Every $\psi\alpha$ -totally continuous function is $\psi\alpha$ -continuous but not conversely.

Proof: Let V be any open set in (Y, σ) . By result 2.2, V is $\psi\alpha$ -open in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, $f^{-1}(V)$ is clopen in (X, τ) which implies that $f^{-1}(V)$ is open in (X, τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) . Therefore f is $\psi\alpha$ -continuous.

Example 4.10 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is $\psi\alpha$ -continuous but not $\psi\alpha$ -totally continuous, since for the $\psi\alpha$ -open set $\{a, c\}$ in (Y, σ) , $f^{-1}(\{a, c\}) = \{a, c\}$ is not clopen in (X, τ) .

Proposition 4.11 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be any function from discrete space (X, τ) into a topological space (Y, σ) . If f is a $\psi\alpha$ -totally continuous function then f is a continuous function.

Proof: Let V be any open set in (Y, σ) . By result 2.2, V is $\psi\alpha$ -open in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, $f^{-1}(V)$ is clopen in (X, τ) which implies that $f^{-1}(V)$ is open in (X, τ) . Therefore f is continuous.

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Proposition 4.12 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be any function from discrete space (X, τ) into a topological space (Y, σ) . If f is a $\psi\alpha$ -totally continuous function, then f is a $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Y, σ) . By result 2.2, V is a $\psi\alpha$ -open set in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, $f^{-1}(V)$ is clopen in (X, τ) which implies that $f^{-1}(V)$ is open in (X, τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) . Therefore f is $\psi\alpha$ -continuous.

Proposition 4.13 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\psi\alpha$ -totally continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a totally $\psi\alpha$ -continuous function. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a totally $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . Since g is totally $\psi\alpha$ -continuous, $g^{-1}(V)$ is $\psi\alpha$ -clopen in (Y, σ) which implies that $g^{-1}(V)$ is $\psi\alpha$ -open in (Y, σ) . Since f is a $\psi\alpha$ -totally continuous function, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X, τ) . By result 2.2, $(g \circ f)^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) . Therefore $g \circ f$ is totally $\psi\alpha$ -continuous.

Proposition 4.14 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\psi\alpha$ -totally continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a totally continuous function. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a totally $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) . Which implies that $g^{-1}(V)$ is open in (Y, σ) . By result 2.2, $g^{-1}(V)$ is $\psi\alpha$ -open in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, $f^{-1}(g^{-1}(V))$ is clopen in (X, τ) . By result 2.2, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -clopen in (X, τ) . Therefore $g \circ f$ is totally $\psi\alpha$ -continuous.

Proposition 4.15 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\psi\alpha$ -totally continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a continuous function. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a totally $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . Since g is continuous, $g^{-1}(V)$ is open in (Y, σ) . By result 2.2, $g^{-1}(V)$ is $\psi\alpha$ -open in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, $f^{-1}(g^{-1}(V))$ is clopen in (X, τ) . By result 2.2, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -clopen in (X, τ) . Therefore $g \circ f$ is totally $\psi\alpha$ -continuous.

Proposition 4.16 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\psi\alpha$ -totally continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a $\psi\alpha$ -continuous function. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a totally $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . Since g is $\psi\alpha$ -continuous, $g^{-1}(V)$ is $\psi\alpha$ -open in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, By result 2.2 $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -clopen in (X, τ) . Therefore $g \circ f$ is totally $\psi\alpha$ -continuous.

Proposition 4.17 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a totally continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a $\psi\alpha$ -totally continuous function. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a $\psi\alpha$ -totally continuous function.

Proof: Let V be any $\psi\alpha$ -open set in (Z, η) . Since g is $\psi\alpha$ -totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) , which implies that $g^{-1}(V)$ is open in (Y, σ) . Since f is a totally continuous function, $(g \circ f)^{-1}(V)$ is clopen in (X, τ) . Therefore $g \circ f$ is $\psi\alpha$ -totally continuous.

Proposition 4.18 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a $\psi\alpha$ -totally continuous function. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a continuous function.

Proof: Let V be any open set in (Z, η) . By result 2.2, V is $\psi\alpha$ -open in (Z, η) . Since g is $\psi\alpha$ -totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) which implies that $g^{-1}(V)$ is open in (Y, σ) . Since f is continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in (X, τ) . Therefore $g \circ f$ is continuous.

Proposition 4.19 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a $\psi\alpha$ -totally continuous function. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . By result 2.2, V is $\psi\alpha$ -open in (Z, η) . Since g is $\psi\alpha$ -totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) which implies that $g^{-1}(V)$ is open in (Y, σ) . Since f is $\psi\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -open in (X, τ) . Therefore $g \circ f$ is $\psi\alpha$ -continuous.

Proposition 4.20 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be two $\psi\alpha$ -totally continuous functions. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is also $\psi\alpha$ -totally continuous function.

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Proof: Let V be any $\psi\alpha$ -open set in (Z, η) . Since g is $\psi\alpha$ -totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) . By result 2.2, $g^{-1}(V)$ is $\psi\alpha$ -clopen in (Y, σ) which implies that $g^{-1}(V)$ is $\psi\alpha$ -open in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X, τ) . Therefore $g \circ f$ is $\psi\alpha$ -totally continuous.

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