

MHD Flow of Stratified Incompressible Viscous Fluid through Porous Medium between Two Semi-Infinite Parallel Plates

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ABSTRACT: The aim of the present paper is to investigate flow of stratified incompressible viscous fluid between two semi-infinite parallel plates. The space between the parallel plates is filled with porous medium. Brinkmann equation is applied to study the fluid flow. Transverse magnetic field is applied perpendicular to the length of the plates. The expressions for fluid velocity and flow rate of the fluid are obtained sophisticatedly. The effects of permeability parameter and magnetic parameter on the fluid velocity and on the flow rate of the fluid are investigated. The results are graphically represented

KEYWORDS: Porous medium, permeability parameter, magnetic parameter incompressible viscous fluid

I. INTRODUCTION

The study of flow through porous medium assumed importance because of the interesting applications in the diverse fields of science, Engineering and Technology. The practical applications are in the percolation of water through soil, extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter disciplinary fields such as biomedical engineering etc. The lung alveolar is an example that finds applications in an animal body. The classical Darcy's law Muskat [1] states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as

$$\vec{v} = -\left(\frac{k}{\mu}\right)\nabla P$$

The law gives good results in the situations when the flow is uni-directional or the flow is at low speed. In general, the specific discharge in the medium need not be always low. As the specific discharge increases, the convective forces get developed and the internal stress generates in the fluid due to its viscous nature and produces distortions in the velocity field. In the case of highly porous medium such as fiber glass, papas of dandelion the flow occurs even in the absence of the pressure gradient.

Modifications for the classical Darcy's law were considered by the Beavers and Joseph [2], Saffman [3] and others. A generalized Darcy's law proposed by Brinkmann [4] is given by

$$O = -\nabla P - \left(\frac{\mu}{K}\right)\vec{v} + \mu \nabla^2 \vec{v}$$

Where μ and K are coefficients of viscosity of the fluid and permeability of the porous medium

II. RELATED WORK

The applications of flows through porous medium bears wide spread interest in Geophysics, biology and medicine. In many of these areas the flow consists of more than one phase, such type of flows find applications in the inter

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disciplinary fields such as bio-medical engineering etc., the flow of blood is one such application. The blood may be represented as Newtonian fluid and the flow of the blood is in two layered. Lightfoot [5], Shukla *et al.* [6] and Chaturani [7]. Bird *et al.* [8] found an exact solution for the laminar flow of two immiscible fluids between two parallel plates. Bhattacharya [9] discussed the flow of immiscible fluids between rigid plates with a time dependent pressure gradient. Vajravelu *et al.* [10] have discussed the effect of magnetic field on unsteady flow of two immiscible conducting fluids between two permeable beds. Transient couette flow in a rotating non-Darcian porous medium parallel plate configuration is studied by Anwar beg *et al.* [11] Kandryzakaria *et al.* [12] discussed magneto hydrodynamics instability of interfacial waves between two immiscible cylindrical fluids.

Earlier Narasimhacharyulu *et al.* [13] studied the problem of two-phase fluid flow between parallel plates with porous lining and Narasimhacharyulu *et al.* [14] examined the flow of micro polar fluid between parallel plates coated with porous lining.

In this present paper we are considering the stratified incompressible viscous fluid between two parallel plates, the space between the plates is filled with porous medium. Transverse magnetic field is applied perpendicular to the length of the plates. The results are graphically represented.

III. MATHEMATICAL FORMULATION OF THE PROBLEM

The flow of stratified incompressible viscous fluid is considered between two semi-infinite parallel plates given by $y = \pm h$. The space between the plates is filled with porous region. The coordinate system is taken such that x-axis lies parallel to the length of the plates and y-axis perpendicular to the length of the plates. The fluid flows under a constant pressure gradient. $G = -\frac{\partial p}{\partial x}$

A transverse magnetic field is applied perpendicular to the flow of the fluid. The induced magnetic effect is negligible in comparison with the transverse magnetic field due to low magnetic Reynolds's number, as a result of slightly conducting fluid [15]. Further the electric force E given by ohm's law $J = (E + V \times B)$ when $B = (H_0, 0, 0)$ and the electrical conductivity is assumed to be a null vector for the simplicity of the problem.

The velocity of the fluid $\vec{V} = (u, 0, 0)$ satisfies the equation of continuity, the physical quantities depend on y only.

The equation of motion is given by

$$\frac{d^2 u}{dy^2} - \frac{u}{k} - \frac{\sigma \beta_0^2}{\mu} u = -\frac{G}{\nu} e^{\beta y}; -h < y < h \quad \dots (2.1)$$

Where $G = -\frac{\partial p}{\partial x}$ is a constant pressure gradient, in the x direction, ν is coefficient of viscosity of the fluid, k is permeability of the porous medium and β is stratification factor

Using the following non-dimensional quantities.

$$u^* = \frac{uh}{\nu}, y^* = \frac{y}{h}, G^* = \frac{Gh^3}{\nu}, \beta_1^2 = \frac{h^2}{K}, M^2 = \frac{\sigma \beta_0^2 h^2}{\mu} \quad \dots (2.2)$$

After removing *, the non-dimensional form of equation of motion is

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$$\frac{d^2u}{dy^2} - \alpha^2 u = -\frac{G}{\nu} e^{\beta y} \quad -1 < y < 1 \tag{2.3}$$

Where $\alpha^2 = \beta_1^2 + M^2$, $\beta_1^2 = \frac{h^2}{K}$, $M^2 = \frac{\sigma \beta_0^2 h^2}{\mu}$

The boundary conditions are given by

$$u = 0 \quad \text{at} \quad y = \pm 1 \quad \}$$

IV. SOLUTION OF THE PROBLEM

Solving the equations (2.3) employing boundary conditions (2.4)

The fluid velocity is given by

$$u = \frac{G}{\nu(\beta^2 - \alpha^2)} \left[\frac{\sinh(\alpha + \beta)e^{\alpha y}}{\sinh 2\alpha} + \frac{\sinh(\alpha - \beta)}{\sinh 2\alpha} e^{-\alpha y} - e^{\beta y} \right] \dots \tag{2.5}$$

Flow rate $Q = \int_{-1}^1 u dy$

$$Q = \frac{G}{\nu(\beta^2 - \alpha^2)} \left[\frac{\sinh(\alpha + \beta)}{\alpha \cosh \alpha} + \frac{\sinh(\alpha - \beta)}{\alpha \cosh \alpha} - 2 \frac{\sinh \beta}{\beta} \right] \dots \tag{2.6}$$

Where $\alpha^2 = \beta_1^2 + M^2$, $\beta_1^2 = \frac{h^2}{K}$, $M^2 = \frac{\sigma \beta_0^2 h^2}{\mu}$

Deductions;

CASE I. MHD flow of incompressible viscous fluid through porous medium between two semi-infinite parallel plates ($\beta = 0$)

The fluid velocity

$$u = \frac{G}{\nu \alpha^2} \left(1 - \frac{\cosh \alpha y}{\cosh \alpha} \right) \dots \tag{2.7}$$

Flow rate

$$Q = \frac{2G}{\nu \alpha^2} \left(1 - \frac{\tanh \alpha}{\alpha} \right) \tag{2.8}$$

Where $\alpha^2 = \beta_1^2 + M^2$, $\beta_1^2 = \frac{h^2}{K}$, $M^2 = \frac{\sigma \beta_0^2 h^2}{\mu}$

CASE II: Flow of stratified incompressible viscous fluid through porous medium between two semi-infinite parallel plates ($M=0$)

The fluid velocity

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$$u = \frac{G}{v(\beta^2 - \alpha^2)} \left[\frac{\sinh(\alpha + \beta)e^{\alpha y}}{\sinh 2\alpha} + \frac{\sinh(\alpha - \beta)}{\sinh 2\alpha} e^{-\alpha y} - e^{\beta y} \right] \dots \quad (2.9)$$

Flow rate

$$Q = \frac{G}{v(\beta^2 - \alpha^2)} \left[\frac{\sinh(\alpha + \beta)}{\alpha \cosh \alpha} + \frac{\sinh(\alpha - \beta)}{\alpha \cosh \alpha} - 2 \frac{\sinh \beta}{\beta} \right] \dots \quad (2.10)$$

$$\text{Where } \alpha^2 = \frac{h^2}{K},$$

CASE III: The flow of incompressible viscous fluid through porous medium between two semi-infinite parallel plates ($\beta=0, M=0$)

Fluid velocity is given by

$$u = \frac{G}{v\alpha^2} \left(1 - \frac{\cosh \alpha y}{\cosh \alpha} \right) \dots \quad (2.12)$$

Flow rate

$$Q = \frac{2G}{v\alpha^2} \left(1 - \frac{\tanh \alpha}{\alpha} \right) \quad \alpha^2 = \frac{h^2}{K}, \quad (2.13)$$

V. CONCLUSION

Fig. 1 shows the variation of velocity with magnetic parameter. As magnetic parameter are increasing fluid velocity is decreasing

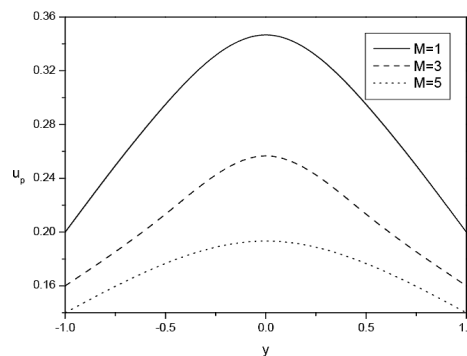


Fig1

In Fig. 2 it is observed that, the effect of the permeability of the porous medium is to decrease velocity in the porous medium with increasing value of K.

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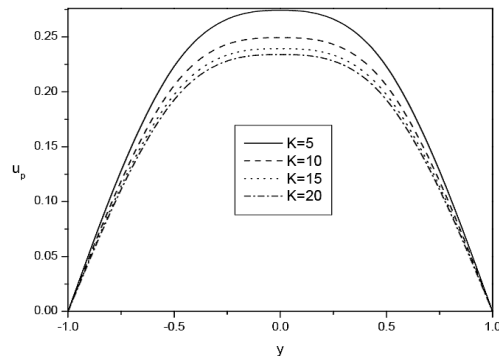


Fig2

From Fig. 3 it is observed that increasing values of magnetic parameter, the values of flow rate are decreasing and also as thickness of the porous medium is increasing, the flow rate is decreasing

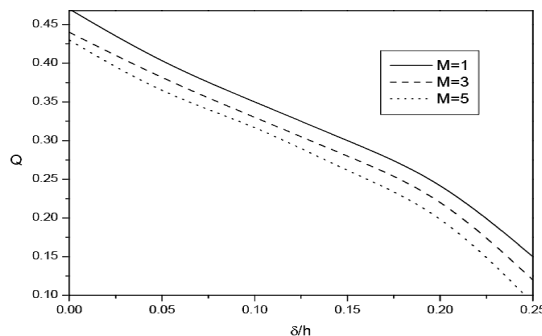


Fig3

From Fig. 4 it is observed that increasing values of permeability parameter, the values of flow rate are decreasing and also as thickness of the porous medium is increasing, the flow rate is decreasing

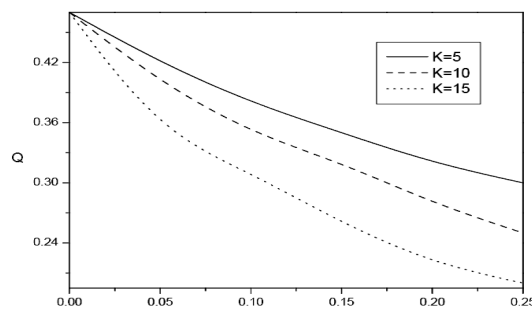


Fig4

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