

Utilization of Jung-Kim-Srivastava Integral Operator over Univalent & Analytic Function

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ABSTRACT: In this paper, using Jung-kim-Shrivastava integral operator, we have introduced new class of analytic and univalent functions in the unit disc $\mathcal{U} = \{z: |z| < 1\}$ and obtain the coefficient estimate, distortion and growth theorem and radius of starlikeness and convexity for the functions belonging to the class. The various results obtained in this paper are sharp.

KEYWORDS: Analytic function, Univalent function, starlike function, convex function, close to convex function, Hadamard product, Jung-Kim-Srivastava Integral operator.

I. INTRODUCTION

Let S stand in favor of mappings of the structure

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \quad k \in \mathbb{N} \quad (1.1)$$

whichever re

gular & one to one in unit disc $\mathcal{U} = \{z/|z| < 1\}$.

Let $S^*(\alpha, \beta)$ exist the subfamily prior to S according afore mappings of the structure (1.1) & fulfilling effective circumstance

$$\left| \frac{\frac{z(I^\sigma f(z))'}{I^\sigma f(z)} - 1}{\frac{\beta z(I^\sigma f(z))'}{I^\sigma f(z)} - \alpha \frac{z(I^\sigma f(z))'}{I^\sigma f(z)}} \right| < \mu, \quad p \in \mathbb{N}, \quad z \in \mathcal{U} \quad (1.2)$$

where $-1 \leq \alpha < \beta \leq 1$ and $0 < \mu \leq 1$.

Recently, Jung, Kim, and Srivastava [5] introduced the following one parameter family of integral operator

$$I^\sigma f(z) = \frac{2^\sigma}{z\Gamma(\sigma)} \int_0^z \left(\log \frac{z}{x} \right)^{\sigma-1} f(x) dx \quad (1.3)$$

For $f(z) \in S$ and $\sigma > 0$ They showed that

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$$I^\sigma f(z) = z + \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^\sigma a_k z^k \tag{1.4}$$

The operator I^σ is closed related to the multiplier transformations studied earlier by Flett [3]. Certain properties of this operator have been investigated by Jung, Kim and Srivastava [4], Uralegaddi and Somanatha [9], Li and author [6]. Using this operator I^σ we shall obtain some basic properties of univalent and analytic functions.

II. COEFFICIENT ESTIMATE

Theorem (i): A function

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \quad a_k \geq 0 \text{ is in } S^*(\alpha, \beta) \text{ if and only if}$$

$$\sum_{k=2}^{\infty} [(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^\sigma a_k \leq \mu(\beta - \alpha) \tag{1.5}$$

Proof : suppose $f(z)$ is in $S^*(\alpha, \beta)$, it is sufficient to show that

$$\left| \frac{\frac{z(I^\sigma f(z))'}{I^\sigma f(z)} - 1}{\frac{\beta z(I^\sigma f(z))'}{I^\sigma f(z)} - \alpha \frac{z(I^\sigma f(z))'}{I^\sigma f(z)}} \right| \leq \mu$$

we have

$$\left| \frac{\frac{z(I^\sigma f(z))'}{I^\sigma f(z)} - 1}{\frac{\beta z(I^\sigma f(z))'}{I^\sigma f(z)} - \alpha \frac{z(I^\sigma f(z))'}{I^\sigma f(z)}} \right| = \left| \frac{\frac{z + \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^\sigma k a_k z^k}{z + \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^\sigma a_k z^k} - 1}{\frac{\beta z + \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^\sigma \beta k a_k z^k}{z + \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^\sigma a_k z^k} - \alpha \frac{\alpha z - \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^\sigma \alpha k a_k z^k}{z + \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^\sigma a_k z^k}} \right|$$

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$$\begin{aligned}
 &= \left| \frac{z + \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^{\sigma} ka_k z^k - z - \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^{\sigma} a_k z^k}{\beta z + \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^{\sigma} \beta ka_k z^k - \alpha z - \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^{\sigma} \alpha ka_k z^k} \right| \\
 &= \left| \frac{\sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^{\sigma} (k-1) a_k z^k}{(\beta - \alpha) z + \sum_{k=2}^{\infty} (\beta - \alpha) \left(\frac{2}{k+2}\right)^{\sigma} ka_k z^k} \right| \leq \mu \\
 &\therefore \left| \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^{\sigma} (k-1) a_k z^k \right| \leq \mu \left| (\beta - \alpha) + \sum_{k=2}^{\infty} (\beta - \alpha) \left(\frac{2}{k+2}\right)^{\sigma} ka_k z^k \right| \\
 &\sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^{\sigma} (k-1) a_k |z^k| - \mu \sum_{k=2}^{\infty} (\beta - \alpha) \left(\frac{2}{k+2}\right)^{\sigma} ka_k |z^k| \leq \mu (\beta - \alpha) \\
 &\sum_{k=2}^{\infty} [(k-1) - \mu (\beta - \alpha) k] \left(\frac{2}{k+2}\right)^{\sigma} a_k |z^k| \leq \mu (\beta - \alpha)
 \end{aligned}$$

Allowing $z \rightarrow 1$ through the real values we obtain

$$\sum_{k=2}^{\infty} [(k-1) - \mu (\beta - \alpha) k] \left(\frac{2}{k+2}\right)^{\sigma} a_k \leq \mu (\beta - \alpha)$$

Conversely
Suppose that

$$\sum_{k=2}^{\infty} [(k-1) - \mu (\beta - \alpha) k] \left(\frac{2}{k+2}\right)^{\sigma} a_k \leq \mu (\beta - \alpha)$$

We have

$$\therefore \left| \sum_{k=2}^{\infty} \left(\frac{2}{k+2}\right)^{\sigma} (k-1) a_k z^k \right| - \mu \left| (\beta - \alpha) + \sum_{k=2}^{\infty} (\beta - \alpha) \left(\frac{2}{k+2}\right)^{\sigma} ka_k z^k \right| \leq 0$$

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$$\therefore \left| \frac{\frac{z(I^\sigma f(z))'}{I^\sigma f(z)} - 1}{\frac{\beta z(I^\sigma f(z))'}{I^\sigma f(z)} - \alpha \frac{z(I^\sigma f(z))'}{I^\sigma f(z)}} \right| \leq \mu$$

$\therefore f(z) \in S^*(\alpha, \beta)$

Hence proved.

Corollary (i): If $f(z) \in S^*(\alpha, \beta)$ then

$$a_k \leq \frac{\mu(\beta - \alpha)}{\left[(k-1) - \mu(\beta - \alpha)k \right] \left(\frac{2}{k+2} \right)^\sigma} \tag{1.6}$$

And the equality holds for

$$f(z) = z + \frac{\mu(\beta - \alpha)}{\left[(k-1) - \mu(\beta - \alpha)k \right] \left(\frac{2}{k+2} \right)^\sigma} z^k \tag{1.7}$$

II. DISTORTION AND GROWTH THEOREM

Theorem (ii): If $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ $a_k \geq 0$ is in $S^*(\alpha, \beta)$ then

$$\begin{aligned} & \left| z| - |z|^2 \frac{\mu(\beta - \alpha)}{\left[(k-1) - \mu(\beta - \alpha)k \right] \left(\frac{2}{k+2} \right)^\sigma} \right| \leq |f(z)| \\ & \leq |z| + |z|^2 \frac{\mu(\beta - \alpha)}{\left[(k-1) - \mu(\beta - \alpha)k \right] \left(\frac{2}{k+2} \right)^\sigma} \end{aligned} \tag{1.8}$$

With the equality hold for

$$f(z) = z + \frac{\mu(\beta - \alpha)}{\left[(k-1) - \mu(\beta - \alpha)k \right] \left(\frac{2}{k+2} \right)^\sigma} z^k$$

Proof : $f(z) \in S^*(\alpha, \beta)$

Then from (1.5)

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$$\sum_{k=2}^{\infty} [(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma} a_k \leq \mu(\beta - \alpha)$$

$$\sum_{k=2}^{\infty} a_k \leq \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}$$

Now $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$

$$\begin{aligned} |f(z)| &\leq |z| + \sum_{k=2}^{\infty} |a_k| |z|^k \leq |z| + |z|^2 \sum_{k=2}^{\infty} |a_k| \\ &\leq |z| + |z|^2 \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}} \end{aligned}$$

Similarly

$$\begin{aligned} |f(z)| &\geq |z| - \sum_{k=2}^{\infty} |a_k| |z|^k \geq |z| - |z|^2 \sum_{k=2}^{\infty} |a_k| \\ &\geq |z| - |z|^2 \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}} \end{aligned}$$

Therefore

$$\begin{aligned} |z| - |z|^2 \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}} &\leq |f(z)| \\ &\leq |z| + |z|^2 \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}} \end{aligned}$$

With the equality hold for

$$f(z) = z + z^2 \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}$$

Theorem (iii) : If $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$, $a_k \geq 0$ is in $S^*(\alpha, \beta)$ then

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$$\begin{aligned}
 1 - |z| \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^\sigma} &\leq |f'(z)| \\
 &\leq 1 + |z| \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^\sigma}
 \end{aligned} \tag{1.9}$$

With the equality hold for

$$f(z) = 1 + z \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^\sigma} \tag{1.10}$$

Proof : $f(z) \in S^*(\alpha, \beta)$

Then from (1.6)

$$\begin{aligned}
 \sum_{k=2}^{\infty} [(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^\sigma a_k &\leq \mu(\beta - \alpha) \\
 \sum_{k=2}^{\infty} a_k &\leq \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^\sigma}
 \end{aligned}$$

Now $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$

$$f'(z) = 1 + \sum_{k=2}^{\infty} a_k k z^{k-1}$$

$$\begin{aligned}
 |f'(z)| &\leq |1| + \sum_{k=2}^{\infty} |a_k| k |z|^{k-1} \leq 1 + |z| \sum_{k=2}^{\infty} |a_k| \\
 &\leq 1 + |z| \frac{\mu(\beta - \alpha)}{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^\sigma}
 \end{aligned}$$

Similarly

$$|f'(z)| \leq |1| + \sum_{k=2}^{\infty} |a_k| k |z|^{k-1} \leq 1 + |z| \sum_{k=2}^{\infty} |a_k|$$

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$$\leq 1 + |z| \frac{\mu(\beta - \alpha)}{\left[(k-1) - \mu(\beta - \alpha)k \right] \left(\frac{2}{k+2} \right)^\sigma}$$

Therefore

$$1 - |z| \frac{\mu(\beta - \alpha)}{\left[(k-1) - \mu(\beta - \alpha)k \right] \left(\frac{2}{k+2} \right)^\sigma} \leq |f'(z)| \leq 1 + |z| \frac{\mu(\beta - \alpha)}{\left[(k-1) - \mu(\beta - \alpha)k \right] \left(\frac{2}{k+2} \right)^\sigma}$$

With the equality hold for

$$f(z) = 1 + z \frac{\mu(\beta - \alpha)}{\left[(k-1) - \mu(\beta - \alpha)k \right] \left(\frac{2}{k+2} \right)^\sigma}$$

IV. RADII OF STARLIKENESS AND CONVEXITY

A mapping $f \in S$ sure be starlike mapping of ordain γ , $0 \leq \gamma < 1$

if $\operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > \gamma$, for all $z \in \mathcal{U}$

A mapping $f \in S$ sure be convex mapping of ordain γ , $0 \leq \gamma < 1$

if $\operatorname{Re}\left(1 + \frac{zf'(z)}{f(z)}\right) > \gamma$, $z \in \mathcal{U}$

Theorem (iv) : If $f(z) \in S^*(\alpha, \beta)$ then f is a starlike mapping of ordain γ , $0 \leq \gamma < 1$ in $|z| < R_1(\alpha, \beta, \mu, \gamma)$ where

$$R_1 = |z| \leq \left(\frac{(1-\gamma) \left[(k-1) - \mu(\beta - \alpha)k \right] \left(\frac{2}{k+2} \right)^\sigma}{(k + \gamma - 2) \mu(\beta - \alpha)} \right)^{k-1} \tag{1.11}$$

Proof : We know that

$$\left| \frac{zf'(z)}{f(z)} - 1 \right| \leq 1 - \gamma \tag{1.12}$$

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$$\begin{aligned}
 zf'(z) - f(z) &= \sum_{k=2}^{\infty} (k-1)a_k z^k \\
 \therefore \left| \frac{zf'(z)}{f(z)} - 1 \right| &= \left| \frac{\sum_{k=2}^{\infty} (k-1)a_k z^k}{z + \sum_{k=2}^{\infty} a_k z^k} \right| \leq \frac{\sum_{k=2}^{\infty} (k-1)|a_k||z|^{k-1}}{1 + \sum_{k=2}^{\infty} |a_k||z|^{k-1}} \leq 1 - \gamma \\
 \sum_{k=2}^{\infty} (k-1)|a_k||z|^{k-1} &\leq (1-\gamma) \left(1 + \sum_{k=2}^{\infty} |a_k||z|^{k-1} \right) \\
 \sum_{k=2}^{\infty} \frac{(k+\gamma-2)}{(1-\gamma)} |a_k||z|^{k-1} &\leq 1 \tag{1.13}
 \end{aligned}$$

From (1.6) we have

$$\sum_{k=2}^{\infty} \frac{[(k-1) - \mu(\beta-\alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}{\mu(\beta-\alpha)} |a_k| \leq 1 \tag{1.14}$$

From (1.13) and (1.14) we get

$$\begin{aligned}
 \frac{(k+\gamma-2)}{(1-\gamma)} |z|^{k-1} &\leq \frac{[(k-1) - \mu(\beta-\alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}{\mu(\beta-\alpha)} \\
 |z|^{k-1} &\leq \frac{(1-\gamma)[(k-1) - \mu(\beta-\alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}{(k+\gamma-2)\mu(\beta-\alpha)} \\
 \therefore |z| &\leq \left(\frac{(1-\gamma)[(k-1) - \mu(\beta-\alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}{(k+\gamma-2)\mu(\beta-\alpha)} \right)^{k-1}
 \end{aligned}$$

that inclusive the confirmation of the *Theorem*.

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Theorem (v) : If $f(z) \in S^*(\alpha, \beta)$ then f is convex mapping of ordain $\gamma, 0 \leq \gamma < 1$ in $|z| < R_2(\alpha, \beta, \mu, \gamma)$ where

$$R_2 = \inf_k \left(\frac{(1-\gamma)[(k-1) - \mu(\beta-\alpha)k] \left(\frac{2}{k+2}\right)^\sigma}{k(k+\gamma-2)\mu(\beta-\alpha)} \right)^{\frac{1}{k-1}} \quad (1.15)$$

Proof : We identify to facilitate f is convex mapping iff zf' is starlike mapping.

We have to show with the intention of

$$\left| \frac{zg'(z)}{g(z)} - 1 \right| \leq 1 - \gamma \quad \text{where } g(z) = zf'(z)$$

$$g(z) = z + \sum_{k=2}^{\infty} ka_k z^k$$

$$zg'(z) = z + \sum_{k=2}^{\infty} k^2 a_k z^k$$

$$\begin{aligned} \therefore zg'(z) - g(z) &= z + \sum_{k=2}^{\infty} k^2 a_k z^k - z - \sum_{k=2}^{\infty} ka_k z^k \\ &= \sum_{k=2}^{\infty} k(k-1) a_k z^k \end{aligned}$$

Now

$$\begin{aligned} \left| \frac{zg'(z)}{g(z)} - 1 \right| &= \left| \frac{zg'(z) - g(z)}{g(z)} \right| \\ &= \frac{\sum_{k=2}^{\infty} k(k-1) |a_k| |z|^k}{z + \sum_{k=2}^{\infty} k |a_k| |z|^k} \end{aligned}$$

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$$= \frac{\sum_{k=2}^{\infty} k(k-1)|a_k||z|^{k-1}}{1 + \sum_{k=2}^{\infty} k|a_k||z|^{k-1}} \leq 1 - \gamma$$

$$\begin{aligned} &\therefore \sum_{k=2}^{\infty} k(k-1)|a_k||z|^{k-1} \\ &\leq (1-\gamma) \left(1 + \sum_{k=2}^{\infty} k|a_k||z|^{k-1} \right) \\ &\sum_{k=2}^{\infty} \frac{k(k+\gamma-2)}{(1-\gamma)} |a_k||z|^{k-1} \leq 1 \end{aligned}$$

From (1.6) we have

$$\sum_{k=2}^{\infty} \frac{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}{\mu(\beta - \alpha)} |a_k| \leq 1 \tag{1.16}$$

From equation (1.5) and (1.16)

$$\begin{aligned} \sum_{k=2}^{\infty} \frac{k(k+\gamma-2)}{(1-\gamma)} |z|^{k-1} &\leq \sum_{k=2}^{\infty} \frac{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}{\mu(\beta - \alpha)} \\ \frac{k(k+\gamma-2)}{(1-\gamma)} |z|^{k-1} &\leq \frac{[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}{\mu(\beta - \alpha)} \\ |z|^{k-1} &\leq \frac{(1-\gamma)[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}{k(k+\gamma-2)\mu(\beta - \alpha)} \\ R_2 &= \inf_k \left(\frac{(1-\gamma)[(k-1) - \mu(\beta - \alpha)k] \left(\frac{2}{k+2}\right)^{\sigma}}{k(k+\gamma-2)\mu(\beta - \alpha)} \right)^{\frac{1}{k-1}} \end{aligned}$$

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V. CONCLUSION

Using Jung-kim-Shrivastava integral operator, we can obtain the coefficient estimate, distortion and growth theorem and radius of starlikeness and convexity of analytic and univalent functions

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k$$

of class $S^*(\alpha, \beta)$. The results obtained in this paper are sharp.

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