

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Vol. 9, Issue 2, February 2020

On Pre Generalized Regular Beta (PGRB) Closed Sets in Topological Spaces

A. Manonmani¹, S. Jayalakshmi²

Assistant Professor, Department of Mathematics, L.R.G Government Arts College (W), Tirupur, Tamilnadu, India¹

Assistant Professor, Department of Mathematics, L.R.G Government Arts College (W), Tirupur, Tamilnadu, India²

Corresponding Author ¹

ABSTRACT: The scope of this paper is to introduce a new type of closed sets called pre generalized regular beta (briefly $pgr\beta$ -closed sets) closed sets in a Topological spaces. Some properties of $pgr\beta$ -closed set in topological spaces are investigated in this connection and they are compared with the existing closed sets. Some properties of $pgr\beta$ -open set are also discussed. Further $pgr\beta$ -continuity, $pgr\beta$ -irresoluteness and $pgr\beta$ - $T_{1/2}$ space are introduced.

KEYWORDS: $r\beta$ open, pre-closure, $pgr\beta$ -closed sets, $pgr\beta$ -continuous function, $pgr\beta$ -irresolute, $pgr\beta$ - $T_{1/2}$ space

I. INTRODUCTION

N. Levine [10] introduced the concept of generalized closed sets in general topology as a generalization of closed sets in a Topological space in the year 1970. M.H. Stone [16] introduced regular open sets in a Topological space. A.S. Mashour, M. E. Abd El-Monsef, S. N. El-Deep and R. A. Mahmoud [13] introduced pre continuity in Topological spaces in the year 1982 and also M. E. Abd El-Monsef, S. N. El-Deep and R. A. Mahmoud [1] studied β -open sets and β -closed sets in 1983. D. Andrijevic [2] introduced Semi-preopen sets in 1986 which is equivalent to β open sets introduced by M. E. Abd. El-Monsef, S. N. El-Deep and R. A. Mahmoud [1]. In this paper, we define and study the properties of $pgr\beta$ -closed sets. Throughout this paper, space (X, τ) (or simply X) always means a topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X , $cl(A)$, $int(A)$ and A^c denote the closure of A , the interior of A and complement of A in X respectively. Moreover in this paper we define $pgr\beta$ open sets, $pgr\beta$ -continuity, $pgr\beta$ -irresoluteness and $pgr\beta$ - $T_{1/2}$ space and few of their properties are studied.

II. PRELIMINARIES

Definition: 2.1.

A subset A of a topological space (X, τ) is called

- a preopen set [13] if $A \subseteq int(cl(A))$ and a preclosed set if $cl(int(A)) \subseteq A$,
- a semi open set [9] if $A \subseteq cl(int(A))$ and a semiclosed set if $int(cl(A)) \subseteq A$
- a semi pre open (or β -open set [1]) set [2] if $A \subseteq cl(int(cl(A)))$ and a α -closed set if $int(cl(int(A))) \subseteq A$,
- a regular open set [16] if $A = int(cl(A))$ and a regular closed set [21] if $A = cl(int(A))$.
- a regular semi-open [6] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.

The pre-closure (resp. β -closure, r -closure) of a subset A of X denoted by $pcl(A)$ (resp. $\beta cl(A)$) (or $\beta cl(A)$) and $rcl(A)$) is defined to be the intersection of all pre-closed sets (resp. β -closed sets and r -closed sets) containing A .

Definition: 2.2.

A subset A of a topological space (X, τ) is called

- a generalized closed set (briefly g -closed) [10] in X if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X ,

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- ii) a generalized preclosed set (briefly gp-closed)[11] in X if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X ,
- iii) a generalized regular closed set (briefly gr-closed)[5] in X if $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X ,
- iv) a regular beta-closed set (briefly r β -closed)[12] in X if $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is β open in X ,
- v) a semi generalized closed set (briefly sg-closed)[4] in X if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X ,
- vi) a generalized semi closed set (briefly gs-closed)[3] in X if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X ,
- vii) a regular pre semi closed set (briefly rps-closed)[15] in X if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular generalized open (briefly rg open) in X
- viii) a generalized star-closed set (briefly g^* -closed)[17] in X if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g open in X ,
- ix) a generalized pre-closed set (briefly g^*p -closed)[18] in X if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g open in X ,
- x) a generalized pre-closed set (briefly gr^* -closed) [8] in X if $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g open in X ,
- xi) a generalized pre-regular set (briefly gpr-closed) [7] in X if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X ,
- xii) a generalized semi pre regular (or generalized beta regular)-closed set ((briefly gspr (or $g\beta r$ -closed)) [13] in X if $\text{spcl}(A) \subseteq U$ (or $\beta\text{cl}(A) \subseteq U$) whenever $A \subseteq U$ and U is regular open in X .

The compliment of above mentioned closed sets in X are their respective open sets in X .

III. PREGENERALIZED REGULAR BETA (PGRB) CLOSED SETS IN TOPOLOGICAL SPACES.

Definition: 3.1.

A subset A of a topological space (X, τ) is called pre generalized regular beta closed [briefly pgr β -closed] in X , if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $r\beta$ -open in (X, τ) .

Proposition: 3.2.

Let (X, τ) be a Topological space, then

- (i) Every closed set in X is pgr β -closed in X ,
- (ii) Every pre-closed set in X is pgr β -closed in X ,
- (iii) Every g closed set in X is pgr β -closed in X ,
- (iv) Every r -closed set in X is pgr β -closed in X ,
- (v) Every gr-closed set in X is pgr β -closed in X ,
- (vi) Every gp-closed set in X is pgr β -closed in X ,
- (vii) Every g^* -closed set in X is pgr β -closed in X ,
- (viii) Every g^*p -closed set in X is pgr β -closed in X ,
- (ix) Every $r\beta$ -closed set in X is pgr β -closed in X ,
- (x) Every pgr β -closed set in X is gpr-closed in X ,
- (xi) Every pgr β -closed set in X is gspr (or $g\beta r$)-closed in X .

Proof.

- (i) Let A be a closed set in X and $A \subseteq U$ where U is $r\beta$ -open in X . Then $A = \text{cl}(A)$. Since $\text{pcl}(A) \subseteq \text{cl}(A) = A \subseteq U$, $\text{pcl}(A) \subseteq U$. Hence A is pgr β -closed in X .
- (ii) Let A be a pre-closed in X and $A \subseteq U$ where U is $r\beta$ -open in X . Then $\text{pcl}(A) = A \subseteq U$. Hence A is pgr β -closed in X .
- (iii) Let A be a g -closed set and $A \subseteq U$ where U is $r\beta$ -open in X . Since $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$, $\text{pcl}(A) \subseteq U$. Hence A is pgr β -closed in X .
- (iv) Let A be a r -closed set in X and $A \subseteq U$ where U is $r\beta$ -open in X . Since $\text{pcl}(A) \subseteq \text{rcl}(A) \subseteq U$, $\text{pcl}(A) \subseteq U$. Hence A is pgr β -closed in X .
- (v) Let A be a gr-closed set in X and $A \subseteq U$ where U is $r\beta$ -open in X . Since $\text{pcl}(A) \subseteq \text{rcl}(A) \subseteq U$, $\text{pcl}(A) \subseteq U$. Also every $r\beta$ open set in X is an open set in X . Hence A is pgr β -closed in X .
- (vi) Let A be a gp-closed set in X and $A \subseteq U$ where U is $r\beta$ -open in X . Here

International Journal of Innovative Research in Science, Engineering and Technology

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$pcl(A) \subseteq U$ and every $r\beta$ open set in X is an open set in X . Hence A is $pgr\beta$ -closed in X .

(vii) Let A be g^* -closed in X and $A \subseteq U$ where U is $r\beta$ -open in X . Then $pcl(A) \subseteq cl(A) \subseteq U$. Also every $r\beta$ open set in X is a g open set in X . Hence A is $pgr\beta$ -closed.

(viii) Let A be g^*p -closed in X and $A \subseteq U$ where U is $r\beta$ -open in X . Then $pcl(A) \subseteq U$. Also every $r\beta$ open set in X is a g open set in X . Hence A is $pgr\beta$ -closed in X .

(ix) Let A be $r\beta$ -closed in X and $A \subseteq U$ where U is $r\beta$ -open in X . Then $pcl(A) \subseteq rcl(A) \subseteq U$. Also every $r\beta$ open set in X is a β open set in X . Hence A is $pgr\beta$ -closed in X .

(x) Let A be $pgr\beta$ -closed in X and $A \subseteq U$ where U is $r\beta$ -open in X . Then $pcl(A) \subseteq U$. Also every regular open set in X is $r\beta$ open in X . Hence A is gpr closed in X .

(xi) Let A be $pgr\beta$ -closed in X and $A \subseteq U$ where U is $r\beta$ -open in X . Then $\beta cl(A) \subseteq pcl(A) \subseteq U$. Also every regular open set in X is $r\beta$ open in X . Hence A is $g\beta r$ -closed in X .

Remark: 3.3

The converse of the above propositions need not be true as seen from the following example.

Example: 3.4

Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$. Then

- (i) $\{a\}$ is $pgr\beta$ -closed in X but not closed in X
- (ii) $\{a\}$ is $pgr\beta$ -closed in X but not pre-closed in X
- (iii) $\{a, c\}$ is $pgr\beta$ -closed in X but not g -closed in X
- (iv) $\{a, d\}$ is $pgr\beta$ -closed in X but not r -closed in X
- (v) $\{a, c, d\}$ is $pgr\beta$ -closed in X but not gr -closed in X
- (vi) $\{a, c\}$ is $pgr\beta$ -closed in X but not gp -closed in X
- (vii) $\{a, d\}$ is $pgr\beta$ -closed in X but not g^* -closed in X
- (viii) $\{b, c, d\}$ is $pgr\beta$ -closed in X but not g^*p -closed in X
- (ix) $\{b\}$ is $pgr\beta$ -closed in X but not $r\beta$ -closed in X
- (x) $\{c, d\}$ is gpr -closed in X but not $pgr\beta$ -closed in X
- (xi) $\{d\}$ is $g\beta r$ -closed in X but not $pgr\beta$ -closed in X

Remark: 3.5

The following example shows that $pgr\beta$ -closed sets in X are independent of (i) β -closed, (ii) rps -closed, (iii) gs -closed, (iv) sg -closed, (v) rs -closed (vi) semi-closed (vii) gr^* -closed in X .

Example: 3.6

Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$. Then

- (i) $\{c\}$ is β -closed in X but not $pgr\beta$ -closed in X , $\{a, c, d\}$ is $pgr\beta$ -closed in X but not β -closed in X .
- (ii) $\{d\}$ is rps -closed in X but not $pgr\beta$ -closed in X , $\{b, c, d\}$ is $pgr\beta$ -closed in X but not rps -closed in X .
- (iii) $\{c\}$ is gs -closed in X but not $pgr\beta$ -closed in X , $\{a, c, d\}$ is $pgr\beta$ -closed in X but not gs -closed in X .
- (iv) $\{d\}$ is sg -closed in X but not $pgr\beta$ -closed in X , $\{b, c, d\}$ is $pgr\beta$ -closed in X but not sg -closed in X .
- (v) $\{c\}$ is rs -closed in X but not $pgr\beta$ -closed in X , $\{a, c\}$ is $pgr\beta$ -closed in X but not rs -closed in X .
- (vi) $\{c\}$ is semi-closed in X but not $pgr\beta$ -closed in X , $\{a, c, d\}$ is $pgr\beta$ -closed in X but not semi-closed in X .
- (vii) $\{d\}$ is gr^* -closed in X but not $pgr\beta$ -closed in X , $\{a, c, d\}$ is $pgr\beta$ -closed in X but not rps -closed in X .

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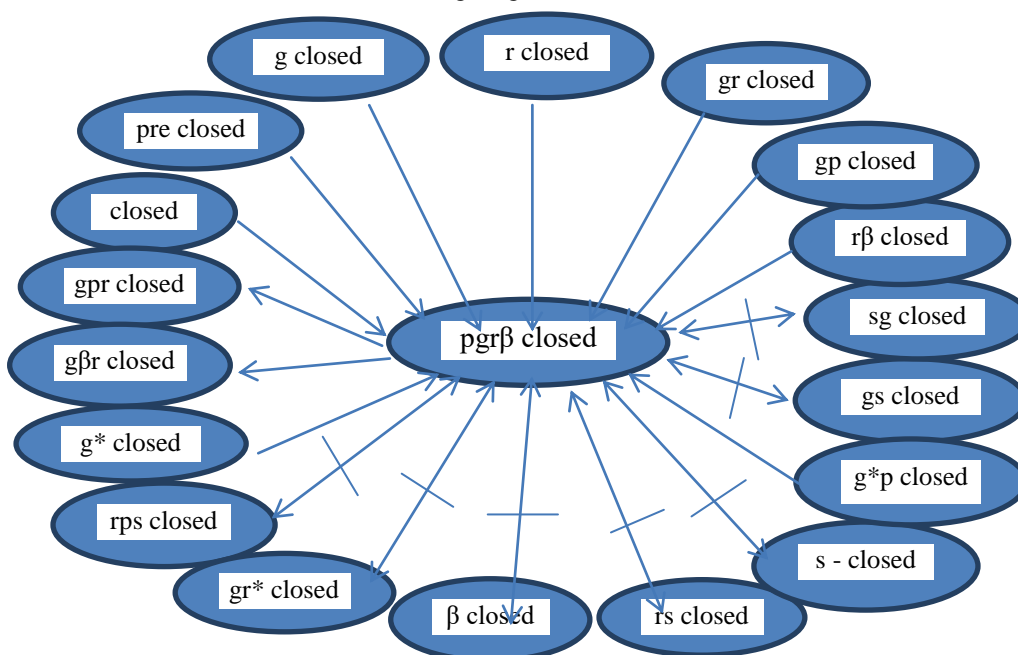
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Remark : 3.7

The above discussions are summarized in the following diagram.



In the above diagram, A \rightarrow B implies A is contained in B
A \leftrightarrow B implies A and B are independent

Remark: 3.8

The union of two $pgr\beta$ -closed sets of a topological space (X, τ) is generally not $pgr\beta$ -closed in X as seen from the following example.

Example: 3.9

Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{a\}, \{b, d\}, \{a, b, d\}, X\}$. Then $\{b\}$ and $\{d\}$ are $pgr\beta$ -closed sets but their union $\{b, d\}$ is not a $pgr\beta$ -closed set in X .

Remark: 3.10

The intersection of two $pgr\beta$ -closed sets of a topological space (X, τ) is generally not $pgr\beta$ -closed in X as seen from the following example.

Example: 3.11

Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$. Then $\{a, d\}$ and $\{b, d\}$ are $pgr\beta$ -closed sets but their intersection $\{d\}$ is not a $pgr\beta$ -closed set in X .

Theorem: 3.12.

If a subset A of X is $pgr\beta$ -closed in X , then $pcl(A) - A$ does not contain any nonempty $r\beta$ -closed set in X .

Proof:

Suppose that A is $pgr\beta$ -closed set in X . We prove the statement by assuming the contradiction. So let U be a nonempty $r\beta$ -closed set such that $U \subseteq pcl(A) - A$. that is $U \subseteq pcl(A) \cap A^c$. Therefore $U \subseteq pcl(A)$ and $U \subseteq A^c$ i.e $U \subseteq X - A$, which

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implies $A \subseteq X - U$. Since U is $r\beta$ -closed, $X - U$ is $r\beta$ open in X . Since A is an $pgr\beta$ -closed set in X , by the definition we have $pcl(A) \subseteq X - U$. So $U \subseteq X - pcl(A)$. Also $U \subseteq pcl(A)$. Therefore $U \subseteq pcl(A) \cap (X - pcl(A)) = \emptyset$, which is a contradiction. Hence $pcl(A) - A$ does not contain any nonempty $r\beta$ closed set in X .

Theorem: 3.13

For an element $x \in X$, the set $X - \{x\}$ is either $pgr\beta$ -closed in X or $r\beta$ open in X .

Proof:

Suppose $X - \{x\}$ is not $r\beta$ open in X . Then X is only $r\beta$ open set containing $X - \{x\}$ and also $pcl(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is $pgr\beta$ -closed set in X .

Theorem: 3.14

If A is a $pgr\beta$ -closed subset of X such that $A \subseteq B \subseteq pcl(A)$, then B is a $pgr\beta$ -closed set in X .

Proof:

Let A be a $pgr\beta$ -closed set of X such that $A \subseteq U \subseteq pcl(A)$. Let $B \subseteq U$ and U be $r\beta$ open set in X . Then $A \subseteq U$. Since A is $pgr\beta$ -closed in X , we have $pcl(A) \subseteq U$. Now $pcl(U) \subseteq pcl(pcl(A)) = pcl(A) \subseteq U$. Therefore U is $pgr\beta$ -closed set in X .

Theorem: 3.13.

Let A be an $pgr\beta$ -closed in (X, τ) . Then A is pre-closed in X if and only if $pcl(A) - A$ is $r\beta$ closed in X .

Proof:

Suppose A is pre-closed in X . Then $pcl(A) = A$ and so $pcl(A) - A = \emptyset$, which is $r\beta$ closed in X . Conversely, suppose $pcl(A) - A$ is $r\beta$ closed in X . Since A is $pgr\beta$ -closed, by Theorem 3.10, $pcl(A) - A$ does not contain any nonempty $r\beta$ closed in X . Then $pcl(A) - A = \emptyset$. Therefore $pcl(A) = A$. Hence A is pre-closed.

Theorem: 3.14.

If X and ϕ are the only $r\beta$ open sets then all the subsets of X are $pgr\beta$ -closed in X .

Proof:

Let A be a subset of X . If $A = \phi$, then A is $pgr\beta$ -closed. If $A \neq \phi$, then X is the only $r\beta$ open set containing A . This implies $pcl(A) \subseteq X$. Hence A is $pgr\beta$ -closed in X .

Theorem: 3.15.

A subset A of a space X is $pgr\beta$ -closed if and only if for each $A \subseteq U$ and U is $r\beta$ open in X , there exists a pre-closed set F such that $A \subseteq F \subseteq U$.

Proof:

Suppose A is a $pgr\beta$ -closed set and $A \subseteq U$ and U is $r\beta$. Then $pcl(A) \subseteq U$. If we put $F = pcl(A)$, then since $A \subseteq pcl(A)$, F is pre-closed and $A \subseteq F \subseteq U$.

Conversely, assume that U is a $r\beta$ open set containing A . Then there exists a pre-closed set F such that $A \subseteq F \subseteq U$. Since $pcl(A)$ is the smallest pre-closed set containing A , we have $A \subseteq pcl(A) \subseteq F$. Also since $F \subseteq U$, we have $pcl(A) \subseteq U$. Hence A is $pgr\beta$ -closed in X .

Theorem: 3.16

If A is pre-closed in X and B is $pgr\beta$ -closed in X , then $A \cup B$ is $pgr\beta$ -closed in X .

Proof:

Let U be a $r\beta$ open set containing $A \cup B$ in X . Then $A \subseteq U$ and $B \subseteq U$. Since B is $pgr\beta$ -closed in X and $B \subseteq U$, we have $pcl(B) \subseteq U$. Then $A \cup B \subseteq A \cup pcl(B) \subseteq U$. Since A is pre-closed in X , we have $A \cup pcl(B)$ is pre-closed in X . Hence there exists a pre-closed set $A \cup pcl(B)$ such that $A \cup B \subseteq A \cup pcl(B) \subseteq U$. Therefore, by Theorem: 3.15. $A \cup B$ is $pgr\beta$ -closed in X .

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(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Theorem: 3.17.

If a subset A of a topological space X is both $r\beta$ -open and $pgr\beta$ -closed in X , then A is pre-closed in X .

Proof:

Suppose A is both $r\beta$ -open and $pgr\beta$ -closed. Then $pcl(A) \subseteq A$. Also, $A \subseteq pcl(A)$. This implies, $A = pcl(A)$. Therefore, A is pre-closed.

4. Pre generalized regular beta open sets in Topological spaces

Definition:4.1.

A subset A of a topological space (X, τ) is called pre generalized regular beta open [briefly $pgr\beta$ -open] in X if A^c is $pgr\beta$ -closed in X .

Proposition:4.2.

Let (X, τ) be a Topological space then

- (i) Every open set in X is $pgr\beta$ -open in X .
- (ii) Every preopen set in X is $pgr\beta$ -open in X .
- (iii) Every g -open set in X is $pgr\beta$ -open in X .
- (iv) Every r -open set in X is $pgr\beta$ -open in X .
- (v) Every gr -open set in X is $pgr\beta$ -open in X .
- (vi) Every gp -open set in X is $pgr\beta$ -open in X .
- (vii) Every g^* -open set in X is $pgr\beta$ -open in X .
- (viii) Every g^*p -open set in X is $pgr\beta$ -open in X .
- (ix) Every $r\beta$ -open set in X is $pgr\beta$ -open in X .
- (x) Every $pgr\beta$ -open set in X is gpr -open in X .
- (xi) Every $pgr\beta$ -open set in X is $g\beta r$ -open in X .

Proof:

- (i) Let A be an open set in X . Then A^c is closed set in X . By (i) of proposition 3.2, A^c is $pgr\beta$ -closed in X . Hence A is $pgr\beta$ -open in X .
- (ii) Let A be a pre-open set in X . Then A^c is pre-closed set in X . By (ii) of proposition 3.2, A^c is $pgr\beta$ -closed in X . Hence A is $pgr\beta$ -open in X .
- (iii) Let A be a g -open set in X . Then A^c is g -closed set in X . By (iii) of proposition 3.2, A^c is $pgr\beta$ -closed in X . Hence A is $pgr\beta$ -open in X .
- (iv) Let A be an r -open set in X . Then A^c is r -closed set in X . By (iv) of proposition 3.2, A^c is $pgr\beta$ -closed in X . Hence A is $pgr\beta$ -open in X .
- (v) Let A be a gr -open set in X . Then A^c is gr -closed set in X . By (v) of proposition 3.2, A^c is $pgr\beta$ -closed in X . Hence A is $pgr\beta$ -open in X .
- (vi) Let A be a gp -open set in X . Then A^c is gp -closed set in X . By (vi) of proposition 3.2, A^c is $pgr\beta$ -closed in X . Hence A is $pgr\beta$ -open in X .
- (vii) Let A be a g^* -open set in X . Then A^c is g^* -closed set. By (vii) of proposition 3.2, A^c is $pgr\beta$ -closed in X . Hence A is $pgr\beta$ -open in X .
- (viii) Let A be a g^*p -open set in X . Then A^c is g^*p -closed set in X . By (viii) of proposition 3.2, A^c is $pgr\beta$ -closed in X . Hence A is $pgr\beta$ -open in X .
- (ix) Let A be an $r\beta$ -open set in X . Then A^c is $r\beta$ -closed set in X . By (ix) of proposition 3.2, A^c is $pgr\beta$ -closed in X . Hence A is $pgr\beta$ -open in X .
- (x) Let A be a $pgr\beta$ -open set in X . Then A^c is $pgr\beta$ -closed set in X . By (x) of proposition 3.2, A^c is gpr -closed in X . Hence A is gpr -open in X .
- (xi) Let A be a $pgr\beta$ -open set in X . Then A^c is $pgr\beta$ -closed set. By (xi) of proposition 3.2, A^c is $g\beta r$ -closed in X . Hence A is $g\beta r$ -open in X .

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Remark: 4.3

The converse of the above propositions need not be true as seen from the following example.

Example: 4.4

Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$. Then

- (i) $\{a\}$ is $\text{pgr}\beta$ -open in X but not open in X
- (ii) $\{b\}$ is $\text{pgr}\beta$ -open in X but not pre-open in X
- (iii) $\{b, c, d\}$ is $\text{pgr}\beta$ -open in X but not g -open in X
- (iv) $\{a, d\}$ is $\text{pgr}\beta$ -open in X but not r -open in X
- (v) $\{b\}$ is $\text{pgr}\beta$ -open in X but not gr -open in X
- (vi) $\{b, c\}$ is $\text{pgr}\beta$ -open in X but not gp -open in X
- (vii) $\{b, c, d\}$ is $\text{pgr}\beta$ -open in X but not g^* -open in X
- (viii) $\{a, c\}$ is $\text{pgr}\beta$ -open in X but not g^*p -open in X
- (ix) $\{b\}$ is $\text{pgr}\beta$ -open in X but not $r\beta$ -open in X
- (x) $\{a, b\}$ is gpr -open in X but not $\text{pgr}\beta$ -open in X
- (xi) $\{a, b, c\}$ is $g\beta r$ -open in X but not $\text{pgr}\beta$ -open in X

Remark: 4.6

The intersection of two $\text{pgr}\beta$ -open sets of a topological space (X, τ) is generally not a $\text{pgr}\beta$ -open in X as seen from the following example.

Example: 4.7

Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{a\}, \{b, d\}, \{a, b, d\}, X\}$. Then $\{a, b, c\}$ and $\{a, c, d\}$ are $\text{pgr}\beta$ -open sets but their intersection $\{a, c\}$ is not an $\text{pgr}\beta$ -open set in X .

Remark: 4.8

The union of two $\text{pgr}\beta$ -open sets of a topological space (X, τ) is generally not a $\text{pgr}\beta$ -open in X as seen from the following example.

Example: 4.9

Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$. Then $\{a\}$ and $\{b\}$ are $\text{pgr}\beta$ -open sets but their union $\{a, b\}$ is not an $\text{pgr}\beta$ -open set in X .

Theorem: 4.10

If a set A is $\text{pgr}\beta$ -open set in X , then $G = X$, whenever G is $r\beta$ open in X and $\text{int}(A) \cup A^C \subset G$.

Proof:

Suppose that A is $\text{pgr}\beta$ -open in X . Let G be $r\beta$ open in X and $\text{int}(A) \cup A^C \subseteq G$. This implies $G^C \subseteq (\text{int}(A) \cup A^C)^C = (\text{int}(A))^C \cap A$ i.e; $G^C \subseteq (\text{int}(A))^C - A^C$. Since $(\text{int}(A))^C = \text{cl}(A^C)$, $G^C \subseteq \text{cl}(A^C) - A^C$. Now G^C is also becomes $r\beta$ open set, since A^C is $\text{pgr}\beta$ -closed in X . We know that $A \subseteq \text{cl}(A)$. So $[\text{cl}(A)]^C \subseteq A^C$, $[\text{cl}(A)]^C - A^C \subseteq \phi$. $G^C = \phi$ and hence $G = X$.

Theorem : 4.11

Every singleton point set in a space X is either $\text{pgr}\beta$ -open in X or $r\beta$ open in X .

Proof:

Let $x \in X$. To prove $\{x\}$ is either $\text{pgr}\beta$ -open in X or $r\beta$ -open in X , it is enough to prove $X - \{x\}$ is either $\text{pgr}\beta$ -closed in X or $r\beta$ -open in X . Suppose $X - \{x\}$ is not an $r\beta$ -open set in X . Then X is the only $r\beta$ -open set containing $X - \{x\}$. This implies, $\text{pcl}(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is $\text{pgr}\beta$ -closed set in X . Therefore $\{x\}$ is $\text{pgr}\beta$ -open in X .

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5.pgrβ-Continuous and pgrβ-Irresolute Functions in Topological Spaces

Definition: 5.1

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called pgrβ- continuous if $f^{-1}(V)$ is pgrβ- closed in (X, τ) for every closed set V of (Y, σ) .

Example: 5.2

Let $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ and $Y = \{a, b, c, d\}$, $\sigma = \{\Phi, \{a\}, \{b, d\}, \{a, b, d\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = d$, $f(b) = c$, $f(c) = b$ and $f(d) = a$. Then f is pgrβ- continuous.

Theorem: 5.3

The composition of two pgrβ- continuous functions need not be pgrβ- continuous.

Example: 5.4

Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\Phi, \{c\}, \{b, c\}, X\}$, $\sigma = \{\Phi, \{b, c\}, Y\}$ and $\eta = \{\Phi, \{a\}, \{b\}, \{a, b\}, Z\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$, and define $g : (Y, \sigma) \rightarrow (Z, \eta)$ by $g(a) = a$, $g(b) = b$, $g(c) = c$ then f and g are pgrβ- continuous. Now $\{c\}$ is closed in (Z, η) , then $(g \circ f)^{-1}(\{c\}) = f^{-1}(g^{-1}(\{c\})) = f^{-1}(\{c\}) = \{c\}$, which is not pgrα- closed in (X, τ) . Hence $g \circ f$ is not pgrβ- continuous.

Definition: 5.5

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called pgrβ- irresolute if $f^{-1}(V)$ is pgrβ- closed in (X, τ) for every pgrβ- closed set V of (Y, σ) .

Example: 5.6

Let $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ and $Y = \{a, b, c, d\}$, $\sigma = \{\Phi, \{a\}, \{b, d\}, \{a, b, d\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(c) = a$, $f(b) = b$. The inverse image of every pgrβ- closed set in Y is pgrβ- closed in X under f . Hence f is pgrβ- irresolute.

Theorem: 5.7

Every pgrβ- irresolute function is pgrβ- continuous.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a pgrβ- irresolute function. Then the inverse image of every pgrβ- closed set in Y is pgrβ- closed in X under f . But every closed set in Y is pgrβ- closed set in Y . Therefore the inverse image of every closed set in Y is pgrβ- closed in X under f . Hence f is pgrβ- continuous.

Remark: 5.8

Converse of the above proposition need not be true as seen from the following example.

Example: 5.8

Consider $X = Y = \{a, b, c\}$, $\tau = \{\Phi, \{a, b\}, X\}$, $\sigma = \{\Phi, \{a\}, \{a, c\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(b) = b$, $f(c) = a$. Here inverse image of all pgrβ- closed sets are not pgrβ- closed set.

Theorem: 5.9

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions, then

- (i) $g \circ f$ is pgrβ- continuous if g is continuous and f is pgrβ- continuous.
- (ii) $g \circ f$ is pgrβ- irresolute, if g is pgrβ- irresolute and f is pgrβ- irresolute.
- (iii) $g \circ f$ is pgrβ- continuous, if g is pgrβ- continuous and f is pgrβ- irresolute.

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Proof:

- (i) Let V be closed in (Z, η) . Then $g^{-1}(V)$ is closed in (Y, σ) , since g is continuous. Since f is $pgr\beta$ -continuous, $f^{-1}(g^{-1}(V))$ is $pgr\beta$ -closed in (X, τ) . That is $(g \circ f)^{-1}(V)$ is $pgr\beta$ -closed in (X, τ) . Hence $g \circ f$ is $pgr\beta$ -continuous.
- (ii) Let V be $pgr\beta$ -closed in (Z, η) . Since g is $pgr\beta$ -irresolute, $g^{-1}(V)$ is $pgr\beta$ -closed in (Y, σ) . Since f is $pgr\beta$ -irresolute, then $f^{-1}(g^{-1}(V))$ is $pgr\beta$ -closed in (X, τ) . That is $(g \circ f)^{-1}(V)$ is $pgr\beta$ -closed in (X, τ) . Hence $g \circ f$ is $pgr\beta$ -irresolute.
- (iii) Let V be closed in (Z, η) . Since g is $pgr\beta$ -continuous then $g^{-1}(V)$ is $pgr\beta$ -closed in (Y, σ) . Since f is $pgr\beta$ -irresolute then $f^{-1}(g^{-1}(V))$ is $pgr\beta$ -closed in (X, τ) . That is $(g \circ f)^{-1}(V)$ is $pgr\beta$ -closed in (X, τ) . Hence $g \circ f$ is $pgr\beta$ -continuous

Definition: 5.10

A Topological space (X, τ) is called $pgr\beta$ - $T_{1/2}$ space if every $pgr\beta$ -closed set is pre closed.

Example: 5.11

Consider $X = \{a, b, c\}, \tau = \{\Phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$. Here every $pgr\beta$ -closed set is pre closed in X . Hence the space (X, τ) is a $pgr\beta$ - $T_{1/2}$ space.

Theorem: 5.12

If $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function, then

- (i) If f is $pgr\beta$ -irresolute and X is $pgr\beta$ - $T_{1/2}$ space, then f is pre irresolute,
(ii) If f is $pgr\beta$ -continuous and X is $pgr\beta$ - $T_{1/2}$ space, then f is pre continuous.

Proof:

(i) Let V be pre closed in Y , then V is $pgr\beta$ -closed in Y . Since f is $pgr\beta$ -irresolute, $f^{-1}(V)$ is $pgr\beta$ -closed in X . Since X is $pgr\beta$ - $T_{1/2}$ space, $f^{-1}(V)$ is pre closed in X . Hence f is pre irresolute.

(ii) Let V be closed in Y . Since f is $pgr\beta$ -continuous, $f^{-1}(V)$ is $pgr\beta$ -closed in X . Since X is $pgr\beta$ - $T_{1/2}$ space, $f^{-1}(V)$ is pre closed in X . Hence f is pre continuous.

Theorem: 5.13

For a Topological space (X, τ) , the following are equivalent.

- (i) X is $pgr\beta$ - $T_{1/2}$ space.
(ii) Every singleton is either regular β -closed or pre open.

Proof :

Suppose $\{x\}$ is not a regular β -closed subset for some $x \in X$. Then $X - \{x\}$ is not regular β -open and hence X is the only regular β -open set containing $X - \{x\}$. Therefore $X - \{x\}$ is $pgr\beta$ -closed. Since (X, τ) is $pgr\beta$ - $T_{1/2}$ space, $X - \{x\}$ is pre closed and then $\{x\}$ is pre open. Hence (i) implies (ii)

Let A be a $pgr\beta$ -closed subset of (X, τ) and $x \in \text{pcl}(A)$. We will show that $x \in A$. If $\{x\}$ is regular β -closed and $x \notin A$, then $\{x\} \in \text{pcl}(A) - A$. Thus $\text{pcl}(A) - A$ contains a nonempty regular β -closed set $\{x\}$, which is a contradiction to Theorem 3.13. So $x \in A$. If $\{x\}$ is pre-open, since $x \in \text{pcl}(A)$ then for every pre-open set U containing x , we have $U \cap A \neq \Phi$. But $\{x\}$ is pre open then $\{x\} \cap A \neq \Phi$. Hence $x \in A$, so in both cases we have $x \in A$. Therefore A is pre-closed set. Hence (ii) implies (i).

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