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# $\beta\alpha$ -Regular Spaces and $\beta\alpha$ -Normal Spaces in Topology

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**ABSTRACT:** : The aim of this paper is to introduce and study some weak forms of regular spaces and weak forms of normal spaces, viz.  $\beta\alpha$ -regular spaces,  $\alpha\beta$ -regular spaces,  $\beta\alpha$ -normal spaces by using  $\beta$ -closed sets and  $\alpha$ -open sets. Also, we studied some related functions like  $\beta\alpha$ -closed functions,  $\beta\alpha$ -continuous functions for preserving these regular spaces and normal spaces.

**KEYWORDS:** semiopen sets,  $\alpha$ -open sets,  $\alpha$ -open sets,  $\beta$ -closed sets,  $\beta$ -open sets, E.D. spaces, PS-spaces,  $\beta\alpha$ -closedness,  $\beta\alpha$ -continuity,  $\beta$ -continuity,  $\beta$ -irresoluteness,  $\alpha$ -irresoluteness

## I. INTRODUCTION

In 1965, O.Njastad [34] introduced and studied the concepts of  $\alpha$ -open sets. In 1983, A.S.Mashhour et al [24] have introduced the concepts of  $\alpha$ -closed sets,  $\alpha$ -continuity,  $\alpha$ -closedness in topological spaces and in 1983, M.E.Abd El-Monsef et al [1] have introduced the concepts of  $\beta$ -open sets and  $\beta$ -continuity in topology. Latter, these  $\beta$ -open sets are recalled as semipreopen sets, which were introduced by D.Andrejevic in [6]. Further these  $\alpha$ -open sets and  $\beta$ -open sets (= semipreopen sets) have been studied by various authors in the literature see [2, 4, 11, 14, 21, 26, 28, 29 & 36]. In 1970 and 1994, respectively Levine [16] and H.Maki et al et al [22] have defined and studied the concept of  $\beta$ -closed sets and  $\alpha\beta$ -closed sets in topology. The aim of this paper is to introduce and study some weak forms of regular spaces and weak forms of normal spaces, viz.  $\beta\alpha$ -regular spaces,  $\alpha\beta$ -regular spaces,  $\beta\alpha$ -normal spaces by using  $\beta$ -closed sets and  $\alpha$ -open sets. Also, we studied some related functions like  $\beta\alpha$ -closed functions,  $\beta\alpha$ -continuous functions for preserving these regular spaces and normal spaces.

## II. PRELIMINARIES

Throughout this paper  $X, Y$  will denote topological spaces on which no separation axioms assumed unless explicitly stated. Let  $f : X \rightarrow Y$  represent a single valued function. Let  $A$  be a subset of  $X$ . The closure and interior of  $A$  are respectively denoted by  $Cl(A)$  and  $Int(A)$ .

The following definitions and results are useful in the sequel.

**Definition 2.1:** A subset  $A$  of  $X$  is called

- (i) semiopen (in short, s-open) set [17] if  $A \subset ClInt(A)$ .
- (ii) preopen (in short, p-open) set [23] if  $A \subset IntCl(A)$ .
- (iii)  $\beta$ -open [1] (=semipreopen [6]) set if  $A \subset ClIntCl(A)$ .
- (iv)  $\alpha$ -open [34] set if  $A \subset IntClInt(A)$

The complement of a s-open (resp. p-open,  $\beta$ -open,  $\alpha$ -open) set is called s-closed [9] (resp. p-closed [12],  $\beta$ -closed [1],  $\alpha$ -closed [24]) set. The family of s-open (resp. p-open,  $\beta$ -open,  $\alpha$ -open) sets of  $X$  is denoted by  $SO(X)$  (resp.  $PO(X)$ ,  $\beta O(X)$ ,  $\alpha O(X)$ ) and that of s-closed (resp. p-closed,  $\beta$ -closed,  $\alpha$ -closed) sets is denoted by  $SF(X)$  (resp.  $PF(X)$ ,  $\beta F(X)$ ,  $\alpha F(X)$ ).

**Definition 2.2:** The intersection of all  $\alpha$ -closed (resp.  $\beta$ -closed) sets containing a subset  $A$  of space  $X$  is called the  $\alpha$ -closure [24] (resp.  $\beta$ -closure [2]) of  $A$  and is denoted by  $\alpha Cl(A)$  (resp.  $\beta Cl(A)$ ).

**Definition 2.3:** The union of all  $\alpha$ -open (resp.  $\beta$ -open) sets which are contained in  $A$  is called the  $\alpha$ -interior [24] (resp. the  $\beta$ -interior [2]) of  $A$  and is denoted by  $\alpha Int(A)$  (resp.  $\beta Int(A)$ ).

**Definition 2.4 [22]:** A subset  $A$  of a space  $X$  is called  $\alpha\beta$ -closed if  $\alpha Cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open set in  $X$ .

**Definition 2.5 [3]:** A space  $X$  is said to be  $\beta$ -regular if for each closed set  $F$  and for each  $x \in X - F$ , there exist two disjoint  $\beta$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $F \subset V$ .

**Definition 2.6 [19]:** A space  $X$  is said to be  $s$ -regular if for each closed set  $F$  and for each  $x \in X-F$ , there exist two disjoint  $s$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $F \subset V$ .

**Definition 2.7 [10]:** A space  $X$  is said to be  $\alpha$ -regular if for each closed set  $F$  and for each  $x \in X-F$ , there exist two disjoint  $\alpha$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $F \subset V$ .

**Definition 2.8 [18]:** A space  $X$  is said to be  $s$ -normal if for any pair of disjoint closed subsets  $A$  and  $B$  of  $X$ , there exist disjoint  $s$ -open sets  $U$  and  $V$  such that  $A \subset U$  and  $B \subset V$ .

**Definition 2.9 [25]:** A space  $X$  is said to be  $\alpha$ -normal if for any pair of disjoint closed subsets  $A$  and  $B$  of  $X$ , there exist disjoint  $\alpha$ -open sets  $U$  and  $V$  such that  $A \subset U$  and  $B \subset V$ .

**Definition 2.10 [21]:** A space  $X$  is said to be  $\beta$ -normal if for any pair of disjoint closed subsets  $A$  and  $B$  of  $X$ , there exist disjoint  $\beta$ -open sets  $U$  and  $V$  such that  $A \subset U$  and  $B \subset V$ .

**Definition 2.11 [8]:** A space  $X$  is said to be submaximal if every dense set of  $X$  is open in  $X$  (i.e., every preopen set of  $X$  is open in  $X$ ).

**Definition 2.12[37]:** A space  $X$  is said to be extremally disconnected (in brief, E.D.,) space if  $Cl(G)$  is open set for each open set  $G$  of  $X$ .

**Definition 2.13[13]:** A space  $X$  is said to be  $\alpha$ -space iff  $\tau = \alpha O(X)$ .

**Definition 2.14[5]:** A space  $X$  is said to be PS-space if every preopen subset of  $X$  is semiopen in it.

**Definition 3.15 [28]:** A topological space  $X$  is said to be  $\beta p$ -regular if for each  $\beta$ -closed set  $F$  of  $X$  and each point  $x$  in  $X - F$ , there exist disjoint  $p$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $F \subset V$ .

**Definition 2.16[32]:** A topological space  $X$  is said to be  $\beta s$ -regular if for each  $\beta$ -closed set  $F$  of  $X$  and each point  $x$  in  $X - F$ , there exist disjoint  $s$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $F \subset V$ .

**Definition 2.17 [32]:** A space  $X$  is said to be  $\beta^*$ -regular if for each  $\beta$ -closed set  $F$  and for each  $x \in X-F$ , there exist disjoint  $\beta$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $F \subset V$ .

**Definition 2.18 [32]:** A space  $X$  is said to be strongly  $\beta^*$ -regular if for each  $\beta$ -closed set  $F$  and each  $x \notin F$ , there exist disjoint open sets  $U$  and  $V$  such that  $F \subset U$  and  $x \in V$ .

**Definition 2.19 [28]:** A space  $X$  is said to be  $\beta p$ -normal if for each pair of disjoint  $\beta$ -closed set  $F$  and  $K$ , there exist disjoint  $p$ -open sets  $U$  and  $V$  such that  $F \subset U$  and  $K \subset V$ .

**Definition 2.20 [32]:** A space  $X$  is said to be  $\beta s$ -normal if for each pair of disjoint  $\beta$ -closed set  $F$  and  $K$ , there exist disjoint  $s$ -open sets  $U$  and  $V$  such that  $F \subset U$  and  $K \subset V$ .

**Definition 2.21 [30&31]:** A space  $X$  is said to be  $\beta^*$ -normal if for each pair of disjoint  $\beta$ -closed set  $F$  and  $K$ , there exist disjoint  $\beta$ -open sets  $U$  and  $V$  such that  $F \subset U$  and  $K \subset V$ .

**Definition 2.22 [28]:** A space  $X$  is said to be strongly  $\beta^*$ -normal if for each pair of disjoint  $\beta$ -closed set  $F$  and  $K$ , there exist disjoint open sets  $U$  and  $V$  such that  $F \subset U$  and  $K \subset V$ .

**Definition 2.23[21]:** A space  $X$  is called  $\beta-T_1$  space if for each pair of distinct points  $x$  and  $y$  of  $X$ , there exist  $\beta$ -open sets  $U$  and  $V$  such that  $x \in U$  &  $y \notin U$  and  $y \in V$  &  $x \notin V$ .

**Definition 2.24 [20]:** A space  $X$  is called  $\alpha-T_2$  space if for each pair of distinct points  $x$  and  $y$  of  $X$ , there exist disjoint  $\alpha$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $y \in V$ .

**Definition 2.25 [21]:** A function  $f: X \rightarrow Y$  is said to be  $\beta$ -irresolute if  $f^{-1}(V)$  is  $\beta$ -open set in  $X$  for each for each  $\beta$ -open set  $V$  in  $Y$ .

**Definition 2.26[20]:** A function  $f: X \rightarrow Y$  is said to be  $\alpha$ -irresolute if  $f^{-1}(V)$  is  $\alpha$ -open set in  $X$  for each for each  $\alpha$ -open set  $V$  in  $Y$ .

**Definition 2.27[10]:** A function  $f: X \rightarrow Y$  is said to be pre- $\alpha$ -open if  $f(V)$  is  $\alpha$ -open set in  $Y$  for each for each  $\alpha$ -open set  $V$  in  $X$ .

**Definition 2.28[10]:** A function  $f: X \rightarrow Y$  is said to be pre- $\alpha$ -closed if  $f(V)$  is  $\alpha$ -closed set in  $Y$  for each for each  $\alpha$ -closed set  $V$  in  $X$ .

**Definition 2.29[21]:** A function  $f: X \rightarrow Y$  is said to be  $M$ - $\beta$ -closed if the image of each  $\beta$ -closed set of  $X$  is  $\beta$ -closed in  $Y$ .

**Lemma 2.30 [13]:** If  $X$  is an  $\alpha$ -space then  $\tau = \alpha O(X)$ .

**Lemma 2.31[24]:** Let  $A$  and  $V$  be subsets of  $X$ . If  $A \in PO(X)$  and  $V \in \alpha O(X)$ , then  $A \cap V$  is  $\alpha$ -open set in the subspace  $A$ .

**Lemma 2.32 [5]:** For a space  $X$  the following conditions are equivalent:

- (i)  $X$  is E.D.-space
- (ii) every  $s$ -open subset is  $p$ -open
- (iii) every  $s$ -open subset is  $\alpha$ -open
- (iv) every  $\beta$ -open subset is  $p$ -open

**Lemma 2.33 [5]:** For a space  $X$  the following conditions are equivalent:



- (i) X is PS-space
- (ii) every p-open subset is s-open
- (iii) every  $\beta$ -open subset is s-open
- (iv) every p-open subset is  $\alpha$ -open

**THEOREM 2.34 [15 & 33]:** In an E.D.-space and submaximal-space X, then  $\tau = \alpha O(X) = SO(X) = PO(X) = \beta O(X)$ .

### III. PROPERTIES OF $\beta\alpha$ -REGULAR SPACES

We, define the following.

**Definition 3.1 :** A topological space X is said to be  $\beta\alpha$ -regular if for each  $\beta$ -closed set F of X and each point x in X - F, there exist disjoint  $\alpha$ -open sets U and V such that  $x \in U$  and  $F \subset V$ .

**Definition 3.2:** A topological space X is said to be  $\alpha\beta$ -regular if for each  $\alpha$ -closed set F of X and each point x in X - F, there exist disjoint  $\beta$ -open sets U and V such that  $x \in U$  and  $F \subset V$ .

Clearly, (i) every  $\beta\alpha$ -regular  $\rightarrow$   $\alpha$ -regular, (ii)  $\beta\alpha$ -regular  $\rightarrow$   $\beta^*$ -regular, (iii) strongly  $\beta^*$ -regular  $\rightarrow$   $\beta\alpha$ -regular, (iv) strongly  $\beta^*$ -regular  $\rightarrow$  regular, (v)  $\beta^*$ -regular  $\rightarrow$   $\alpha\beta$ -regular and (vi)  $\beta\alpha$ -regular  $\rightarrow$   $\beta$ s-regular as well as  $\beta$ p-regular.

We, recall the following.

**Lemma 3.3 [ 2 ]:** If A is subset of X and  $B \in \beta O(X)$  such that  $A \cap B = \emptyset$  then  $\beta Cl(A) \cap B = \emptyset$ .

We, prove the following.

**Theorem 3.4 :** For a topological space X the following statements are equivalent ;

- (a) X is  $\beta\alpha$ -regular
- (b) For each  $x \in X$  and for each  $\beta$ -open set U containing x there exists a  $\alpha$ -open set V containing x such that  $x \in V \subset \alpha Cl(V) \subset U$ .
- (c) For each  $\beta$ -closed set F of X,  $\bigcap \{ \alpha Cl(V) \mid F \subset V \text{ and } V \in \alpha O(X) \} = F$
- (d) For each nonempty subset A of X and each  $U \in \beta O(X)$  if  $A \cap U \neq \emptyset$  then there exists  $V \in \alpha O(X)$  such that  $A \cap V \neq \emptyset$  and  $\alpha Cl(V) \subset U$
- (e) For each nonempty subset A of X and each  $F \in \beta F(X)$  if  $A \cap F = \emptyset$  then there exists  $V, W \in \alpha O(X)$  such that  $A \cap V \neq \emptyset, F \subset W$  and  $V \cap W = \emptyset$ .

**Proof :** (a)  $\Rightarrow$  (b) Let X be  $\beta\alpha$ -regular space. Let  $x \in X$  and U be  $\beta$ -open set containing x implies  $(X - U)$  is  $\beta$ -closed such that  $x \notin (X - U)$ . Therefore by (a) there exists two  $\alpha$ -open sets V and W such that  $x \in V$  and  $(X - U) \subset W \Rightarrow (X - W) \subset U$ . Since  $V \cap W = \emptyset \Rightarrow \alpha Cl(V) \cap W = \emptyset \Rightarrow \alpha Cl(V) \subset (X - W) \subset U$ . Therefore,  $x \in V \subset \alpha Cl(V) \subset U$ .

(b)  $\Rightarrow$  (c) Let F be a  $\beta$ -closed subset of X and  $x \notin F$ , then  $(X - F)$  is  $\beta$ -open set containing x. By (b) there exists  $\alpha$ -open set U such that  $x \in U \subset \alpha Cl(U) \subset (X - F)$  implies  $F \subset (X - \alpha Cl(U)) \subset (X - U)$  i.e  $F \subset V \subset (X - U)$  where  $V = (X - \alpha Cl(U)) \in \alpha O(X)$  and  $x \notin V$  that implies  $x \notin \alpha Cl(V)$  implies  $x \notin \bigcap \{ \alpha Cl(V) \mid F \subset V \in \alpha O(X) \}$ . Hence,  $\bigcap \{ \alpha Cl(V) \mid F \subset V \in \alpha O(X) \} = F$

(c)  $\Rightarrow$  (d) A be a subset of X and  $U \in \beta O(X)$  such that  $A \cap U \neq \emptyset \Rightarrow$  there exists  $x_0 \in X$  such that  $x_0 \in A \cap U$ . Therefore  $(X - U)$  is  $\beta$ -closed set not containing  $x_0 \Rightarrow x_0 \notin \beta Cl(X - U)$ . By (c), there exists  $W \in \alpha O(X)$  such that  $(X - U) \subset W \Rightarrow x_0 \notin \alpha Cl(W)$ . Put  $V = (X - \alpha Cl(W))$ , then V is  $\alpha$ -open set containing  $x_0 \Rightarrow A \cap V \neq \emptyset$  and  $\alpha Cl(V) \subset \alpha Cl(X - \beta Cl(W)) \subset \alpha Cl(X - W)$ . Therefore,  $\alpha Cl(V) \subset \alpha Cl(X - W) \subset U$ .

(d)  $\Rightarrow$  (e) Let A be a nonempty subset of X and F be  $\beta$ -closed set such that  $A \cap F = \emptyset$ . Then  $(X - F)$  is  $\beta$ -open in X and  $A \cap (X - F) \neq \emptyset$ . Therefore by (d), there exist  $V \in \alpha O(X)$  such that  $A \cap V \neq \emptyset$  and  $\alpha Cl(V) \subset (X - F)$ . Put  $W = (X - \alpha Cl(V))$  then  $W \in \alpha O(X)$  such that  $F \subset W$  and  $W \cap V = \emptyset$ .

(f)  $\Rightarrow$  (a) Let  $x \in X$  be arbitrary and F be  $\beta$ -closed set not containing x. Let  $A = (X - F)$  be a nonempty  $\beta$ -open set containing x then by (e), there exist disjoint  $\alpha$ -open sets V and W such that  $F \subset W$  and  $A \cap V \neq \emptyset \Rightarrow x \in V$ . Thus X is a  $\beta\alpha$ -regular.

**Theorem 3.5 :** In a topological space X following statements are equivalent;

- (a) X is  $\beta\alpha$ -regular
- (b) for each  $\beta$ -open set U of X containing x there exists  $\alpha$ -open set V such that  $x \in V \subset \alpha Cl(V) \subset U$



**Proof :** (a)  $\Rightarrow$  (b) Let  $x \in X$  and  $U$  be  $\beta$ -open set of  $X$  containing  $x \Rightarrow (X - U)$  is  $\beta$ -closed set not containing  $x$ . As  $X$  is  $\beta\alpha$ -regular, there exist disjoint  $\alpha$ -open sets  $V$  and  $W$  such that  $x \in V$  and  $(X-U) \subset W \Rightarrow (X-W) \subset U$ . As  $V \cap W = \emptyset \Rightarrow \alpha Cl(V) \cap W = \emptyset \Rightarrow \alpha Cl(V) \subset (X-W) \subset U$ . Hence  $x \in V \subset \alpha Cl(V) \subset U$ .

(b)  $\Rightarrow$  (a) Let for each  $x \in X$ ,  $F$  be  $\beta$ -closed set not containing  $x$ , therefore  $(X-F)$  is  $\beta$ -open set containing  $x$  hence from (b) there exists  $\alpha$ -open set  $V$  such that  $x \in V \subset \alpha Cl(V) \subset (X-F)$ . Let  $U = (X - \alpha Cl(V))$  then  $U$  is  $\alpha$ -open set such that  $F \subset U$ ,  $x \in V$  and  $U \cap V = \emptyset$ . Thus there exists disjoint  $\alpha$ -open sets  $U$  and  $V$  such that  $x \in V$  and  $F \subset U$ . Therefore  $X$  is  $\beta\alpha$ -regular.

In [ 28] the following are proved :

**Lemma 3.6 :** For a space  $X$ , the following are true :

- (i) If  $X$  is  $\beta^*$ -regular space then it is  $\beta$ -regular.
- (ii) If  $X$  is strongly  $\beta^*$ -regular space then it is  $\beta^*$ -regular space.

**Theorem 3.7 :** For a topological space  $X$  the following statements are equivalent ;

- (a)  $X$  is  $\alpha\beta$ -regular
- (b) For each  $x \in X$  and for each  $\alpha$ -open set  $U$  containing  $x$  there exists a  $\beta$ -open set  $V$  containing  $x$  such that  $x \in V \subset \beta Cl(V) \subset U$ .
- (c) For each  $\alpha$ -closed set  $F$  of  $X$ ,  $\cap \{ \beta Cl(V) / F \subset V \text{ and } V \in \beta O(X) \} = F$
- (d) For each nonempty subset  $A$  of  $X$  and each  $U \in \alpha O(X)$  if  $A \cap U \neq \emptyset$  then there exists  $V \in \beta O(X)$  such that  $A \cap V \neq \emptyset$  and  $\beta Cl(V) \subset U$
- (e) For each nonempty subset  $A$  of  $X$  and each  $F \in \alpha F(X)$  if  $A \cap F = \emptyset$  then there exists  $V, W \in \beta O(X)$  such that  $A \cap V \neq \emptyset$ ,  $F \subset W$  and  $V \cap W = \emptyset$ .

The routine proof of the Theorem is omitted.

We, prove the following.

**Theorem 3.8 :**  $\beta-T_1$  space and  $\beta\alpha$ -regular space is  $\alpha-T_2$  space.

**Proof :** Let  $X$  be  $\beta-T_1$  space and  $\beta\alpha$ -regular space. As  $X$  is  $\beta-T_1$  space, every singleton set  $\{x\}$  is  $\beta$ -closed set for all  $x \in X$ .  $X$  being  $\beta\alpha$ -regular and  $\{x\}$  is a  $\beta$ -closed subset of  $X$  and  $y$  be any point of  $X - \{x\}$ , then  $x \neq y$ . By definition of  $\beta\alpha$ -regularity, there exist disjoint  $\alpha$ -open sets  $G$  and  $H$  such that  $\{x\} \subset G$  and  $y \in H$ . This implies that  $x \in G$  and  $y \in H$ . Therefore,  $X$  is  $\alpha-T_2$  space.

We, prove some subspace theorems in the following.

**Theorem 3.9 :** If  $X$  be a  $\beta\alpha$ -regular space and  $G \in PO(X)$ , then  $G$  is  $\beta\alpha$ -regular as subspace.

**Proof :** Let  $F$  be a  $\beta$ -closed set of  $G$  and  $x \in (G-F)$  then there exists a  $\beta$ -closed set  $H$  of  $X$  such that  $F = G \cap H$  and  $x \notin H$ . Since  $X$  is  $\beta\alpha$ -regular, therefore for each  $\beta$ -closed set  $H$  of  $X$  and  $x \notin H$ , there exist  $\alpha$ -open sets  $U_x$  and  $V_H$  of  $X$  such that  $x \in U_x$  and  $H \subset V_H$  with  $U_x \cap V_H = \emptyset$ . Now, put  $A = U_x \cap G$  and  $B = V_H \cap G$  then  $A$  and  $B$  are  $\alpha$ -open subsets of  $G$  by Lemma-2.31 such that  $x \in A$  and  $F \subset B$  with  $A \cap B = \emptyset$ . This shows that  $G$  is  $\beta\alpha$ -regular space.

Next, we prove some preservation theorems in the following.

**Theorem 3.10 :** If  $f : X \rightarrow Y$  is a pre- $\alpha$ -open,  $\beta$ -irresolute bijection and  $X$  is  $\beta\alpha$ -regular space, then  $Y$  is  $\beta\alpha$ -regular.

**Proof :** Let  $F$  be any  $\beta$ -closed subset of  $Y$  and  $y \in Y$  with  $y \notin F$ . Since  $f$  is  $\beta$ -irresolute,  $f^{-1}(F)$  is  $\beta$ -closed set in  $X$ . Again,  $f$  is bijective, let  $f(x) = y$ , then  $x \notin f^{-1}(F)$ . Since  $X$  is  $\beta\alpha$ -regular, there exist disjoint  $\alpha$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $f^{-1}(F) \subset V$ . Since  $f$  is pre- $\alpha$ -open bijection, we have  $y \in f(U)$  and  $F \subset f(V)$  and  $f(U) \cap f(V) = f(U \cap V) = \emptyset$  and  $f(U)$  and  $f(V)$  are  $\alpha$ -open set in  $Y$ . Hence,  $Y$  is  $\beta\alpha$ -regular space.

We, recall the following.

**Definition 3.11 [ 28]:** A function  $f : X \rightarrow Y$  is said to be always- $\beta$ -closed if the image of each  $\beta$ -closed subset of  $X$  is  $\beta$ -closed set in  $Y$ .

Now, we prove the following.

**Theorem 3.12 :** If  $f : X \rightarrow Y$  is an always  $\beta$ -closed,  $\alpha$ -irresolute injection and  $Y$  is  $\beta\alpha$ -regular space, then  $X$  is  $\beta\alpha$ -regular.

**Proof :** Let  $F$  be any  $\beta$ -closed set of  $X$  and  $x \notin F$ . Since  $f$  is an always  $\beta$ -closed injection,  $f(F)$  is  $\beta$ -closed set in  $Y$  and  $f(x) \notin f(F)$ . Since  $Y$  is  $\beta\alpha$ -regular space and so there exist disjoint  $\alpha$ -open sets  $U$  and  $V$  in  $Y$  such that  $f(x) \in U$  and  $f(F) \subset V$ . By hypothesis,  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $\alpha$ -open sets in  $X$  with  $x \in f^{-1}(U)$ ,  $F \subset f^{-1}(V)$  and  $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ . Hence,  $X$  is  $\beta\alpha$ -regular space.

We, define the following.

**Definition 3.13 :** A function  $f : X \rightarrow Y$  is said to be  $\beta\alpha$ -continuous if the inverse image of each  $\beta$ -open set of  $Y$  is  $\alpha$ -open set in  $X$ .

Next, we give the following.

**Theorem 3.14 :** If  $f : X \rightarrow Y$  is an always  $\beta$ -closed,  $\beta\alpha$ -continuous injection and  $Y$  is  $\beta$ -regular space, then  $X$  is  $\beta\alpha$ -regular.

**Note 3.15 :** In an  $\alpha$ -space, (i) regular spaces and  $\alpha$ -regular spaces coincide each other. (ii) strongly  $\beta^*$ -regular spaces and  $\beta\alpha$ -regular spaces coincide each other. see **Lemma-2.30**.

**Note 3.16 :** Usually, every  $\beta\alpha$ -regular space  $\rightarrow$   $\beta s$ -regular space, but in an E.D.-space, every  $\beta s$ -regular space  $\rightarrow$   $\beta\alpha$ -regular space, see **Lemma 2.32**.

**Note 3.17 :** Usually, every  $\beta\alpha$ -regular space  $\rightarrow$   $\beta p$ -regular space, but in a PS-space, every  $\beta p$ -regular space  $\rightarrow$   $\beta\alpha$ -regular space, see **Lemma 2.33**.

**Note 3.18 :** Usually, every regular space  $\rightarrow$   $\alpha$ -regular space  $\rightarrow$   $p$ -regular space  $\rightarrow$   $\beta$ -regular space and every regular space  $\rightarrow$   $\alpha$ -regular space  $\rightarrow$   $s$ -regular space  $\rightarrow$   $\beta$ -regular space but in an E.D.-space and submaximal space, Regular space =  $\alpha$ -regular space =  $s$ -regular space =  $p$ -regular space =  $\beta$ -regular space, see **Lemma 2.34**.

#### IV. $\beta\alpha$ -NORMAL SPACES

We, define the following.

**Definition 4.1 :** A function  $f : X \rightarrow Y$  is said to be  $\beta\alpha g$ -continuous if for each  $\beta$ -closed set  $F$  of  $Y$ ,  $f^{-1}(F)$  is  $\alpha g$ -closed set in  $X$ .

It is obvious that a function  $f : X \rightarrow Y$  is  $\beta\alpha g$ -continuous if and only if  $f^{-1}(V)$  is  $\alpha g$ -open in  $X$  for each  $\beta$ -open set  $V$  of  $Y$ .

**Definition 4.2:** A function  $f : X \rightarrow Y$  is said to be  $\beta\alpha g$ -closed if for each  $\beta$ -closed set  $F$  of  $X$ ,  $f(F)$  is  $\alpha g$ -closed set in  $Y$ .

We, recall the following.

**Definition 4.3[ 10] :** A function  $f : X \rightarrow Y$  is said to be :

- (i) pre- $\alpha g$ -continuous if the inverse image of each  $\alpha$ -closed set  $F$  of  $Y$  is  $\alpha g$ -closed in  $X$ .
- (ii) pre- $\alpha g$ -closed if the image of each  $\alpha$ -closed set of  $X$  is  $\alpha g$ -closed in  $Y$ .

Clearly, (i) every  $\beta\alpha g$ -continuous function is pre- $\alpha g$ -continuous,

(ii) every  $\beta\alpha g$ -closed function is pre- $\alpha g$ -closed, since in both cases every  $\alpha$ -closed set is  $\beta$ -closed set.

We, prove the following.

**Theorem 4.4 :** A surjective function  $f : X \rightarrow Y$  is  $\beta\alpha g$ -closed if and only if for each subset  $B$  of  $Y$  and each  $\beta$ -open set  $U$  of  $X$  containing  $f^{-1}(B)$ , there exists a  $\alpha g$ -open set  $V$  of  $Y$  such that  $B \subset V$  and  $f^{-1}(V) \subset U$ .

**Proof :** Suppose  $f$  is  $\beta\alpha g$ -closed function. Let  $B$  be any subset of  $Y$  and  $U$  be any  $\beta$ -open set in  $X$  containing  $f^{-1}(B)$ . Put  $V = Y - f(X - U)$ . Then,  $V$  is  $\alpha g$ -open set in  $Y$  such that  $B \subset V$  and  $f^{-1}(V) \subset U$ .

Conversely, let  $F$  be any  $\beta$ -closed set of  $X$ . Put  $B = Y - f(F)$ , then we have  $f^{-1}(B) \subset X - F$  and  $X - F$  is  $\beta$ -open in  $X$ . There exists a  $\alpha$  $g$ -open set  $V$  of  $Y$  such that  $B = Y - f(F) \subset V$  and  $f^{-1}(V) \subset X - F$ . Therefore, we obtain that  $f(F) = Y - V$  and hence  $f(F)$  is  $\alpha$  $g$ -closed set in  $Y$ . This shows that  $f$  is  $\beta\alpha$  $g$ -closed function.

We, define the following.

**Definition 4.5** [ ]: A function  $f : X \rightarrow Y$  is said to be strongly  $\beta$ -closed if the image of each  $\beta$ -closed set of  $X$  is closed in  $Y$ .

**Definition 4.6** : A function  $f : X \rightarrow Y$  is said to be always  $\alpha$  $g$ -closed if the image of each  $\alpha$  $g$ -closed set of  $X$  is  $\alpha$  $g$ -closed in  $Y$ .

**Theorem 4.7:** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. Then the composition function  $g \circ f : X \rightarrow Z$  is  $\beta\alpha$  $g$ -closed if  $f$  and  $g$  satisfy one of the following conditions :

- (i)  $f$  is  $\beta\alpha$  $g$ -closed and  $g$  is always  $\alpha$  $g$ -closed.
- (ii)  $f$  is strongly  $\beta$ -closed and  $g$  is  $\alpha$  $g$ -closed.

**Proof :** (i) Let  $H$  be any  $\beta$ -closed set in  $X$  and  $f$  is  $\beta\alpha$  $g$ -closed, then  $f(H)$  is  $\alpha$  $g$ -closed set in  $Y$ . Again,  $g$  is always  $\alpha$  $g$ -closed function and  $f(H)$  is  $\alpha$  $g$ -closed set in  $Y$ , then  $g \circ f(H)$  is  $\alpha$  $g$ -closed set in  $Z$ . This shows that  $g \circ f$  is  $\beta\alpha$  $g$ -closed function.

(ii) Let  $F$  be any  $\beta$ -closed set in  $X$  and  $f$  is strongly  $\beta$ -closed function, then  $f(F)$  is closed set in  $Y$ . Again,  $g$  is  $\alpha$  $g$ -closed function and  $f(F)$  is closed set in  $Y$ , then  $g \circ f(F)$  is  $\alpha$  $g$ -closed set in  $Z$ . This shows that  $g \circ f$  is  $\beta\alpha$  $g$ -closed function.

We, define the following.

**Definition 4.8** : A function  $f : X \rightarrow Y$  is said to be  $\alpha$  $g\beta$ -closed if for each  $\alpha$  $g$ -closed set  $F$  of  $X$ ,  $f(F)$  is  $\beta$ -closed set in  $Y$ .

**Definition 4.9** : A function  $f : X \rightarrow Y$  is said to be strongly  $\alpha$  $g$ -closed if for each  $\alpha$  $g$ -closed set  $F$  of  $X$ ,  $f(F)$  is closed set in  $Y$ .

**Theorem 4.10** : Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. Then,

- (i) if  $f$  is  $\beta$ -closed and  $g$  is  $\beta\alpha$  $g$ -closed, then  $g \circ f$  is  $\alpha$  $g$ -closed.
- (ii) if  $f$  is  $\beta\alpha$  $g$ -closed and  $g$  is strongly  $\alpha$  $g$ -closed, then  $g \circ f$  is strongly  $\beta$ -closed.
- (iii) if  $f$  is  $M$ - $\beta$ -closed and  $g$  is  $\beta\alpha$  $g$ -closed, then  $g \circ f$  is  $\beta\alpha$  $g$ -closed.
- (iv) if  $f$  is always  $\alpha$  $g$ -closed and  $g$  is  $\alpha$  $g\beta$ -closed, then  $g \circ f$  is  $\alpha$  $g\beta$ -closed.

**Proof :** (i). Let  $H$  be a closed set in  $X$ , then  $f(H)$  be  $\beta$ -closed set in  $Y$  since  $f$  is  $\beta$ -closed function. Again,  $g$  is  $\beta\alpha$  $g$ -closed and  $f(H)$  is  $\beta$ -closed set in  $Y$  then  $g(f(H)) = g \circ f(H)$  is  $\alpha$  $g$ -closed set in  $Z$ . Thus,  $g \circ f$  is  $\alpha$  $g$ -closed function.

(ii) Let  $F$  be any  $\beta$ -closed set in  $X$  and  $f$  is  $\beta\alpha$  $g$ -closed function, then  $f(F)$  is  $\alpha$  $g$ -closed set in  $Y$ . Again,  $g$  is strongly  $\alpha$  $g$ -closed and  $f(F)$  is  $\alpha$  $g$ -closed set in  $Y$ , then  $g \circ f(F)$  is closed set in  $Z$ . This shows that  $g \circ f$  is strongly  $\beta$ -closed function.

(iii) Let  $H$  be any  $\beta$ -closed set in  $X$  and  $f$  is  $M$ - $\beta$ -closed function then  $f(H)$  is  $\beta$ -closed set in  $Y$ . Again,  $g$  is  $\beta\alpha$  $g$ -closed function and  $f(H)$  is  $\beta$ -closed set in  $Y$ , then  $g \circ f(H)$  is  $\alpha$  $g$ -closed set in  $Z$ . Therefore,  $g \circ f$  is  $\beta\alpha$  $g$ -closed function.

(iv) Let  $H$  be any  $\alpha$  $g$ -closed set in  $X$  and  $f$  is always  $\alpha$  $g$ -closed function, then  $f(H)$  be  $\alpha$  $g$ -closed set in  $Y$ . Again,  $g$  is  $\alpha$  $g\beta$ -closed function and  $f(H)$  is  $\alpha$  $g$ -closed set in  $Y$ , then  $g \circ f(H)$  is  $\beta$ -closed set in  $Z$ . This shows that  $g \circ f$  is  $\alpha$  $g\beta$ -closed function.

**Theorem 4.11** : Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions and let the composition function  $g \circ f : X \rightarrow Z$  is  $\beta\alpha$  $g$ -closed. Then, the following hold :

- (i) if  $f$  is  $\beta$ -irresolute surjection, then  $g$  is  $\beta\alpha$  $g$ -closed.
- (ii) if  $g$  is  $\alpha$  $g$ -irresolute injection, then  $f$  is  $\beta\alpha$  $g$ -closed.

**Proof :** (i) Let  $F$  be a  $\beta$ -closed set in  $Y$ . Since  $f$  is  $\beta$ -irresolute surjective,  $f^{-1}(F)$  is  $\beta$ -closed set in  $X$  and  $(g \circ f)(f^{-1}(F)) = g(F)$  is  $\alpha$  $g$ -closed set in  $Z$ . This shows that  $g$  is  $\beta\alpha$  $g$ -closed function.

(ii) Let  $H$  be a  $\beta$ -closed set in  $X$ . Then,  $g \circ f(H)$  is  $\alpha$  $g$ -closed set in  $Z$ . Again,  $g$  is  $\alpha$  $g$ -irresolute injective,  $g^{-1}(g \circ f(H)) = f(H)$  is  $\alpha$  $g$ -closed set in  $Y$ . This shows that  $f$  is  $\beta\alpha$  $g$ -closed function.

We, define the following.

**Definition 4.12:** A space  $X$  is said to be  $\beta\alpha$ -normal if for any pair of disjoint  $\beta$ -closed sets  $A$  and  $B$  of  $X$ , there exist disjoint  $\alpha$ -open sets  $U$  and  $V$  such that  $A \subset U$  and  $B \subset V$ .

**Definition 4.13:** A space  $X$  is said to be  $\alpha\beta$ -normal if for any pair of disjoint  $\alpha$ -closed sets  $A$  and  $B$  of  $X$ , there exist disjoint  $\beta$ -open sets  $U$  and  $V$  such that  $A \subset U$  and  $B \subset V$ .

We, recall the following.

Clearly, (i) every  $\beta\alpha$ -normal  $\rightarrow$   $\alpha$ -normal, (ii)  $\beta\alpha$ -normal  $\rightarrow$   $\beta^*$ -normal, (iii) strongly  $\beta^*$ -normal  $\rightarrow$   $\beta\alpha$ -normal, (iv) strongly  $\beta^*$ -normal  $\rightarrow$  normal, (v)  $\beta^*$ -normal  $\rightarrow$   $\alpha\beta$ -normal and (vi)  $\beta\alpha$ -normal  $\rightarrow$   $\beta$  $s$ -normal as well as  $\beta$  $p$ -normal.

We , recall the following.

**Lemma 4.14 [22] :** A subset A of a space X is  $\alpha g$ -open iff  $F \subset \alpha Int(A)$  whenever  $F \subset A$  and F is closed set in X.

**Lemma 4.15 [4] :** If X is submaximal and E.D. space , then every  $\beta$ -open set in X is open set.

We, characterize the  $\beta\alpha$ -normal spaces in the following.

**Theorem 4.16 :** The following statements are equivalent for a submaximal and E.D., space X :

- (i) X is  $\beta\alpha$ -normal space,
- (ii) For any pair of disjoint  $\beta$ -closed sets A , B of X , there exist disjoint  $\alpha g$ -open sets U , V such that  $A \subset U$  and  $B \subset V$ ,
- (iii) For any  $\beta$ -closed set A and any  $\beta$ -open set V containing A , there exists a  $\alpha g$ -open set U such that  $A \subset U \subset \alpha Cl(U) \subset V$ .

Proof : (i)  $\rightarrow$  (ii) . Obvious , since every  $\alpha$ -open set is  $\alpha g$ -open set.

(ii) $\rightarrow$ (iii) . Let A be any  $\beta$ -closed set and V be any  $\beta$ -open set containing A . Since A and  $(X -V)$  are disjoint  $\beta$ -closed sets of X , there exist  $\alpha g$ -open sets U and W of X such that  $A \subset U$  and  $(X-V) \subset W$  and  $U \cap W = \emptyset$  . By Lemmas – 4.12 and 4.13 , we have  $(X -V) \subset \alpha Int(W)$  . Since  $U \cap \alpha Int(W) = \emptyset$  , we have  $\alpha Cl(U) \cap \alpha Int(W) = \emptyset$  and hence  $\alpha Cl(U) \subset X - \alpha Int(W) \subset V$  . Thus , we obtain that  $A \subset U \subset \alpha Cl(U) \subset V$  .

(iii) $\rightarrow$  (i). Let A and B be any disjoint  $\beta$ -closed sets of X . Since  $(X-B)$  is  $\beta$ -open set containing A, there exists a  $\alpha g$ -open set G such that  $A \subset G \subset \alpha Cl(G) \subset (X-B)$ . Then by Lemmas-4.12 and 13 ,  $A \subset \alpha Int(G)$  . Put  $U = \alpha Int(G)$  and  $V = X - \alpha Cl(G)$ . Then , U and V are disjoint  $\alpha$ -open sets such that  $A \subset U$  and  $B \subset V$  . Therefore, X is  $\beta\alpha$ -normal space.

We, recall the following.

**Definition 4.17[11]:** A subset A of a space X is called  $g\beta$ -closed (=  $gsp$ -closed) set if  $\beta Cl(A) \subset U$ , whenever  $A \subset U$  and U is open set in X.

The complement of a  $g\beta$ -closed set is called  $g\beta$ -open set in X.

We, give the following.

**Theorem 4.18:** The following statements are equivalent for a submaximal and E.D., space X :

- (i) X is  $\alpha\beta$ -normal space,
- (ii) For any pair of disjoint  $\alpha$ -closed sets A , B of X , there exist disjoint  $g\beta$ -open sets U , V such that  $A \subset U$  and  $B \subset V$ ,
- (iii) For any  $\alpha$ -closed set A and any  $\alpha$ -open set V containing A , there exists a  $g\beta$ -open set U such that  $A \subset U \subset \beta Cl(U) \subset V$ .

We prove the following .

**Theorem 4.19 :** Every  $\beta-T_1$  space and  $\beta\alpha$ -normal space is  $\beta\alpha$ -regular space.

**Proof :** Let X be  $\beta-T_1$  space and  $\beta\alpha$ -normal space . Let F be any  $\beta$ -closed set in X and  $x \notin F \Rightarrow x \in X - F$  . As X is  $\beta-T_1$  space,  $\{x\}$  is  $\beta$ -closed set for all  $x \in X$ . Thus , F and  $\{x\}$  are two disjoint  $\beta$ -closed sets of X. Since X is  $\beta\alpha$ -normal space , there exist disjoint  $\alpha$ -open sets G and H in X such that  $\{x\} \subset G$  and  $F \subset H$  i.e.,  $x \in G$  and  $F \subset H$  , this shows that X is  $\beta\alpha$ -regular space.

Now , we prove some subspace theorem for  $\beta\alpha$ -normality in the following

**Theorem 4.20:** Let X be  $\beta\alpha$ -normal space and  $Y \in PO(X)$  ,then Y is  $\beta\alpha$ -normal space.

Proof : Let X be  $\beta\alpha$ -normal space and Y be an p-open subset of X. Let  $A_Y$  and  $B_Y$  be disjoint  $\beta$ -closed subsets of Y. Therefore ,  $A_Y = Y \cap A$  and  $B_Y = Y \cap B$  where A and B are disjoint  $\beta$ -closed subsets of X. As X is  $\beta\alpha$ -normal , there exist disjoint  $\alpha$ -open subsets G and H of X such that  $A \subset G$  and  $B \subset H$  which implies that :  $Y \cap A \subset Y \cap G$  ,  $Y \cap B \subset Y \cap H$  where  $Y \cap G$  and  $Y \cap H$  are  $\alpha$ -open subsets of Y by Lemma- 2.31 with  $(Y \cap G) \cap (Y \cap H) = Y \cap (G \cap H) = \emptyset$ .

Therefore , Y is  $\beta\alpha$ -normal space.

**Theorem 4.21 :** If  $f : X \rightarrow Y$  is a  $\beta$ -irresolute ,  $\beta\alpha g$ -closed surjection and X is  $\beta\alpha$ -normal, then Y is  $\beta\alpha$ -normal space.

**Proof :** Let A and B be any disjoint  $\beta$ -closed sets of Y. Then,  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint  $\beta$ -closed sets of X since f is  $\beta$ -irresolute function . Since X is  $\beta\alpha$ -normal , there exist disjoint  $\alpha$ -open sets U and V in X such that  $f^{-1}(A) \subset U$  and  $f^{-1}(B) \subset V$  . By Th.4.4 , there exist  $\alpha g$ -open sets G and H of Y such that  $A \subset G$  ,  $B \subset H$  ,  $f^{-1}(G) \subset U$  and  $f^{-1}(H) \subset V$  . Then , we have  $f^{-1}(G) \cap f^{-1}(H) = \emptyset$  and hence  $G \cap H = \emptyset$  . It follows from Th.4.14 that space Y is  $\beta\alpha$ -normal.

Finally, we introduce the following .

**Definition 4.22:** A space X is said to be  $\beta\beta$ -normal if for any pair of disjoint  $\beta$ -closed sets A and B of X ,there exist disjoint  $\beta$ -open sets U and V such that  $A \subset U$  and  $B \subset V$ .

**Note 4.23 :**  $\beta\beta$ -normal space (=  $\beta^*$ -normal space due to Navalagi[ 30&31]) and (= semipre-normal space due to Navalagi et al [29] ).



The purpose of introducing the above spaces is to compare with  $\beta$ s-normal spaces due to Navalagi et al [ 32 ],  $\beta$ p-normal spaces due to Navalagi [ 28 ] and  $\beta\alpha$ -normal spaces due to present paper itself.

**Note 4.24 :** In an  $\alpha$ -space , (i) normal spaces and  $\alpha$ -normal spaces coincide each other.(ii) strongly  $\beta^*$ -normal spaces and  $\beta\alpha$ -normal spaces coincide each other, see **Lemma-2.30**.

**Note 4.25 :** Usually, every  $\beta\alpha$ -normal space  $\rightarrow$   $\beta$ s-normal space , but in an E.D.-space , every  $\beta$ s-normal space  $\rightarrow$   $\beta\alpha$ -normal space , see **Lemma 2. 32**.

**Note 4.26 :** Usually, every  $\beta\alpha$ -normal space  $\rightarrow$   $\beta$ p-normal space , but in a PS-space , every  $\beta$ p-normal space  $\rightarrow$   $\beta\alpha$ -normal space , see **Lemma 2. 33** .

**Note 4.27 :** Usually, every normal space  $\rightarrow$   $\alpha$ -normal space  $\rightarrow$  p-normal space  $\rightarrow$   $\beta$ -normal space and every normal space  $\rightarrow$   $\alpha$ -normal space  $\rightarrow$  s-normal space  $\rightarrow$   $\beta$ -normal space but in an E.D.-space and submaximal space, Normal space =  $\alpha$ -normal space = s-normal space = p-normal space =  $\beta$ -normal space , see **Lemma 2. 34** .

## V.CONCLUSION

Finally, we introduce the following .

**DEFINITION 5.1 (=Definition 4.20):** A space X is said to be  $\beta\beta$ -normal if for any pair of disjoint  $\beta$ -closed sets A and B of X ,there exist disjoint  $\beta$ -open sets U and V such that  $A \subset U$  and  $B \subset V$ .

**Note 5.2 :**  $\beta\beta$ -normal space (=  $\beta^*$ -normal space due to Navalagi[30&31 ]) and (= semipre-normal space due to Navalagi et al [29 ]).

The purpose of introducing the above spaces is to compare with  $\beta$ s-normal spaces due to Navalagi et al [ 32 ],  $\beta$ p-normal spaces due to Navalagi [28 ] and  $\beta\alpha$ -normal spaces due to present paper itself.

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