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Solution of Fractional Programming Problem by Simplex Method

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ABSTRACT: This paper we study a new method for solving Fractional Programming Problem, and where the constraint functions are in the form of linear inequalities. The proposed method is based mainly upon simplex method, which is very easy to understand and apply. This can be illustrated with the help of numerical examples.

KEYWORDS: Fractional programming, Simplex method.

I. INTRODUCTION

The linear fractional programming (LFP) problem has attracted the interest of many researches due to its application in many important fields such as production planning, financial and corporate planning, health care and hospital planning. Several methods were suggested for solving this problem such as the variable transformation method [1]. Algorithms for solving linear fractional programming problems are well known by many authors. Charnes-Copper [1] replaces a linear fractional program by one equivalent linear program, in which one extra constraint and one extra variable has been added. The usual simplex algorithm computes the optimum solution. Isbell-Marlow and Martos [3, 4] find the solution of a sequence of linear programs. Wagner-Yuan [8] showed that, when the feasible set is bounded. Chdhas-Rachael [2] solves a system of linear of inequalities in which the objective function is expressed as one of the constraint along with the given set of linear constraints of the problem. ResentlyTantawy [7] has suggested a feasible direction approach and a duality approach to solve a linear fractional programming problem. In this paper we to find the solution of fractional programming problems. For this solution, we use simplex method.

II. PRELIMINARIES

Definition.2.1: The Mathematical form of Linear Fractional Programming Problem (LFPP) is as follows:

$$\text{Max } Z \text{ or } \text{Min } Z = \frac{(c^T X + \alpha)}{(d^T + \beta)}$$

Subject to:

$$AX \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b, \quad X \geq 0$$

Where,

- i. A is an $m \times n$ matrix,
- ii. b is an $m \times 1$ vector,
- iii. c, d and X are $n \times 1$ vectors,
- iv. α and β are scalars,
- v. c^T and d^T are transpose of c and d respectively.

Definition.2.2: A slack or surplus value is reported for each of the constraints. The term "slack" applies to less than or equal constraints, and the term "surplus" applies to greater than or equal constraints. If a constraint is binding, then the corresponding slack or surplus value will equal zero.

Definition.2.3: A feasible solution is a set of values for the decision variables that satisfies all of the constraints in an optimization problem. The set of all feasible solutions defines the feasible region of the problem



1. Algorithm :

Step 1: First, observes whether all the right side constants of the constraints are non-negative. If not, it can be changed into positive value on multiplying both the sides of the constraints by -1 .

Step 2: Next converts the inequality constraints to equations by introducing the non-negative slack or surplus variables. The coefficients of slack or surplus variables are always taken zero in the objective function.

Step 3: constructs the simplex table by using the following notations.

Let X_B be the initial basic feasible solution of the given problem such that

$$\begin{aligned} BX_B &= b \\ X_B &= bB^{-1} \end{aligned}$$

where $B = (b_1, b_2, \dots, b_r, b_s, \dots, b_m)$.

Further suppose that

$$\begin{aligned} Z_1 &= c'_B X_B + \alpha \\ Z_2 &= d'_B X_B + \beta \end{aligned}$$

where c'_B and d'_B are the vectors having their components as the coefficients associated with the basic variables in the numerator and denominator of the objective function respectively.

Step 4: Now, compute the 'net evaluation' Δ_j for each variable x_j by the formula

$$\Delta_j = \frac{Z_j^{(1)} - c_j}{Z_j^{(2)} - d_j} = \frac{c_B x_j + \alpha - c_j}{d_B x_j + \beta - d_j}$$

Step 5: If all $\Delta_j \geq 0$, the optimal solution is obtained.

Step 6: If optimal solution is an integer (or mixed integer) solution then we get the required solution otherwise use Branch and Bound method.

2. Numerical Example:

Example 4.1

$$\max Z = \frac{2x_1 + 4x_2 + 1}{2x_1 + 5x_2 + 6}$$

Subject to:

$$\begin{aligned} x_1 + 2x_2 &\leq 2 \\ x_1 + x_2 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Solution :

$$\max Z = \frac{2x_1 + 4x_2 + 1}{2x_1 + 5x_2 + 6}$$

Subject to:

$$\begin{aligned} x_1 + 2x_2 &\leq 2 \\ x_1 + x_2 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

After Adding slack variables S_1 and S_2 the constraints become

$$\begin{aligned} x_1 + 2x_2 + S_1 &= 2 \\ x_1 + x_2 + S_2 &= 4 \\ x_1, x_2, S_1, S_2 &\geq 0 \end{aligned}$$

Rewrite

$$\begin{aligned} x_1 + 2x_2 + S_1 + 0S_2 &= 2 \\ x_1 + x_2 + 0S_1 + S_2 &= 4 \\ x_1, x_2, S_1, S_2 &\geq 0 \end{aligned}$$



Table 3.1

			c_j	2	4	0	0	Min Ratio $\frac{x_{ij}}{y_{ij}}$
			d_j	2	5	0	0	
d_B	c_B	X_B	B	x_1	x_2	S_1	S_2	
0	0	S_1	2	1	2	1	0	$2/2 = 1 \rightarrow$
0	0	S_2	4	1	1	0	1	$4/1 = 4$
	$Z_1 = 1$	$Z_j^{(1)} - c_j$		-1	-3	1	1	
	$Z_2 = 6$	$Z_j^{(2)} - d_j$		4	1	6	6	
	$Z = \frac{1}{6}$	Δ_j		$-\frac{1}{4}$	$-\frac{3}{1} \uparrow$	$\frac{1}{6}$	$\frac{1}{6}$	

Where $Z_1 = c_B^{(1)} X_B + \alpha$

$$Z_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} + 1 = 0 + 1 = 1$$

$$Z_2 = d_B^{(1)} X_B + \beta$$

$$Z_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} + 6 = 0 + 6 = 6$$

$$\Delta_j = \frac{Z_j^{(1)} - c_j}{Z_j^{(2)} - d_j}$$

Here most negative is - 3 and minimum ratio is 1. So x_2 enter in the basis and S_1 leave the basis.

Table 3.2

			c_j	2	4	0	0	Min Ratio $\frac{x_{ij}}{y_{ij}}$
			d_j	2	5	0	0	
d_B	c_B	X_B	B	x_1	x_2	S_1	S_2	
5	4	x_2	1	$\frac{1}{2}$	1	$\frac{1}{2}$	0	
0	0	S_2	3	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	
	$Z_1 = 5$	$Z_j^{(1)} - c_j$		1	1	3	1	
	$Z_2 = 11$	$Z_j^{(2)} - d_j$		$\frac{13}{2}$	6	$\frac{17}{2}$	6	
	$Z = \frac{5}{11}$	Δ_j		$\frac{2}{13}$	$\frac{1}{6}$	$\frac{6}{17}$	$\frac{1}{6}$	



Where $Z_1 = c_B^{(1)} X_B + \alpha$

$$Z_1 = \binom{4}{0} \binom{1}{3} + 1 = 5$$

$$Z_2 = d_B^{(1)} X_B + \beta$$

$$Z_2 = \binom{5}{0} \binom{1}{3} + 6 = 5 + 6 = 11$$

$$\Delta_j = \frac{Z_j^{(1)} - c_j}{Z_j^{(2)} - d_j}$$

Here $x_1 = 0, x_2 = 1$ and $Z = \frac{Z_1}{Z_2} = \frac{5}{11}$ Since all $\Delta_j \geq 0$, therefore the current solution is the optimal basic feasible solution.

Example 4.2:

$$\max Z = \frac{x_1 + 2x_2}{2x_1 - x_2 + 1}$$

Subject to:

$$\begin{aligned} -x_1 + 2x_2 &\leq 1, \\ x_1 + x_2 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Solution :

$$\max Z = \frac{x_1 + 2x_2}{2x_1 - x_2 + 1}$$

Subject to:

$$\begin{aligned} -x_1 + 2x_2 &\leq 1 \\ x_1 + x_2 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

After Adding slack variables S_1 and S_2 the constraints become

$$\begin{aligned} -x_1 + 2x_2 + S_1 &= 1 \\ x_1 + x_2 + S_2 &= 4 \\ x_1, x_2, S_1, S_2 &\geq 0 \end{aligned}$$

Rewrite

$$\begin{aligned} -x_1 + 2x_2 + S_1 + 0S_2 &= 1 \\ x_1 + x_2 + 0S_1 + S_2 &= 4 \\ x_1, x_2, S_1, S_2 &\geq 0 \end{aligned}$$

Table 3.3

		c_j	1	2	0	0	Min Ratio $\frac{x_{ij}}{y_{ij}}$	
		d_j	2	-1	0	0		
d_B	c_B	X_B	B	x_1	x_2	S_1	S_2	
0	0	S_1	1	-1	2	1	0	1/2 →
0	0	S_2	4	1	1	0	1	4/1 = 4
	$Z_1 = 0$	$Z_j^{(1)} - c_j$		-1	-2	0	0	
	$Z_2 = 1$	$Z_j^{(2)} - d_j$		-1	2	1	1	
	$Z = 0$	Δ_j		1	-1 ↑	1	1	



Where $Z_1 = c_B^{(1)} X_B + \alpha$

$$Z_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix} + 0 = 0$$

$$Z_2 = d_B^{(1)} X_B + \beta$$

$$Z_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix} + 1 = 0 + 1 = 1$$

$$\Delta_j = \frac{Z_j^{(1)} - c_j}{Z_j^{(2)} - d_j}$$

Here most negative is -1 and minimum ratio is $\frac{1}{2}$. So x_2 enter in the basis and S_1 leave the basis.

Table 3.4

				c_j	1	2	0	0	Min Ratio $\frac{x_{ij}}{y_{ij}}$
				d_j	2	-1	0	0	
d_B	c_B	X_B	B	x_1	x_2	S_1	S_2		
-1	2	x_2	$\frac{1}{2}$	$-\frac{1}{2}$	1	$\frac{1}{2}$	0		
0	0	S_2	3	$\frac{3}{2}$	0	$-\frac{1}{2}$	1		
	$Z_1 = 1$	$Z_j^{(1)} - c_j$		-2	0	1	0		
	$Z_2 = \frac{1}{2}$	$Z_j^{(2)} - d_j$		$-\frac{1}{2}$	1	$\frac{1}{2}$	1		
	$Z = 2$	Δ_j		4	0	2	0		

Where $Z_1 = c_B^{(1)} X_B + \alpha$

$$Z_1 = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + 0 = 1$$

$$Z_2 = d_B^{(1)} X_B + \beta$$

$$Z_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + 1 = \frac{1}{2}$$

$$\Delta_j = \frac{Z_j^{(1)} - c_j}{Z_j^{(2)} - d_j}$$

Here $x_1 = 0, x_2 = \frac{1}{2}$ and $Z = \frac{Z_1}{Z_2} = 2$ Since all $\Delta_j \geq 0$, therefore the current solution is the Optimal basic feasible solution.

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