



e-ISSN: 2319-8753 | p-ISSN: 2320-6710

IJIRSET

International Journal of Innovative Research in
SCIENCE | ENGINEERING | TECHNOLOGY

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 10, Issue 1, January 2021

ISSN INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA

Impact Factor: 7.512

9940 572 462

6381 907 438

ijirset@gmail.com

www.ijirset.com



Allied ℓ -Continuous Functions in Topology

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ABSTRACT: In 1970, Gentry and Hoyle have defined and studied the notion of c -continuity in topological spaces. Later, Long et al and Gauld have studied some more properties of c -continuity in the literature. In 1981, Kohli has defined and studied a class of functions called I -continuous functions in topological spaces which is stronger than c -continuity. In 1963, 1965, 1982 and 1983 N. Levine, O. Njastad, Mashhour et al and M.E. Abd El-Monsef et al have defined and studied the concepts of semiopen sets, α -open sets, preopen sets and β -open sets in topology. The purpose of this paper is to define and study the new classes of functions called I_p -continuous functions, I_s -continuous functions, I_α -continuous functions & I_β -continuous functions and also, we obtain their basic properties.

2010 M.S.C. : 54 A05, 54C08, 54D15

KEYWORDS AND PHRASES : preopen sets, α -open sets, semiopen sets, β -open sets, compact subsets, precontinuity, c -continuity, I -continuity and M -precontinuity.

I. INTRODUCTION

In 1970, Gentry and Hoyle [22] have defined and studied the new class of functions called c -continuous functions. Later, in 1974 & 1975, Long et al [25 & 26] have studied further properties of c -continuous functions and defined a new class of functions called c^* -continuous functions in topological spaces. Again, in 1978 Gauld [5] has studied some more properties of c -continuous functions via cocompact topologies. In 1981, Kohli [23] has defined and studied a class of functions called I -continuous functions in topological spaces which is stronger than c -continuity. In 1963, 1965, 1982 and 1983 N. Levine [24], O. Njastad [42], Mashhour et al [29] and M.E. Abd El-Monsef et al [1] have defined and studied the concepts of semiopen sets, α -open sets, preopen sets and β -open sets in topology. The purpose of this paper is to define and study the new classes of functions called I_p -continuous functions, I_s -continuous functions, I_α -continuous functions & I_β -continuous functions and also, we obtain their basic properties.

II. PRELIMINARIES

Throughout the present paper, spaces (X, τ) and (Y, σ) (or simply, X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated and $f: X \rightarrow Y$ denotes a single valued function of a space (X, τ) into a space (Y, σ) . A be a subset of space X . The closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$ respectively. A subset A of a space X is called regular open (resp. regular closed) if $A = Int Cl(A)$ (resp. $A = Cl Int(A)$). The following definitions and results are useful in the sequel

Definition 2.1: A subset A of a space X is said to be

- (i) α -open [42] set if $A \subset Int Cl Int(A)$.
- (ii) semiopen [24] set if $A \subset Cl Int(A)$.
- (iii) preopen [29] set if $A \subset Int Cl A$.
- (iv) semipreopen [5] (= β -open [1]) set if $A \subset Cl Int Cl(A)$.

The complement of an α -open (resp. semiopen, preopen, β -open) set in X is called α -closed [30] (resp. semiclosed [6] pre-closed [16], β -closed [2]). The family of all α -open (resp. semiopen, preopen, β -open) sets in a space X is denoted by $\alpha O(X)$ (resp. $SO(X)$, $PO(X)$, $\beta O(X)$).

Definition 2.2: The intersection of all preclosed (resp. semiclosed, α -closed, β -closed) sets containing A is called the preclosure [16] (resp. the semiclosure [12], the α -closure [30], β -closure [2]) of A and is denoted by $pCl(A)$ (resp. $sCl(A)$, $\alpha Cl(A)$, $\beta Cl(A)$).



The union of all pre-open (resp. semiopen , α -open , β -open) sets which are contained in A is called pre-interior (resp. semi-interior , α -interior , β -interior) of A and is denoted by $pInt(A)$ [31&33](resp. $sInt(A)$ [13&34] , $\alpha Int(A)$ [35] , $\beta Int(A)$ [2]).

Definition 2.3 : A function $f : X \rightarrow Y$ is called :

- (i) precontinuous [29] if the inverse image of each open set U of Y is preopen set in X.
- (ii) semicontinuous [24] if the inverse image of each open set U of Y is semiopen set in X.
- (iii) α -continuous [30] if the inverse image of each open set U of Y is α -open set in X.
- (iv) β -continuous [1] if the inverse image of each open set U of Y is β -open set in X.

Definition 2.4[32] : A function $f : X \rightarrow Y$ is said to be M-preopen (resp. M-preclosed) if the image of each preopen (resp. preclosed) subset of X is preopen (resp. preclosed) subset in Y.

Definition 2.5[27] : A function $f : X \rightarrow Y$ is said to be pre- β -open (resp. pre- β -closed) if the image of each β -open (resp. β -closed) subset of X is β -open (resp. β -closed) subset in Y.

Definition 2.6[14] : A function $f : X \rightarrow Y$ is said to be pre- α -open (resp. pre- α -closed) if the image of each α -open (resp. α -closed) subset of X is α -open (resp. α -closed) subset in Y.

Definition 2.7[11&17] : A function $f : X \rightarrow Y$ is said to be presemiopen (resp. presemiclosed) if the image of each semiopen (resp. semiclosed) subset of X is semiopen (resp. semiclosed) subset in Y.

Definition 2.8[22] : A function $f : X \rightarrow Y$ is said to be c-continuous if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ and having compact complement , there exists an open set U containing x such that $f(U) \subset V$.

Theorem 2.9[22,Th. 1] : Let $f : X \rightarrow Y$ be a function .Then the following statements are equivalent :

- (i) f is c-continuous.
- (ii) If V is an open subset of Y with compact complement , then $f^{-1}(V)$ is open subset of X.
These statements are implied by :
- (iii) If F is a compact subset of Y , then $f^{-1}(F)$ is closed subset of X and, moreover , if Y is Hausdorff, then all the above statements : (i)-(iii) are equivalent.

Theorem 2.10 : Let $f : X \rightarrow Y$ be a function . Then, f is c-continuous iff :

- (i) The inverse image of each open subset of Y having compact complement is open in X [25].
- (ii) The inverse image of each closed compact subset of Y is closed in X [26].

Definition 2.11[23] : A function $f : X \rightarrow Y$ is called I-continuous if for each point $x \in X$ and each open set V containing $f(x)$ and having Lindeloff complement there is an open set U containing x such that $f(U) \subset V$.

Theorem 2.12[23, Th.2.1]: Let $f : X \rightarrow Y$ be a function from a topological space X into a topological space Y. The following are equivalent :

- (i) f is I-continuous.
- (ii) If V is an open subset of Y having Lindeloff complement, then $f^{-1}(V)$ is an open subset of X.
- (iii) If F is a closed Lindeloff subset of Y, then $f^{-1}(F)$ is closed in X.
- (iv) For each $x \in X$ and each net $\{x_\alpha\}_{\alpha \in D}$ which converges to x, the net $\{f(x_\alpha)\}_{\alpha \in D}$ is eventually in each open set containing $f(x)$ and having Lindeloff complement.

Definition 2.13[10&38] : A function $f : X \rightarrow Y$ is said to be c-semicontinuous if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ and having compact complement , there exists an semiopen set U containing x such that $f(U) \subset V$.

Definition 2.14[39]: A function $f : X \rightarrow Y$ is said to be c-precontinuous if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ and having compact complement , there exists an preopen set U containing x such that $f(U) \subset V$.

Definition 2.15[40] : A function $f : X \rightarrow Y$ is said to be c - α -continuous if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ and having compact complement, there exists an α -open set U containing x such that $f(U) \subset V$.

Definition 2.16[41] : A function $f : X \rightarrow Y$ is said to be c - β -continuous if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ and having compact complement, there exists an β -open set U containing x such that $f(U) \subset V$.

Definition 2.17[32]: Let X be a space and $\{x_\alpha\}_{\alpha \in D}$ be a net in X . Let $x \in X$ then $\{x_\alpha\}_{\alpha \in D}$ pre-converges to x , denoted by $\{x_\alpha\}_{\alpha \in D} \overset{p}{\rightarrow} x$ iff it is eventually in every preopen set containing x .

Definition 2.18[3]: Let X be a space and $\{x_\alpha\}_{\alpha \in D}$ be a net in X . Let $x \in X$ then $\{x_\alpha\}_{\alpha \in D}$ s -converges to x , denoted by $\{x_\alpha\}_{\alpha \in D} \overset{s}{\rightarrow} x$ iff it is eventually in every semiopen set containing x .

Definition 2.19 : A space X is said to be :

- (i) submaximal [8] if every dense subset of it is open (i.e. if $PO(X) = \tau$ [46]).
- (ii) an extremely disconnected (E.D.) [47] if closure of each open set is open in it (i.e. if $A \in \tau$ for each $A \in RF(X)$ [19]).
- (iii) PS-space [4] if every preopen set of X is semiopen in X .
- (iv) α -space [18] if every α -open set of X is open in X (i.e. $\tau = \alpha O(X)$).
- (v) β -space [9] if every β -open set of X is open in X (i.e. $\tau = \beta O(X)$).

Remark 2.20: It is known that, $\tau \subset \alpha O(X) \subset SO(X) \subset \beta O(X)$ and $\tau \subset \alpha O(X) \subset PO(X) \subset \beta O(X)$.

Theorem 2. 21 [18]: In an α -space X , we have $\tau = \alpha O(X)$.

Theorem 2.22 [4]: For a space X the following conditions are equivalent :

- (i) X is an E.D. - space ,
- (ii) $SO(X) \subset PO(X)$ (i.e. each semiopen subset is preopen),
- (iii) $SO(X) \subset \alpha O(X)$ (i.e. each semiopen subset is α -open),
- (iv) $\beta O(X) \subset PO(X)$ (i.e. each β -open subset is preopen).

Theorem 2.23 [4]: A space X is PS-space if it satisfies the following equivalent conditions :

- (i) $PO(X) \subset SO(X)$ (i.e. each preopen subset is semiopen),
- (ii) $\beta O(X) \subset SO(X)$ (i.e. each β -open subset is semiopen),
- (iii) $PO(X) \subset \alpha O(X)$ (i.e. each preopen subset is α -open).

III. PROPERTIES OF lp -CONTINUOUS FUNCTIONS

We, define the following.

Definition 3.1 : A function $f : X \rightarrow Y$ is said to be lp -continuous if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ and having Lindeloff complement, there exists an preopen set U containing x such that $f(U) \subset V$.

Clearly, every l -continuous function is lp -continuous & every lp -continuous function is c -precontinuous .

We, recall the following.

Definition 3.2 [44] : Let $x \in X$. A set $U \subset X$ is called a pre-neighbourhood (in brief, pre-nbd) of x in X if there exists $A \in PO(X)$ such that $x \in A \subset U$.

Theorem 3.3 [33]: A subset of a space X is preopen if it is a pre-neighborhood of each of its points.

We, prove the following .

Theorem 3.4 : Let $f : X \rightarrow Y$ be a function .Then the following statements are equivalent :

- (i) f is lp -continuous .
- (ii) If V is an open subset of Y with Lindelöf complement, then $f^{-1}(V)$ is preopen subset of X .
- (iii) If F is a closed Lindelöf subset of Y , then $f^{-1}(F)$ is preclosed subset of X and,



Proof follows by Theorem 2.12 above.

Easy proof of the following is omitted.

Lemma 3.5 : A function $f : X \rightarrow Y$ is said to be **lp**-continuous if the inverse image of each open subset of Y having Lindelöf complement is preopen in X .

Lemma 3.6 : A function $f : X \rightarrow Y$ is said to be **lp**-continuous if the inverse image of each closed Lindelöf subset of Y is preclosed in X .

We, recall the following.

Lemma 3.7[43]: If $A \in SO(X)$ and $V \in PO(X)$, then $A \cap V \in PO(A)$.

Lemma 3.8 [16]: If $A \in \alpha O(X)$ and $V \in PO(X)$ then $A \cap V \in PO(A)$.

Next, we state the following.

Theorem 3.9 : If $f : X \rightarrow Y$ is **lp**-continuous function and A be an semiopen subset of X , then $f/A : A \rightarrow Y$ is also **lp**-continuous .

Theorem 3.10 : If $f : X \rightarrow Y$ is **lp**-continuous function and A be an α -open subset of X , then $f/A : A \rightarrow Y$ is also **lp**-continuous .

Theorem 3.11 : If $f : X \rightarrow Y$ is preirresolute and $g : Y \rightarrow Z$ is **lp**-continuous , then $g \circ f$ is **lp**-continuous .

Proof. Let U be an open subset of Z having Lindelöf complement. Then, $g^{-1}(U)$ is preopen set in Y , since g is **lp**-continuous. Again, as f is preirresolute & $g^{-1}(U)$ is preopen subset of Y , $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is preopen subset in X . This shows that $g \circ f$ is **lp**-continuous function.

Corollary 3.12 : If $f : X \rightarrow Y$ is precontinuous and $g : Y \rightarrow Z$ is **l**-continuous , then $g \circ f$ is **lp**-continuous .

We, recall the following.

Definition 3.13[36] : A function $f : X \rightarrow Y$ is said to be **p**-continuous if the inverse image of each preopen set of Y is open in X .

We state the following

Theorem 3.14: If $f : X \rightarrow Y$ is **p**-continuous and $g : Y \rightarrow Z$ is **lp**-continuous , then $g \circ f$ is **l**-continuous .

We, define the following.

Definition 3.15 : A function $f : X \rightarrow Y$ is said to be α **p**-continuous if the inverse image of each α -open set of Y is preopen in X .

We state the following

Theorem 3.16: If $f : X \rightarrow Y$ is α **p**-continuous and $g : Y \rightarrow Z$ is **l** α -continuous , then $g \circ f$ is **lp**-continuous .

We, recall the following.

Definition 3.17 [21] : A function $f : X \rightarrow Y$ is said to be contra pre-continuous if the inverse image of each open set of Y is preclosed in X .

We, state the following.

Theorem 3.18 : Let $f : X \rightarrow Y$ be contra-pre-continuous function and $g : Y \rightarrow Z$ be **l**-continuous function .Then, $g \circ f$ is **lp**-continuous .

Theorem 3.19 : Let $f : X \rightarrow Y$ be either M -preopen or M -preclosed surjection and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is **lp**-continuous .Then, g is **lp**-continuous .



Proof : Suppose f is M -preopen (resp. M -preclosed) surjection and V be an open subset with Lindelöf complement (resp. V be a closed Lindelöf subset) in Z . Since $g \circ f$ is lp -continuous , $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is preopen (resp. preclosed) subset in X by Th.3.5 & Th.3.6. Since f is M -preopen (resp. M -preclosed) surjection , then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is preopen (resp. preclosed) set in Y and consequently, g is lp -continuous function in either case. We, recall the following.

Definition 3.20[36] : A function $f : X \rightarrow Y$ is said to be p -open (resp. p -closed) if the image of each preopen (resp. preclosed) subset of X is open (resp. closed) in Y .

Theorem 3.21 : Let $f : X \rightarrow Y$ be either p -open or p -closed surjection and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is lp -continuous .Then, g is l -continuous .

Proof is similar to that of Theorem -5.17 above.

In view of Th.2.23(i) & (ii) , respectively , we state the following.

Remark 3.22 : (i) Every lp -continuous function is ls -continuous (also See Remark-4.23(i))

(ii) Every lp -continuous function is $l\alpha$ -continuous (also See Remark-5.20(ii))

In view of Th.2.23 (i) above (i.e. in an PS-space , every preopen subset is semiopen & every preopen subset is an α -open, resp.) , so , we have the following.

Theorem 3.23 : Let $f : X \rightarrow Y$ be a presemiopen surjection with an PS.-space X and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is lp -continuous .Then, g is ls -continuous .

Proof : Suppose f is presemiopen surjective function with an PS.-space X and V be an open subset with Lindelöf complement in Z . Since $g \circ f$ is lp -continuous , $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is preopen subset in X . Since a space X is PS.-space assumed and hence $(g \circ f)^{-1}(V)$ is semiopen subset in X by Th.2.20(i) . Also, as f is presemiopen and surjective function , $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is semiopen subset in Y and consequently, g is ls -continuous .

Theorem 3.24 : Let $f : X \rightarrow Y$ be pre- α -open surjection with a PS -space X and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is lp -continuous .Then, g is $l\alpha$ -continuous [See Th.-5.22]..

In view of the fact that an arbitrary union of semiopen (resp. pre-open) sets is semiopen [13](resp. pre-open[33]) , we have the following :

Theorem 3.25 : If X and Y are two topological spaces and $X = A \cup B$, where A and B are semiopen subsets of X and $f : X \rightarrow Y$ is a function such that $f|_A$ and $f|_B$ are lp -continuous , then f is lp -continuous .

Proof : Assume that A and B are semiopen subsets in X . Let U be an open subset of Y with Lindelöf complement. Then, we have $f^{-1}(U) = (f|_A)^{-1}(U) \cup (f|_B)^{-1}(U)$, each of which is preopen by Lemma-3.7 . Thus, $f^{-1}(U)$ is preopen in X and hence f is lp -continuous.

Again ,in view of the fact that an arbitrary union of α -open (resp. preopen) sets is α -open [[35](resp. preopen[33]) . So , by virtue of Lemma-3.7 , we have the following :

Theorem 3.26 : If X and Y are two topological spaces and $X = A \cup B$, where A and B are α -open subsets of X and $f : X \rightarrow Y$ is a function such that $f|_A$ and $f|_B$ are lp -continuous , then f is lp -continuous .

We , define the following.

Definition 3.27 : A function $f : X \rightarrow Y$ is said to be lp - irresolute if for each $x \in X$ and each preopen set $V \subset Y$ containing $f(x)$ and having Lindeloff complement , there exists an preopen set U containing x such that $f(U) \subset V$.

Definition 3.28 : A function $f : X \rightarrow Y$ is said to be c -pre-irresolute if for each $x \in X$ and each preopen set $V \subset Y$ containing $f(x)$ and having compact complement , there exists an preopen set U containing x such that $f(U) \subset V$.

Remark 3.29 : (i)Every lp -irresolute function is lp -continuous

(ii)Every c -preirresolute function is c -continuous

(iii)Every lp -irresolute function is c -pre-irresolute .



We, prove the following .

Theorem 3.30 : Let $f : X \rightarrow Y$ be a function .Then the following statements are equivalent :

- (i) f is **Ip**-irresolute .
- (ii) If V is an preopen subset of Y with Lindelöf complement , then $f^{-1}(V)$ is preopen subset of X .
- (iii) If F is a preclosed Lindelöf subset of Y , then $f^{-1}(F)$ is preclosed subset of X and,
- (iv) For each $x \in X$ and each net $\{x_\lambda\}_{\lambda \in D}$ which p -converges to x , the net $\{f(x_\lambda)\}_{\lambda \in D}$ is eventually in each preopen set containing $f(x)$ and having Lindeloff complement.

Proof follows by Theorem 2.12 above.

III. PROPERTIES OF **Is**-CONTINUOUS FUNCTIONS

We, define the following.

Definition 4.1 : A function $f : X \rightarrow Y$ is said to be **Is**-continuous if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ and having Lindeloff complement , there exists an semiopen set U containing x such that $f(U) \subset V$.

Clearly, every **Is**-continuous function is c -semicontinuous .

We, recall the following.

Definition 4.2 [7] : Let $x \in X$. A subset $U \subset X$ is called semi neighbourhood of x in X if there exists $A \in SO(X)$ such that $x \in A \subset U$.

Theorem 4.3 [13&34] : A subset of a space X is semi open iff it is a semi-neighbourhood of each of its points.

We, prove the following .

Theorem 4.4 : Let $f : X \rightarrow Y$ be a function .Then the following statements are equivalent :

- (i) f is **Is**-continuous .
- (ii) If V is an open subset of Y with Lindelöf complement , then $f^{-1}(V)$ is semiopen subset of X .
- (iii) If F is a closed Lindelöf subset of Y , then $f^{-1}(F)$ is semiclosed subset of X and,

Proof follows by Theorem 2.12 above.

Easy proof of the following is omitted.

Lemma 4.5 : A function $f : X \rightarrow Y$ is said to be **Is**-continuous if the inverse image of each open subset of Y having Lindelöf complement is semiopen in X .

Lemma 4.6 : A function $f : X \rightarrow Y$ is said to be **Is**-continuous if the inverse image of each closed Lindelöf subset of Y is semiclosed in X .

We, recall the following.

Lemma 4.7[43] : If $A \in PO(X)$ and $V \in SO(X)$, then $A \cap V \in SO(A)$.

Next, we state the following.

Theorem 4.8 : If $f : X \rightarrow Y$ is **Is**-continuous function and A be an preopen subset of X , then $f/A : A \rightarrow Y$ is also **Is**-continuous .

Theorem 4.9 : If $f : X \rightarrow Y$ is irresolute and $g : Y \rightarrow Z$ is **Is**-continuous , then $g \circ f$ is **Is**-continuous .

Proof. Let U be an open subset of Z having Lindelöf complement. Then, $g^{-1}(U)$ is semiopen set in Y , since g is **Is**-continuous. Again, as f is irresolute & $g^{-1}(U)$ is semiopen subset of Y , $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is semiopen subset in X . This shows that $g \circ f$ is **Is**-continuous function.

Corollary 4.10 : If $f : X \rightarrow Y$ is semicontinuous and $g : Y \rightarrow Z$ is l -continuous , then $g \circ f$ is **Is**-continuous .



We, define the following.

Definition 4.11 : A function $f : X \rightarrow Y$ is said to be *sst*- continuous if the inverse image of each semiopen set of Y is open in X .

We state the following

Theorem 4.12: If $f : X \rightarrow Y$ is *sst*-continuous and $g : Y \rightarrow Z$ is *ls*-continuous, then $g \circ f$ is *l*-continuous.

We, define the following.

Definition 4.13 : A function $f : X \rightarrow Y$ is said to be α -continuous if the inverse image of each α -open set of Y is semiopen in X .

We state the following

Theorem 4.14: If $f : X \rightarrow Y$ is α -continuous and $g : Y \rightarrow Z$ is *l* α -continuous, then $g \circ f$ is *ls*-continuous.

We, recall the following.

Definition 4.15 [15] : A function $f : X \rightarrow Y$ is said to be *contra*-semi-continuous if the inverse image of each open set of Y is *semiclosed* in X .

We, state the following.

Theorem 4.16 : Let $f : X \rightarrow Y$ be *contra*- semi-continuous function and $g : Y \rightarrow Z$ be *l*-continuous function. Then, $g \circ f$ is *ls*-continuous.

Theorem 4.17 : Let $f : X \rightarrow Y$ be either *pre*semiopen or *pre*semiclosed surjection and let $g : Y \rightarrow Z$ be any function such that g is *ls*-continuous. Then, $f \circ g$ is *ls*-continuous.

Proof : Suppose f is *pre*semiopen (resp. *pre*semiclosed) surjection and V be an open subset with Lindelöf complement (resp. V be a closed Lindelöf subset) in Z . Since $g \circ f$ is *ls*-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semiopen (resp. *semiclosed*) subset in X by Th.4.5 & Th.4.6. Since f is *pre*semiopen (resp. *pre*semiclosed) surjection, then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is semiopen (resp. *semiclosed*) set in Y and consequently, g is *ls*-continuous function in either case.

We, define the following.

Definition 4.18 : A function $f : X \rightarrow Y$ is said to be *sts*-open (resp. *sts*-closed) if the image of each semiopen (resp. *semiclosed*) subset of X is open (resp. *closed*) in Y .

Theorem 4.19 : Let $f : X \rightarrow Y$ be either *sts*-open or *sts*-closed surjection and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is *ls*-continuous. Then, g is *l*-continuous.

Proof is similar to that of Theorem -5.17 above.

In view of Th.2.22(ii) & (iii), respectively, we state the following.

Remark 4.20 : (i) Every *ls*-continuous function is *lp*-continuous

(ii) Every *ls*-continuous function is *l* α -continuous (to See Remark-5.20(i))

In view of Th.2.23(ii) & (iii) &, respectively, we state the following.

Remark 4.21 : (i) Every *lp*-continuous function is *ls*-continuous

(ii) Every *lp* β -continuous function is *ls*-continuous (to See Remark-6.16(ii))

In view of Th.2.22 (ii)&(iii) above (i.e. in an E.D.-space, every semiopen subset is *pre*open & every semiopen subset is an α -open, resp.), so, we have the following.

Theorem 4.22 : Let $f : X \rightarrow Y$ be a *M*-pre open surjection with an E.D.-space X and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is *ls*-continuous. Then, g is *lp*-continuous.

Proof : Suppose f is *M*-preopen surjective function with an E.D.-space X and V be an open subset with Lindelöf complement in Z . Since $g \circ f$ is *ls*-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semiopen subset in X . Since a space X is



E.D.-space assumed and hence $(gof)^{-1}(V)$ is preopen subset in X by Th.2.19(ii). Also, as f is M -preopen and surjective function, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is preopen subset in Y and consequently, g is **lp**-continuous.

Remark 4.23 : For the Case of Th.2.22(iii), to See Th. 5.21.

In view of Th.2.23 (i) above (i.e. in a PS-space, every preopen subset is semiopen), so, we have the following.

Theorem 4.24 : Let $f : X \rightarrow Y$ be presemiopen surjection with a PS-space X and let $g : Y \rightarrow Z$ be any function such that gof is **lp**-continuous. Then, g is **ls**-continuous.

Proof : Suppose f is presemiopen surjective function with a PS-space X and V be an open subset with Lindelöf complement in Z . Since gof is **lp**-continuous, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is preopen subset in X . Since a space X is PS-space and hence $(gof)^{-1}(V)$ is semiopen subset in X by Th.2.20(i). Also, as f is presemiopen and surjective function, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is semiopen subset in Y and consequently, g is **ls**-continuous.

Remark 4.25 : For the Case of Th.2.23(ii), to See Th. 6.18.

In view of the fact that an arbitrary union of preopen (resp. semiopen) sets is preopen (resp. semiopen) [] & [], we have the following :

Theorem 4.26 : If X and Y are two topological spaces and $X = A \cup B$, where A and B are preopen subsets of X and $f : X \rightarrow Y$ is a function such that $f|_A$ and $f|_B$ are **ls**-continuous, then f is **ls**-continuous.

Proof : Assume that A and B are preopen subsets in X . Let U be an open subset of Y with Lindelöf complement. Then, we have $f^{-1}(U) = (f|_A)^{-1}(U) \cup (f|_B)^{-1}(U)$, each of which is semiopen by Lemma-4.6. Thus, $f^{-1}(U)$ is semiopen in X and hence f is **ls**-continuous.

Again, in view of the fact that an arbitrary union of α -open (resp. semiopen) sets is α -open [35](resp. semiopen[13&34]). So, by virtue of Lemma-4.7, we have the following :

Theorem 4.27 : If X and Y are two topological spaces and $X = A \cup B$, where A and B are α -open subsets of X and $f : X \rightarrow Y$ is a function such that $f|_A$ and $f|_B$ are **ls**-continuous, then f is **ls**-continuous.

We, define the following.

Definition 4.28 : A function $f : X \rightarrow Y$ is said to be **I**-irresolute if for each $x \in X$ and each semiopen set $V \subset Y$ containing $f(x)$ and having Lindelöf complement, there exists a semiopen set U containing x such that $f(U) \subset V$.

Definition 4.29 : A function $f : X \rightarrow Y$ is said to be **c**-irresolute if for each $x \in X$ and each semiopen set $V \subset Y$ containing $f(x)$ and having compact complement, there exists a semiopen set U containing x such that $f(U) \subset V$.

Remark 4.30 : (i) Every **I**-irresolute function is **ls**-continuous

(ii) Every **c**-irresolute function is **c**-continuous

(iii) Every **I**-irresolute function is **c**-irresolute.

We, prove the following.

Theorem 4.31: Let $f : X \rightarrow Y$ be a function. Then the following statements are equivalent :

- (i) f is **I**-irresolute.
- (ii) If V is a semiopen subset of Y with Lindelöf complement, then $f^{-1}(V)$ is semiopen subset of X .
- (iii) If F is a semiclosed Lindelöf subset of Y , then $f^{-1}(F)$ is semiclosed subset of X and,
- (iv) For each $x \in X$ and each net $\{x_\lambda\}_{\lambda \in D}$ which s -converges to x , the net $\{f(x_\lambda)\}_{\lambda \in D}$ is eventually in each semiopen set containing $f(x)$ and having Lindelöf complement.

Proof follows by Theorem 2.12 above.



IV. PROPERTIES OF $\mathbf{l\alpha}$ -CONTINUOUS FUNCTIONS

We, define the following.

Definition 5.1 : A function $f : X \rightarrow Y$ is said to be $\mathbf{l\alpha}$ -continuous if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ and having Lindelöf complement, there exists an α -open set U containing x such that $f(U) \subset V$.

Clearly, every $\mathbf{l\alpha}$ -continuous function is $c\text{-}\alpha$ -continuous.

We, recall the following.

Definition 5.2 [35] : Let $x \in X$. A set $U \subset X$ is called a α -neighbourhood (in brief, $\alpha\text{-nbd}$) of x in X if there exists $A \in \alpha O(X)$ such that $x \in A \subset U$.

Theorem 5.3[35] : A subset of a space X is α -open if it is a α -neighborhood of each of its points.

We, prove the following.

Theorem 5.4 : Let $f : X \rightarrow Y$ be a function. Then the following statements are equivalent :

- (i) f is $\mathbf{l\alpha}$ -continuous.
- (ii) If V is an open subset of Y with Lindelöf complement, then $f^{-1}(V)$ is α -open subset of X .
- (iii) If F is a closed Lindelöf subset of Y , then $f^{-1}(F)$ is α -closed subset of X and,

Proof follows by Theorem 2.12 above.

Easy proof of the following is omitted.

Lemma 5.5 : A function $f : X \rightarrow Y$ is said to be $\mathbf{l\alpha}$ -continuous if the inverse image of each open subset of Y having Lindelöf complement is α -open in X .

Lemma 5.6 : A function $f : X \rightarrow Y$ is said to be $\mathbf{l\alpha}$ -continuous if the inverse image of each closed Lindelöf subset of Y is α -closed in X .

We, recall the following.

Lemma 5.7[28] : If $A \in PO(X)$ and $V \in \alpha O(X)$, then $A \cap V \in \alpha O(A)$.

Lemma 5.8[45] : If $A \in SO(X)$ and $V \in \alpha O(X)$, then $A \cap V \in \alpha O(A)$.

Next, we state the following.

Theorem 5.9 : If $f : X \rightarrow Y$ is $\mathbf{l\alpha}$ -continuous function and A be an preopen subset of X , then $f/A : A \rightarrow Y$ is also $\mathbf{l\alpha}$ -continuous.

Theorem 5.10 : If $f : X \rightarrow Y$ is $\mathbf{l\alpha}$ -continuous function and A be an semiopen subset of X , then $f/A : A \rightarrow Y$ is also $\mathbf{l\alpha}$ -continuous.

Theorem 5.11 : If $f : X \rightarrow Y$ is α -irresolute and $g : Y \rightarrow Z$ is $\mathbf{l\alpha}$ -continuous, then $g \circ f$ is $\mathbf{l\alpha}$ -continuous.

Proof. Let U be an open subset of Z having Lindelöf complement. Then, $g^{-1}(U)$ is α -open set in Y , since g is $\mathbf{l\alpha}$ -continuous. Again, as f is α -irresolute, $g^{-1}(U)$ is α -open subset of Y , $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is α -open subset in X . This shows that $g \circ f$ is $\mathbf{l\alpha}$ -continuous function.

Corollary 5.12 : If $f : X \rightarrow Y$ is α -continuous and $g : Y \rightarrow Z$ is \mathbf{l} -continuous, then $g \circ f$ is $\mathbf{l\alpha}$ -continuous.

We, define the following.

Definition 5.13 : A function $f : X \rightarrow Y$ is said to be α s-continuous if the inverse image of each α -open set of Y is open in X .

We state the following

Theorem 5.14 : If $f : X \rightarrow Y$ is α s-continuous and $g : Y \rightarrow Z$ is $\mathbf{l\alpha}$ -continuous, then $g \circ f$ is \mathbf{l} -continuous.



We, recall the following.

Definition 5.17 [20] : A function $f : X \rightarrow Y$ is said to be contra α -continuous if the inverse image of each open set of Y is α -closed in X .

We, state the following.

Theorem 5.18 : Let $f : X \rightarrow Y$ be contra α -continuous function and $g : Y \rightarrow Z$ be I -continuous function .Then, $g \circ f$ is $I\alpha$ -continuous .

Theorem 5.19 : Let $f : X \rightarrow Y$ be either pre- α -open or pre- α -closed surjection and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is $I\alpha$ -continuous .Then, g is $I\alpha$ -continuous .

Proof : Suppose f is pre- α -open (resp. pre- α -closed) surjection and V be an open subset with Lindelöf complement (resp. V be a closed Lindelöf subset) in Z . Since $g \circ f$ is $I\alpha$ -continuous , $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is α -open (resp. α -closed) subset in X by Th.5.4 & Th.5.5. Since f is pre- α -open (resp.pre- α -closed) surjection , then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is α -open (resp. α -closed) set in Y and consequently, g is $I\alpha$ -continuous function in either case.

We, define the following.

Definition 5.20 : A function $f : X \rightarrow Y$ is said to be strongly α -open (resp. strongly α -closed) if the image of each α -open (resp. α -closed) subset of X is open (resp. closed) in Y .

Theorem 5.21 : Let $f : X \rightarrow Y$ be either strongly α -open or strongly α -closed surjection and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is $I\alpha$ -continuous .Then, g is I -continuous .

Proof is similar to that of Theorem -5.17 above.

In view of Th.2.22 (iii) & Th.2.23(iii) , respectively , we state the following.

Remark 5.22 : (i) Every I_s -continuous function is $I\alpha$ -continuous

(ii) Every I_p -continuous function is $I\alpha$ -continuous

In view of Th.2.22 (iii) above (i.e. in an E.D.-space , every semiopen subset is α -open) , so , we have the following.

Theorem 5.23 : Let $f : X \rightarrow Y$ be a pre- α -open surjection with an E.D.-space X and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is I_s -continuous .Then, g is $I\alpha$ -continuous .

Proof : Suppose f is pre- α -open surjective function with an E.D.-space X and V be an open subset with Lindelöf complement in Z . Since $g \circ f$ is I_s -continuous , $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semiopen subset in X . Since a space X is E.D.-space assumed and hence $(g \circ f)^{-1}(V)$ is α -open subset in X by Th.2.19(iii) . Also, as f is pre- α -open and surjective function , $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is α -open subset in Y and consequently, g is $I\alpha$ -continuous .

In view of Th.2.23 (iii) above (i.e. in a PS-space , every preopen subset is α -open) , so , we have the following.

Theorem 5.24 : Let $f : X \rightarrow Y$ be pre- α -open surjection with a PS -space X and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is I_p -continuous .Then, g is $I\alpha$ -continuous .

Proof : Suppose f is pre- α -open surjective function with a PS-space X and V be an open subset with Lindelöf complement in Z . Since $g \circ f$ is I_p -continuous , $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is preopen subset in X . Since a space X is PS-space and hence $(g \circ f)^{-1}(V)$ is α -open subset in X by Th.2.20(iii) . Also, as f is pre- α -open and surjective function , $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is α -open subset in Y and consequently, g is $I\alpha$ -continuous .

In view of the fact that an arbitrary union of preopen (resp. α -open) sets is preopen [33](resp. α -open[35]), we have the following :

Theorem 5.25 : If X and Y are two topological spaces and $X = A \cup B$, where A and B are preopen subsets of X and $f : X \rightarrow Y$ is a function such that $f|_A$ and $f|_B$ are $I\alpha$ -continuous , then f is $I\alpha$ -continuous .



Proof : Assume that A and B are preopen subsets in X . Let U be an open subset of Y with Lindelöf complement. Then, we have $f^{-1}(U) = (f|_A)^{-1}(U) \cup (f|_B)^{-1}(U)$, each of which is α -open by Lemma-5.6. Thus, $f^{-1}(U)$ is α -open in X and hence f is α -continuous.

Again, in view of the fact that an arbitrary union of semiopen (resp. α -open) sets is semiopen [13&34] (resp. α -open [35]). So, by virtue of Lemma-5.7, we have the following :

Theorem 5.26 : If X and Y are two topological spaces and $X = A \cup B$, where A and B are semiopen subsets of X and $f : X \rightarrow Y$ is a function such that $f|_A$ and $f|_B$ are α -continuous, then f is α -continuous.

We, define the following.

Definition 5.27 : A function $f : X \rightarrow Y$ is said to be α -irresolute if for each $x \in X$ and each α -open set $V \subset Y$ containing $f(x)$ and having Lindelöf complement, there exists an α -open set U containing x such that $f(U) \subset V$.

Definition 5.28 : A function $f : X \rightarrow Y$ is said to be c - α -irresolute if for each $x \in X$ and each α -open set $V \subset Y$ containing $f(x)$ and having compact complement, there exists an α -open set U containing x such that $f(U) \subset V$.

Remark 5.29 : (i) Every α -irresolute function is α -continuous
 (ii) Every c - α -irresolute function is c - α -continuous
 (iii) Every α -irresolute function is c - α -irresolute ..

We, define the following.

Definition 5.30 : Let X be a space and $\{x_\lambda\}_{\lambda \in D}$ be a net in X . Let $x \in X$ then $\{x_\lambda\}_{\lambda \in D}$ α -converges to x , denoted by $\{x_\lambda\}_{\lambda \in D} \xrightarrow{\alpha} x$ iff it is eventually in every preopen set containing x .

We, prove the following .

Theorem 5.31: Let $f : X \rightarrow Y$ be a function. Then the following statements are equivalent :

- (i) f is α -irresolute .
- (ii) If V is an α -open subset of Y with Lindelöf complement, then $f^{-1}(V)$ is α -open subset of X .
- (iii) If F is a α -closed Lindelöf subset of Y , then $f^{-1}(F)$ is α -closed subset of X and,
- (iv) For each $x \in X$ and each net $\{x_\lambda\}_{\lambda \in D}$ which α -converges to x , the net $\{f(x_\lambda)\}_{\lambda \in D}$ is eventually in each α -open set containing $f(x)$ and having Lindelöf complement.

Proof follows by Theorem 2.12 above.

V. PROPERTIES OF $\mathbb{I}\beta$ -CONTINUOUS FUNCTIONS

We, define the following.

Definition 6.1 : A function $f : X \rightarrow Y$ is said to be $\mathbb{I}\beta$ -continuous if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ and having Lindelöf complement, there exists an β -open set U containing x such that $f(U) \subset V$.

Clearly, every $\mathbb{I}\beta$ -continuous function is c - β -continuous .

We, recall the following.

Definition 6.2 [37] : Let $x \in X$. A set $U \subset X$ is called a semipre-neighbourhood ($=\beta$ -neighbourhood) (in brief, β -nbd) of x in X if there exists $A \in \beta O(X)$ such that $x \in A \subset U$.

Theorem 6.3 [37] : A subset of a space X is β -open if it is a β -neighborhood of each of its points.

We, prove the following .

Theorem 6.4: Let $f : X \rightarrow Y$ be a function. Then the following statements are equivalent :

- (i) f is $\mathbb{I}\beta$ -continuous .
- (ii) If V is an open subset of Y with Lindelöf complement, then $f^{-1}(V)$ is β -open subset of X .
- (iii) If F is a closed Lindelöf subset of Y , then $f^{-1}(F)$ is β -closed subset of X and,

Proof follows by Theorem 2.12 above.

Easy proof of the following is omitted.

Lemma 6.5 : A function $f : X \rightarrow Y$ is said to be $\mathbf{I}\beta$ -continuous if the inverse image of each open subset of Y having Lindelöf complement is β -open in X .

Lemma 6.6 : A function $f : X \rightarrow Y$ is said to be $\mathbf{I}\beta$ -continuous if the inverse image of each closed Lindelöf subset of Y is β -closed in X .

We, recall the following.

Lemma 6.7[1] : If $A \in \alpha O(X)$ and $V \in \beta O(X)$, then $A \cap V \in \beta O(A)$.

Next, we prove the following.

Theorem 6.8 : If $f : X \rightarrow Y$ is $\mathbf{I}\beta$ -continuous function and A be an α -open subset of X , then $f/A : A \rightarrow Y$ is also $\mathbf{I}\beta$ -continuous .

Easy proof of the Theorem follows by Lemma – 6.6 above.

Theorem 6.9 : If $f : X \rightarrow Y$ is β -irresolute and $g : Y \rightarrow Z$ is $\mathbf{I}\beta$ -continuous, then gof is $\mathbf{I}\beta$ -continuous .

Proof. Let U be an open subset of Z having Lindelöf complement. Then, $g^{-1}(U)$ is β -open set in Y , since g is $\mathbf{I}\beta$ -continuous. Again, as f is β -irresolute, $g^{-1}(U)$ is β -open subset of Y , $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is β -open subset in X . This shows that gof is $\mathbf{I}\beta$ -continuous function.

Corollary 6.10 : If $f : X \rightarrow Y$ is β -continuous and $g : Y \rightarrow Z$ is \mathbf{I} -continuous, then gof is $\mathbf{I}\beta$ -continuous .

We, define the following.

Definition 6.11 : A function $f : X \rightarrow Y$ is said to be $st\beta$ -continuous if the inverse image of each β -open set of Y is open in X .

Theorem 6.12: If $f : X \rightarrow Y$ is $st\beta$ -continuous and $g : Y \rightarrow Z$ is $\mathbf{I}\beta$ -continuous, then gof is \mathbf{I} -continuous .

Proof follows from Th.-6.9 and Def.-6.10.

We, recall the following.

Definition 6.13 [9] : A function $f : X \rightarrow Y$ is said to be contra β -continuous if the inverse image of each open set of Y is β -closed in X .

We, state the following.

Theorem 6.14 : Let $f : X \rightarrow Y$ be contra β -continuous function and $g : Y \rightarrow Z$ be \mathbf{I} -continuous function. Then, gof is $\mathbf{I}\beta$ -continuous .

Theorem 6.15 : Let $f : X \rightarrow Y$ be either pre- β -open or pre- β -closed surjection and let $g : Y \rightarrow Z$ be any function such that gof is $\mathbf{I}\beta$ -continuous. Then, g is $\mathbf{I}\beta$ -continuous .

Proof : Suppose f is pre- β -open (resp. pre- β -closed) surjection and V be an open subset with Lindelöf complement (resp. V be a closed Lindelöf subset) in Z . Since gof is $\mathbf{I}\beta$ -continuous, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is β -open (resp. β -closed) subset in X by Lemma-6.5 & Lemma-6.6. Since f is pre- β -open (resp. pre- β -closed) surjection, then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is β -open (resp. β -closed) set in Y and consequently, g is $\mathbf{I}\beta$ -continuous function in either case.

We, define the following.

Definition 6.16 : A function $f : X \rightarrow Y$ is said to be $st\beta$ -open (resp. $st\beta$ -closed) if the image of each β -open (resp. β -closed) subset of X is open (resp. closed) in Y .

Theorem 6.17 : Let $f : X \rightarrow Y$ be either $st\beta$ -open or $st\beta$ -closed surjection and let $g : Y \rightarrow Z$ be any function such that gof is $\mathbf{I}\beta$ -continuous. Then, g is \mathbf{I} -continuous .

Proof follows by Theorem -6.15 above.

In view of Th.2.22(iv) & Th.2.23 (ii), respectively, we state the following.



Remark 6.18 : (i) Every $\mathbf{I\beta}$ -continuous function is $\mathbf{I\mu}$ -continuous
 (ii) Every $\mathbf{I\beta}$ -continuous function is $\mathbf{I\sigma}$ -continuous

In view of Th.2.22 (iv) above (i.e. in an E.D.-space, every β -open subset is preopen), so, we have the following.

Theorem 6.19 : Let $f : X \rightarrow Y$ be a \mathbf{M} -Preopen surjection with an E.D.-space X and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is $\mathbf{I\beta}$ -continuous. Then, g is $\mathbf{I\mu}$ -continuous.

Proof : Suppose f is \mathbf{M} -preopen surjective function with an E.D.-space X and V be an open subset with Lindelöf complement in Z . Since $g \circ f$ is $\mathbf{I\beta}$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is β -open subset in X . Since a space X is E.D.-space assumed and hence $(g \circ f)^{-1}(V)$ is preopen subset in X by Th.2.19 (iv). Also, as f is \mathbf{M} -preopen and surjective function, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is preopen subset in Y and consequently, g is $\mathbf{I\mu}$ -continuous.

In view of Th.2.23 (ii) above (i.e. in a PS-space, every β -open subset is semiopen), so, we have the following.

Theorem 6.20 : Let $f : X \rightarrow Y$ be presemiopen surjection with a PS-space X and let $g : Y \rightarrow Z$ be any function such that $g \circ f$ is $\mathbf{I\beta}$ -continuous. Then, g is $\mathbf{I\sigma}$ -continuous.

Proof : Suppose f is presemiopen surjective function with a PS-space X and V be an open subset with Lindelöf complement in Z . Since $g \circ f$ is $\mathbf{I\beta}$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is β -open subset in X . Since a space X is PS-space and hence $(g \circ f)^{-1}(V)$ is semiopen subset in X by Th.2.20 (ii). Also, as f is presemiopen and surjective function, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is semiopen subset in Y and consequently, g is $\mathbf{I\sigma}$ -continuous.

In view of the fact that an arbitrary union of β -open (resp. α -open) sets is β -open [1&37](resp. α -open[35]), we have the following :

Theorem 6.21 : If X and Y are two topological spaces and $X = A \cup B$, where A and B are α -open subsets of X and $f : X \rightarrow Y$ is a function such that $f|_A$ and $f|_B$ are $\mathbf{I\beta}$ -continuous, then f is $\mathbf{I\beta}$ -continuous.

Proof : Assume that A and B are α -open subsets in X . Let U be an open subset of Y with Lindelöf complement. Then, we have $f^{-1}(U) = (f|_A)^{-1}(U) \cup (f|_B)^{-1}(U)$, each of which is β -open by Lemma-6.6. Thus, $f^{-1}(U)$ is β -open in X and hence f is $\mathbf{I\beta}$ -continuous.

We, define the following.

Definition 6.22 : A function $f : X \rightarrow Y$ is said to be $\mathbf{I\beta}$ -irresolute if for each $x \in X$ and each β -open set $V \subset Y$ containing $f(x)$ and having Lindelöf complement, there exists an β -open set U containing x such that $f(U) \subset V$.

Definition 6.23 : A function $f : X \rightarrow Y$ is said to be c - $\mathbf{I\beta}$ -irresolute if for each $x \in X$ and each β -open set $V \subset Y$ containing $f(x)$ and having compact complement, there exists an β -open set U containing x such that $f(U) \subset V$.

Remark 6.24: (i) Every $\mathbf{I\beta}$ -irresolute function is $\mathbf{I\beta}$ -continuous
 (ii) Every c - $\mathbf{I\beta}$ -irresolute function is c - $\mathbf{I\beta}$ -continuous
 (iii) Every $\mathbf{I\beta}$ -irresolute function is c - $\mathbf{I\beta}$ -irresolute ..

Clearly, every $\mathbf{I\beta}$ -irresolute function is c - $\mathbf{I\beta}$ -irresolute.

We, define the following.

Definition 6.25: Let X be a space and $\{x_\lambda\}_{\lambda \in D}$ be a net in X . Let $x \in X$ then $\{x_\lambda\}_{\lambda \in D}$ β -converges to x , denoted by $\{x_\lambda\}_{\lambda \in D} \xrightarrow{\beta} x$ iff it is eventually in every β -open set containing x .

We, prove the following.

Theorem 6.26 : Let $f : X \rightarrow Y$ be a function. Then the following statements are equivalent :

- (i) f is $\mathbf{I\beta}$ -irresolute.
- (ii) If V is an β -open subset of Y with Lindelöf complement, then $f^{-1}(V)$ is β -open subset of X .
- (iii) If F is a β -closed Lindelöf subset of Y , then $f^{-1}(F)$ is β -closed subset of X and,
- (iv) For each $x \in X$ and each net $\{x_\lambda\}_{\lambda \in D}$ which β -converges to x , the net $\{f(x_\lambda)\}_{\lambda \in D}$ is eventually in each β -open set containing $f(x)$ and having Lindelöf complement.

Proof follows by Theorem 2.12 above.

VI. CONCLUSION

We have defined various allied I -irresolute functions and c -irresolute functions at the of, **respective, Sections** above, one can establish their some more properties analogous to that of allied I -continuous functions and c -continuous functions.

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