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# Static and Dynamic Analysis of Homogeneous Isotropic Rectangular Plates with Linearly Varying Thickness

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**ABSTRACT:** In the recent past, there has been a phenomenal increase in the use of plates of uniform/non-uniform thickness due to the desirability of lightweight, high strength, corrosion resistance and high-temperature performance requirements in modern technology. Plates of various materials are widely used in many engineering structures and machines. Their design requires an accurate determination of natural frequencies and mode shapes. As an important numerical computational method, the finite difference method has broad applications in various engineering problems. In this paper, rectangular isotropic plates of linearly varying thickness in one direction are analyzed using finite difference method. Various combinations of boundary conditions are taken and the variation is studied. The static deflections are found out with reasonable accuracy by adopting a suitable mesh size. Dynamic analysis is carried out to find out frequency parameters for the first three modes of vibration. The aspect ratio is varied and the results are compared with those available in literature and are found to be in close agreement. In this paper, I presented the analysis and numerical results for free transverse vibration of thin rectangular plates having arbitrarily varying non-homogeneity with the in-plane coordinates along the two concurrent edges. For finding the general governing differential equation first used the Kirchhoff's plate theory by considering their assumptions. After finding the governing equation for the plate. For the non- homogeneity a linear variation for Young's modulus and density of the plate has been assumed. Finite difference method (FDM) has been used to obtain the eigen value problem for such model plate for two different boundary condition at the edge namely (i) CCCC fully clamped (ii) CSCS two opposites are clamped and other two are simply supported. By solving these eigen value problem using MATLAB, the lowest three eigen value have been reported as the first three natural frequencies for the three mode of vibration. The effect of various plate parameters such on the vibration characteristics has been analysed. Three dimensional mode shape has been plotted.

**KEYWORDS:** Non-homogeneity, flexural rigidity, FDM, Non-dimensional frequency parameters

## I. INTRODUCTION

### 1.1 Background of the Study

A plate can be defined as a structural component having significant dimensions in two directions. It is seen as a continuum, in which one dimension is much smaller than the other two dimensions. A plate is flat (i.e. it has level surface) before loading as contrasted from a shell which is curved before loading. It is described as thin if the ratio of its thickness to the smaller span length is less than 1/20, otherwise it is thick. When the stiffness and strength of a plate in all directions is identical, the plate is said to be isotropic, otherwise it is orthotropic. Most plates are either rectangular, circular, or any other geometry in shape (Chakraverty, 2009). Plates are integral parts of any structure. They occur as shear walls, floor panels and shelves. In steel structures, plates occur as components of I-, H-, T-, or Channel sections. In structural hollow sections, sheets used to enclose lift shafts, or walls or cladding in framed structures, are plates.

Every structural component is designed to carry loads. The loads that act on plates are either static or dynamic. These loads can either be in-plane or out-of-plane. In-plane loads cause stretching of the plate when tensile, and buckling

when compressive. Out-of-plane loads cause bending of the plate. When the load is not suddenly applied to the plate, and it does not change with time and position, it is said to be a static load. A load whose magnitude, direction and/or position varies with time is a dynamic load. Forces suddenly applied to structures, are termed shock or impact loads, and result in dynamic loading. Shock loading is produced by a sudden application of force or motion to a structural member, whereas impact loading results from the collision of bodies. When the time of application of a load, is equal to or smaller than the largest natural period of vibration of the structural component, shock or impact loading is produced.

The effects of dynamic loads on structures include reduction in strength, initiation and propagation of fatigue cracks, destruction and breakage of certain sensitive equipment, feeling of danger and insecurity in the minds of occupants of the structure, and in extreme cases, collapse leading to loss of lives and property.

Structural dynamics deals with time-dependent motions of structures and the analyses of the internal forces associated with them, with the objective of determining the effects of vibrations on the performance of the structure. Usually, a dynamic force acts to modify the static stress and strain fields as well as the resistance properties of a structural material. The problem is that dynamic behaviour is influenced by a larger number of parameters than static behaviour. And so, assessing the structural response to dynamic loads is not easy.

Researchers in the past, like Euler, Bernoulli and Timoshenko, have been able to formulate a governing differential equation describing the response of a rectangular thin isotropic plate subjected to a dynamic load (Szilard, 2004; Ventsel & Krauthammer, 2001). This resulted into a fourth order partial differential equation. Obtaining the exact solution to this equation has been the subject of research for engineers and mathematicians for several years. The existing conventional methods of obtaining solutions are the equilibrium approach developed by Euler, the energy approach and the numerical approach. Navier's and Levy's methods are examples of equilibrium approach. The energy approach includes the Ritz, Raleigh-Ritz, Galerkin, minimum potential energy methods, etc. Examples of the numerical approach are finite difference, finite strip, Runge-Kutta and finite element methods, among others. Each of these methods has limitations, problems, and most of them only give approximate solutions.

The present work is done to fill this gap of isotropic plate with the uniform thickness. For such plate model, the governing differential equation of motion has been solved employing the Finite Difference Method (FDM). Further, this consideration for non-homogeneity together with aspect ratio of the plate shows their effect on the natural frequencies for the all possible combination of the classical boundary condition at the four edge of the plate material. But, in this paper linear model for non-homogeneity of the isotropic material have been considered assuming that the Young's modulus and density are varying in same distinct manner along the both in plane coordinates. The non-homogeneity arises due to arbitrary variation in Young's modulus and density. All of these investigation have been taking poison's ratio  $\nu$  as constant. This type of non-homogeneity arises during the fibre reinforced plastic structure, which use fibre of different strength properties along two mutually perpendicular directions along the edges of the plate.

## II. LITERATURE REVIEW

### 2.1 Plate Dynamics

The objective of dynamics is to determine the effect of vibrations on the performance of the plate.

Ventsel and Krauthammer (2001) considered two kinds of motion of plates, namely, free and forced vibrations. Free or natural vibration occurs in the absence of applied loads, but may be initiated by applying initial conditions to the plate. Free vibration analysis deals with natural characteristics of the plate, and these natural vibrations occur at discrete frequencies, depending on the geometry and material properties of the plates. Forced vibration results from an application of time-dependent loads. There are two kinds of forced vibration, namely harmonic and transient vibrations. A harmonic response occurs when a periodic force is applied to the plate, while a transient response occurs when the applied force is not a periodic force.

Ventsel and Krauthammer (2001) stated that the dynamics of plates can be modelled mathematically by partial differential equations based on Newton's laws or by integral equations based on the consideration of virtual work.

They stated that in practice, only the lateral vibration is of interest; the effects of extensional vibrations in the middle plane may be neglected. So, inertia forces associated with the lateral translation of the plate are considered. Damping of vibrations is caused by either internal friction or by the surrounding media.

Ventsel and Krauthammer (2001) stated that structural damping has little or no effect on the natural frequencies and the steady-state deflection constants of the vibration, and so can be safely ignored in the analysis.

Kalita and Dutta (2013) studied the different mode frequencies for free vibration of isotropic plates using the ANSYS computer package. Several plate types involving a combination of three different boundary conditions, namely, clamped, simply-supported and free edge conditions were considered. The different types of plates considered are SSSS, SCSC, SCSS, SCSF, SSSF, CCCC, CCCS, CCCF, CCSS, CCSF, CCFF, CSCF, CSSF, CSFF, CFSF, CFCF, and CFFF (where C, S and F are clamped, simply-supported and free edges respectively). The analysis for isotropic plate was carried out for uniform thickness and different aspect ratios ( $a/b = 0.4, 1.0, 1.5$  and  $2.0$ ).

Ezeh, Ibearugbulem, Njoku, and Ettu (2013) presented a free vibration analysis of thin rectangular flat isotropic plate. The peculiar shape function for SSSS plate in vibration was obtained by substituting the boundary conditions that satisfy the support modes of the plate into the Taylor series formulated shape function. The peculiar shape function obtained was used in the Galerkin's functional to derive the fundamental natural frequency equation for SSSS plate. From the comparison between the present solution and those from previous research works, it can be inferred that the Taylor series formulated shape function used for SSSS plate is a close approximation of the exact shape function. Also, the Galerkin's functional as derived therein is a good variational method, which gave solutions that are upper bound.

Ezeh, Ibearugbulem, and Ebirim (2014a) presented a theoretical formulation based on Taylor series shape function and application of Ritz method for the free vibration analysis of plate with three edges clamped and one freed (i. e. CCCF and CCFC plates). The polynomial shape function derived was substituted into the potential energy functional, which was minimized to obtain the fundamental natural frequency. This research focused on aspect ratios that vary from 0.1 to 2.0 with 0.1 increments. The results obtained were compared with previous research works; for example, for aspect ratio of 1, the value of non dimensional natural frequency parameter is 24.6475.

Since we know that, as the technology is increasing rapidly, the requirement of the advanced material is also at a high demand due to their extensive use in the engineering applications, especially for the performance in the high temperature environment. Because these material are stiffer, stronger and corrosion resistance. This literature provides free vibration analysis of thin rectangular isotropic plate having arbitrarily varying non-homogeneity with the inplane coordinates along the two concurrent edges.

The study of non-homogeneous material with fair amount of accuracy is of practical importance from the view point of design engineers. In this regard, the earliest model for the non-homogeneity was proposed by Bose [1] in which Young's modulus and density of the plate material are supposed to vary with the radius vector i.e.  $(E, \rho) = (0, 0, E, \rho)r$ . An approximate solution is obtained using Ritz method. The frequency parameter  $\Omega$  is found to increase with the increase in non-homogeneity parameter as well as taper parameter, while it decreases with the increase in density parameter. Later on, Biswas [2] developed a model for non-homogeneity considering the exponential variation for torsional rigidity  $Z \propto 1/\mu\mu\mu$  and the material density  $Z \propto 1/\mu\rho = \rho$ , where  $0 < \mu, \rho$  and  $\mu$  are constants. DQM (Differential Quadrature Method) has been used to analyse the free vibration of non-homogeneous orthotropic rectangular plate of parabolic varying thickness resting on Winkler foundation.

Das and Mishra [3] deals with the transverse vibration of elliptical and circular plate using two dimensionally boundary characteristics orthogonal polynomial using Rayleigh Ritz method by considered the non-homogeneity of the plate characterized by taking Young's modulus  $(E, \rho) Z = E_0(1 - \alpha\sigma\sigma\sigma Z)$  and density  $(\rho) Z = \rho_0(1 - \beta\sigma\sigma\sigma Z)$ ,  $0 \leq \rho \leq 1$  and  $\alpha$  is an integer greater than 3. Rao et al. [4] proposed a model for non-homogeneous isotropic thin plate by assuming linear variation for Young's modulus and density given by  $E = E_0(1 + \alpha x)$  and  $\rho = \rho_0(1 + \beta x)$ ,  $\alpha, \beta$  being constant.

The free transverse vibration of non-homogeneous circular plate of linearly varying thickness on the basis of classical plate theory has been analysed with help of Tomar JS, Gupta DC and Jain NC [5]. In which The frequency parameter increases with increase the rate of increase for free plate is higher than simply-supported plate but lesser

than clamped plate. The Natural frequencies for free vibration of non-homogeneous elliptic and circular plates using two-dimensional orthogonal polynomials has been analyse with help of Chakravety and Pety [6]. In this paper the non- homogeneity is taken by considering linear variation of the Young's Modulus and parabolic variation of the density of the material.

The first five natural frequencies have been analysed in this paper. Effect of non-homogeneity on asymmetric vibrations of non-uniform circular plates has been analysed by Seema Sharma, R. Lal, Neelam Singh [7] where the non-homogeneity arises due to exponential variation of the Young's modulus as well as density parameters along the radial direction. The first three natural frequencies are found with using of Ritz method. The effect of the taper parameters and the density parameters has been analysed in this paper. Free transverse vibrations of nonhomogeneous orthotropic rectangular plates with bilinear thickness variation resting on Winkler foundation are presented by Yazuvendrakumar, R. Lal [8] by using two dimensional boundary characteristics orthogonal polynomial using Rayleigh Ritz method on the basis of classical plate theory. Orthogonal polynomial has been generated by using Gram-Schmidt. The non-homogeneity is considering by taking the linear variation of the Young's modulus as well as density. The two dimensional thickness variation is considered by taken Cartesian product of linear variation. In the previous all literature papers there is assumption taken that the Poisson's ratio is constant throughout. However, in the two studies Rossi and Laura [9] it has been found that effect of the Poisson's ratio on the frequency is not appreciable and reported as considerably less than 1%. Free vibration analysis of clamped orthotropic plate material using Lagrangian multiplier technique by R. L Ram Kumar et al., [10].

Transverse shear deformation effects were included using a higher-order plate theory. The validated free vibration analysis has also been used to develop an analysis to predict the dynamic response of clamped orthotropic plates subjected to low-velocity impact by a hard object. Effect of non-homogeneity parameters, aspect ratio and thickness variation together with foundation parameter on thenatural frequencies has been illustrated for the first three modes of vibration for four different combinations of clamped, simply supported and free edges correct to four decimal places. The hierarchical finite element method is used to determine the natural frequencies and modes of a flat, rectangular plate by Bardell [11] Ten different boundary conditions—including free edges and point supports—are considered in this paper. Extensive results are presented for each case (including the variation of frequency with the aspect ratio and the Poisson ratio), and these are shown to be in very good agreement with the work of other investigators. This confirms both the applicability and accuracy of solution of the HFEM to problem of this type.

Kantorovich method has also been used to find the natural frequencies of orthotropic rectangular plates Sakata et al., [12]. In this method the natural frequencies are obtained by the successive reduction of the plate partial differential equation and by solving the results. The reduction of the partial differential equation is obtained by assuming an approximate solution satisfying the boundary condition along the one direction and then apply the Kantorovich method. Vibration analysis of laminated composite plates on elastic foundations by Ishan kucukrendeci and Hasan Kucuk [13]. In this study the composite plates consist of layers and layer are arranged at different angles. And this plate supported by the springs of five different points. The finite element method is used to find the natural frequencies of this plate model. The boundary condition of the plate having three different conditions i.e clamped, simply supported and free edge. MATLAB is used to solve the equation.

Free vibration analysis of rectangular plates with variable thickness M. Huang [14]. In this paper an approximate method which is based on the Green function of a ate plate is used. The Green function of a rectangular plate with arbitrary variable thickness is obtained as a discrete form of solution for the deflection of the plate with concentrated load. The solution is obtained at each discrete point which are equally distributed on the plate. After applying the green function, the free vibration of the plate converted into the eigen value problem of matrix. The convergence an accuracy of the numerical solution for the natural frequencies parameters calculated by the proposed method.

### III. PROPOSED METHODOLOGY

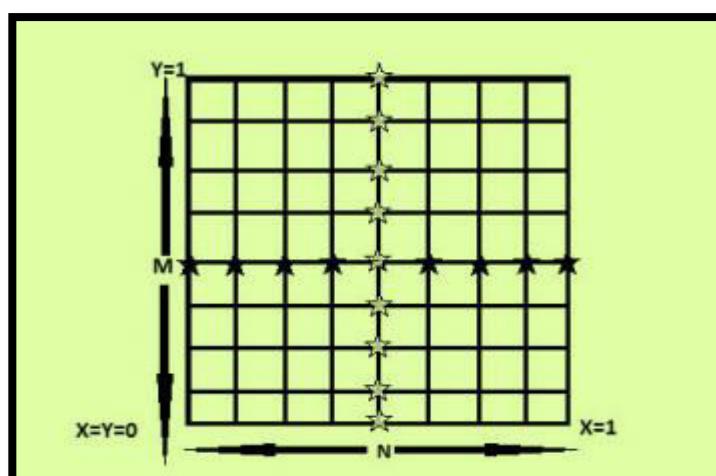
#### 3.1 METHODOLOGY

All together there are 21 combinations of simple boundary conditions (i.e., either clamped (C), simply supported (SS), or free (F)) for rectangular plates. Frequency parameter does not depend upon the Poisson'sratio until at least

one of the edge of the plate is free. However because D contains 9 , the frequency themselves depends upon 9 for all cases.

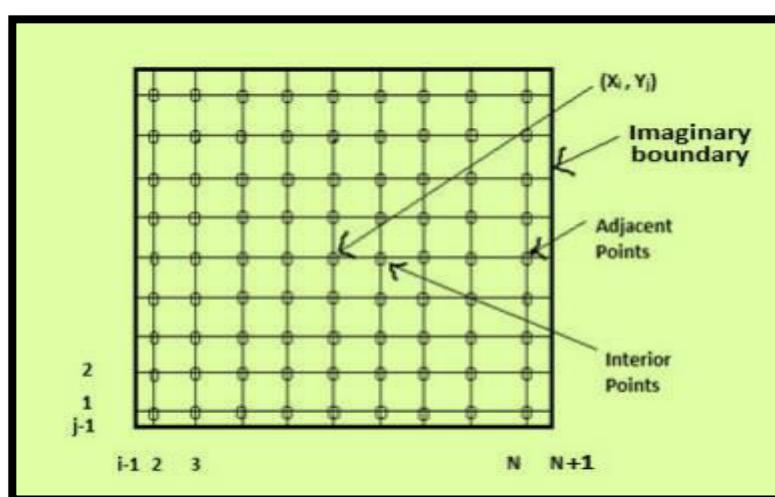
### 3.1.1 Method of Solution Using FDM Technique

Finite difference method is important tool for the analysis of structures. Due to FDM technique a sustained line of development of methods of nonlinear analysis of structural element and frames. This method is also extended to use in composite frames and structure. A good correlation with this experiment has been obtained for the deflection. Also the problem of nonlinear response of the structure has been successfully solved. Since we know that the FDM technique is a numerical method to find the approximate solution of a differential equation. It is very helpful to find the solution of partial differential equation. In this method at first the problem has to discretized. This is done by dividing the problem domain into a uniform grid. Therefore, it is also known as discretization method. For this plate model divided the model into  $M \times N$  grid points. The length and the width of each grid is  $\Delta X$  and  $\Delta Y$  respectively.



(Figure 1 Grid point distribution on a rectangular domain)

For applying the method FDM, one has to divide the computational domain of the plate  $\{0 \leq X \leq 1, 0 \leq Y \leq 1\}$  by drawing the line parallel to X -axis and Y - axis respectively. The total number of functional value in the whole domain is  $N \times M$ . Equally space grid points have been considered for the analysis of the plate model.



(Figure 2 Demonstration of interior points, adjacent points and boundary of a rectangular plates)

Figure (2) shows that number of grid points vary from  $(i-1)$  to  $N$  in  $x$ -direction and  $(j-1)$  to  $M$  in the  $y$ -direction. Let choose a grid of coordinate  $(X_i Y_j)$  and displacement function at that coordinate will be  $W(i, j)$ . The equation (3.16) have the 4th order partial differential term of two variables. Now for solving the equation (3.16) by FDM technique we have to find the value of partial differential part in the form of forward difference.

e.g.

$$\begin{aligned}\frac{\partial^3 W}{\partial X^3} &= \frac{W(i+2, j) - W(i-2, j) - 2\{W(i+1, j) - W(i-1, j)\}}{\Delta X^3}, \\ \frac{\partial^3 W}{\partial Y^3} &= \frac{W(i, j+2) - W(i, j-2) - 2\{W(i, j+1) - W(i, j-1)\}}{\Delta Y^3} \\ \frac{\partial^3 W}{\partial X \partial Y^2} &= \frac{W(i+1, j+1) + W(i+1, j-1) - \Gamma_1 - W(i-1, j+1) - W(i-1, j-1)}{\Delta X \Delta Y^2} \\ \frac{\partial^3 W}{\partial X^2 \partial Y} &= \frac{W(i+1, j+1) + W(i-1, j+1) - \Gamma_2 - W(i+1, j-1) - W(i-1, j-1)}{\Delta X^2 \Delta Y}\end{aligned}\tag{4.1}$$

(4.2)

$$\begin{aligned}\frac{\partial^4 W}{\partial X^2 \partial Y^2} &= \frac{W(i+1, j+1) + W(i+1, j-1) - \Gamma_1 - \Gamma_2 + W(i-1, j+1) + W(i-1, j-1) + 4W(i, j)}{\Delta X^2 \Delta Y^2} \\ \Gamma_1 &= 2\{W(i+1, j) - W(i-1, j)\} \\ \Gamma_2 &= 2\{W(i, j+1) - W(i, j-1)\}\end{aligned}$$

$$\begin{aligned}\frac{\partial^4 W}{\partial X^4} &= \frac{W(i+2, j) - W(i-2, j) - 4\{W(i+1, j) - W(i-1, j)\} + 6W(i, j)}{\Delta X^4} \\ \frac{\partial^4 W}{\partial Y^4} &= \frac{W(i, j+2) - W(i, j-2) - 4\{W(i, j+1) - W(i, j-1)\} + 6W(i, j)}{\Delta Y^4}\end{aligned}\tag{4.3}$$

Now put the required above value in the equation (2.16) we get

$$W(i, j) = -\frac{R}{S}\tag{4.4}$$

Where,

$$\begin{aligned}
 R &= \Delta_1(R_1) + \Delta_2(R_2) + \Delta_3(R_3) + \Delta_4(R_4) + \Delta_5(R_5) \\
 S &= \Delta_1(S_1) - S_2 + \Delta_5 S_3 + \Delta_6 \\
 R_1 &= (\lambda^4 \Psi_1 + 2\lambda^2 \Psi_2 + \Psi_3), \\
 R_2 &= (\Psi_4 + \lambda^2 \Psi_5), R_3 = (\lambda^4 \Psi_6 + \lambda^2 \Psi_7) \\
 R_4 &= (\Phi_1 + 9\lambda^2 \Phi_2), R_5 = (9\lambda^2 \Phi_1 + \lambda^4 \Phi_3) \\
 S_1 &= \left( \frac{6}{\Delta X^4} + \frac{8\lambda^2}{\Delta X^2 \Delta Y^2} + \frac{6\lambda^2}{\Delta Y^4} \right), S_2 = \left( \frac{2\Delta_4}{\Delta X^2} + \frac{29\lambda^2}{\Delta Y^2} + \right) \\
 S_3 &= \left( \frac{2\Delta_5 9\lambda^2}{\Delta X^2} \right), S_4 = \left( \frac{6\Delta_5 \lambda^4}{\Delta Y^4} \right)
 \end{aligned} \tag{4.5}$$

$$\begin{aligned}
 \Delta_1 &= (1 + \alpha_1 X^p + \alpha_2 Y^q), \\
 \Delta_2 &= 2\alpha_1 p X^{p-1}, \Delta_3 = 2\alpha_2 q Y^{q-1} \\
 \Delta_4 &= p(p-1)\alpha_1 X^{p-2} \\
 \Delta_5 &= q(q-1)\alpha_1 Y^{q-2} \\
 \Delta_6 &= \Omega^2 (1 + \beta_1 X^r + \beta_2 Y^s)
 \end{aligned} \tag{4.6}$$

$$\begin{aligned}
 \Psi_1 &= \frac{\partial^4 W}{\partial Y^4} - \frac{6W(i,j)}{\Delta Y^4} \\
 \Psi_2 &= \frac{\partial^4 W}{\partial X^2 \partial Y^2} - \frac{4W(i,j)}{\Delta X^2 \Delta Y^2} \\
 \Psi_3 &= \frac{\partial^4 W}{\partial X^4} - \frac{6W(i,j)}{\Delta X^4} \\
 \Psi_4 &= \frac{\partial^3 W}{\partial X^3}, \Psi_5 = \frac{\partial^3 W}{\partial X \partial Y^2} \\
 \Psi_6 &= \frac{\partial^3 W}{\partial Y^3}, \Psi_7 = \frac{\partial^3 W}{\partial X^2 \partial Y} \\
 \Phi_1 &= \frac{W(i+1,j) + W(i-1,j)}{\Delta X^2} \\
 \Phi_2 &= \frac{W(i,j+1) + W(i,j-1)}{\Delta Y^2} \\
 \Phi_3 &= \frac{W(i,j+2) - 4\{W(i,j+1) + W(i,j-1)\} + W(i,j-2)}{\Delta X^2}
 \end{aligned} \tag{4.7}$$

Similarly, the discretized boundary condition along the four edge of the rectangular plates can be written as

**For clamped edge**

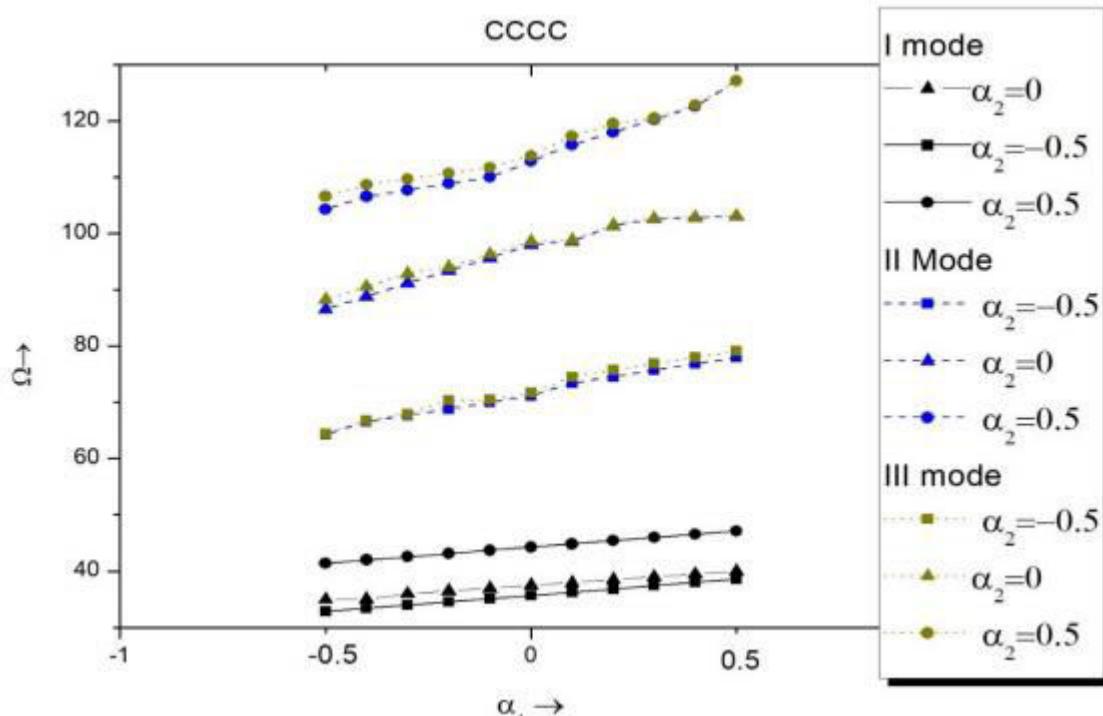
$$\begin{aligned} W(i,j) &= \frac{\partial W}{\partial X} = \frac{W(i+1,j) - W(i,j)}{\Delta X} = 0 \text{ at } X = 0 \text{ and } X = 1 \\ W(i,j) &= \frac{\partial W}{\partial Y} = \frac{W(i,j+1) - W(i,j)}{\Delta Y} = 0 \text{ at } Y = 0 \text{ and } Y = 1 \end{aligned} \quad (4.8)$$

**for simply supported edge**

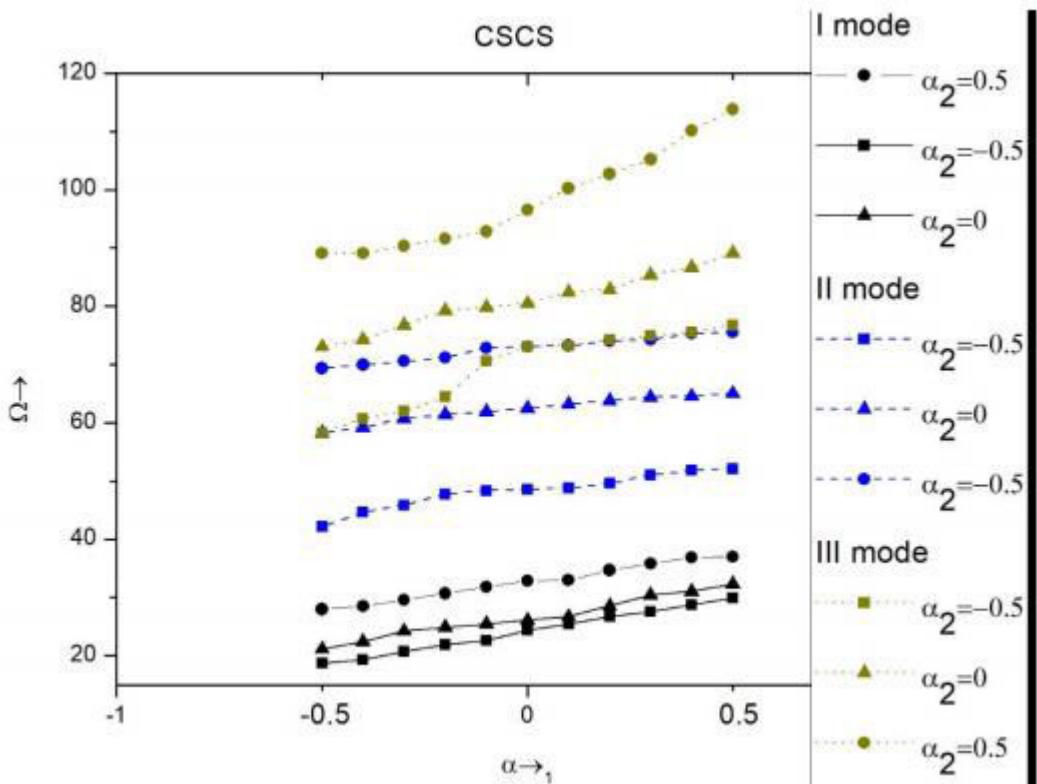
$$\begin{aligned} W(i,j) &= \frac{\partial^2 W}{\partial X^2} = \frac{W(i+1,j) + W(i-1,j) - 2W(i,j)}{\Delta X^2} = 0 \text{ at } X = 0 \text{ and } X = 1 \\ W(i,j) &= \frac{\partial^2 W}{\partial Y^2} = \frac{W(i,j+1) + W(i,j-1) - 2W(i,j)}{\Delta Y^2} = 0 \text{ at } Y = 0 \text{ and } Y = 1 \end{aligned} \quad (4.9)$$

With using of the boundary condition in the converted governing equation of the plate we get the solution of this plate model.

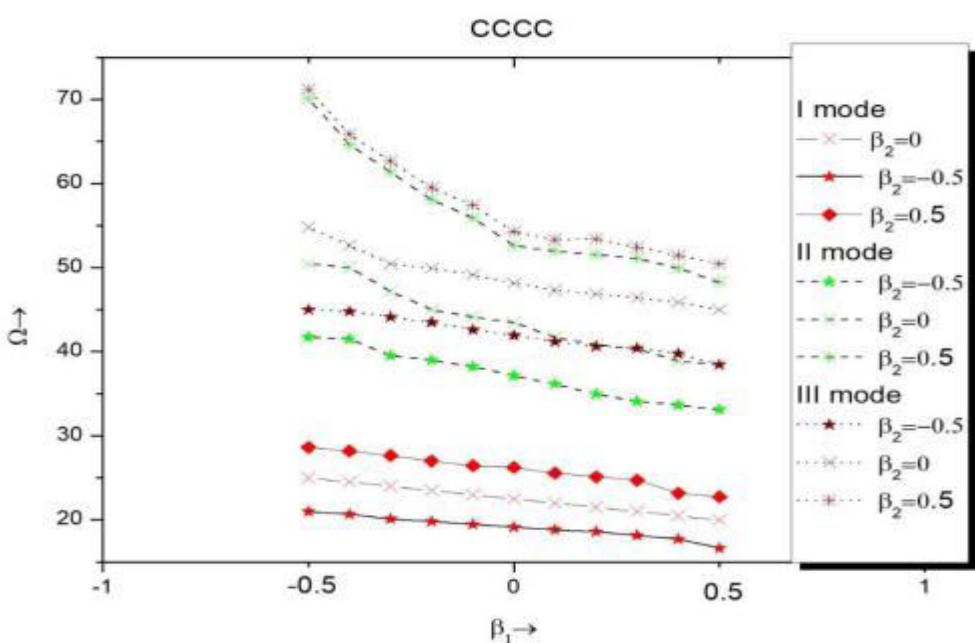
### 3.2 RESULTS



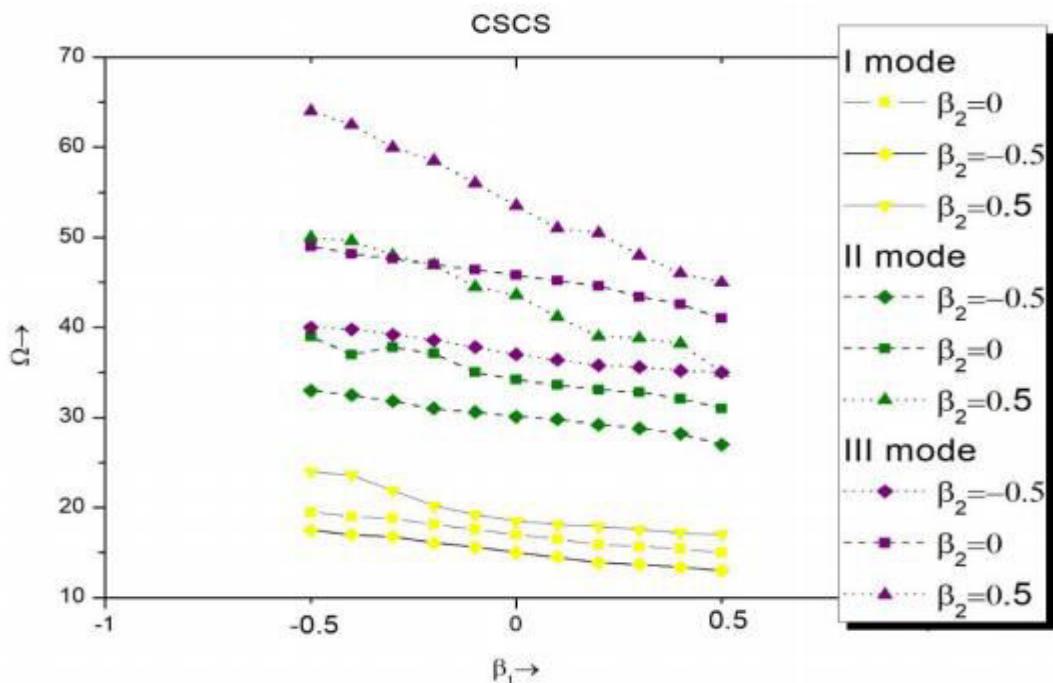
(Figure 3 Frequency parameters  $\Omega$  versus non-homogeneity parameters  $\alpha_1$  for a square CCCC plate for  $\beta\beta=-0.5$ , for the first, second and third mode respectively)



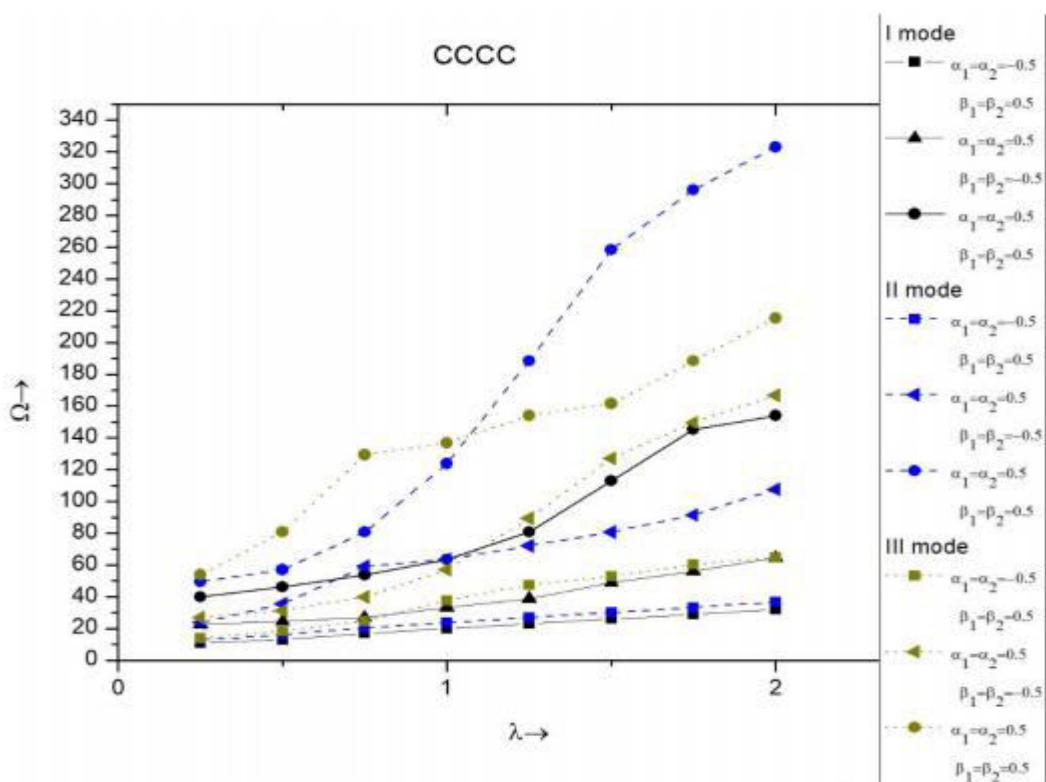
(Figure 4 Frequency parameters  $\Omega$  versus non-homogeneity parameters  $\alpha_1$  for a square CSCS plate for  $\beta\beta=-0.5$ , for the first, second and third mode respectively)



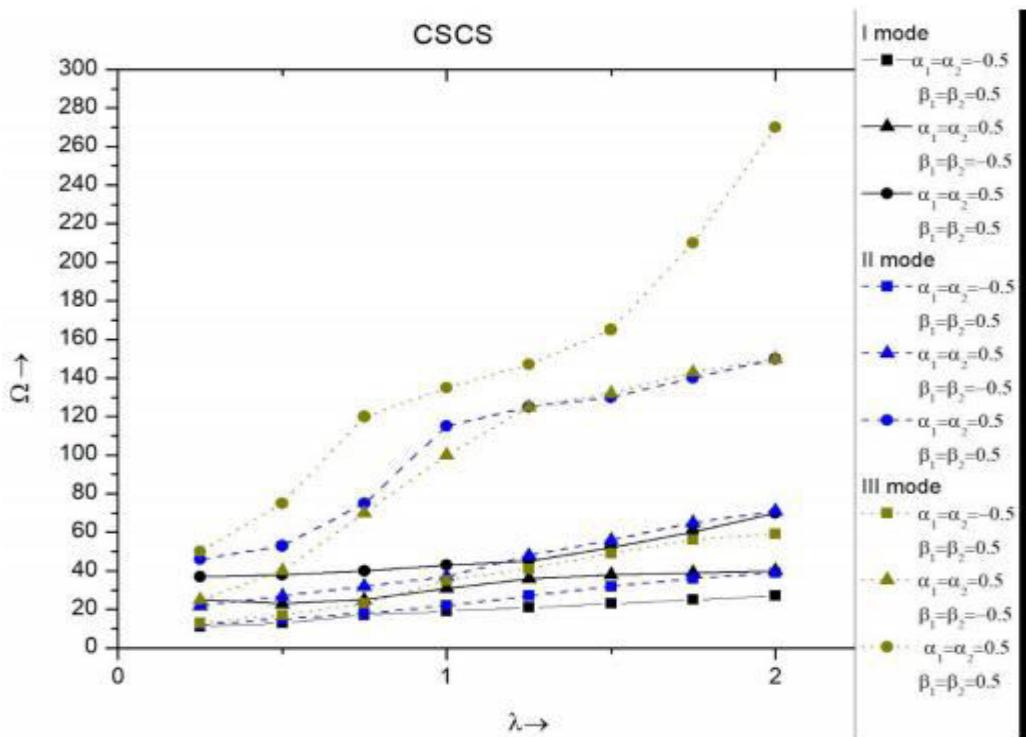
(Figure 5 Frequency parameters  $\Omega$  versus non-homogeneity parameters  $\beta_1$  for a CCCC square  $\alpha\alpha=-0.5$ , for the first, second and third mode respectively)



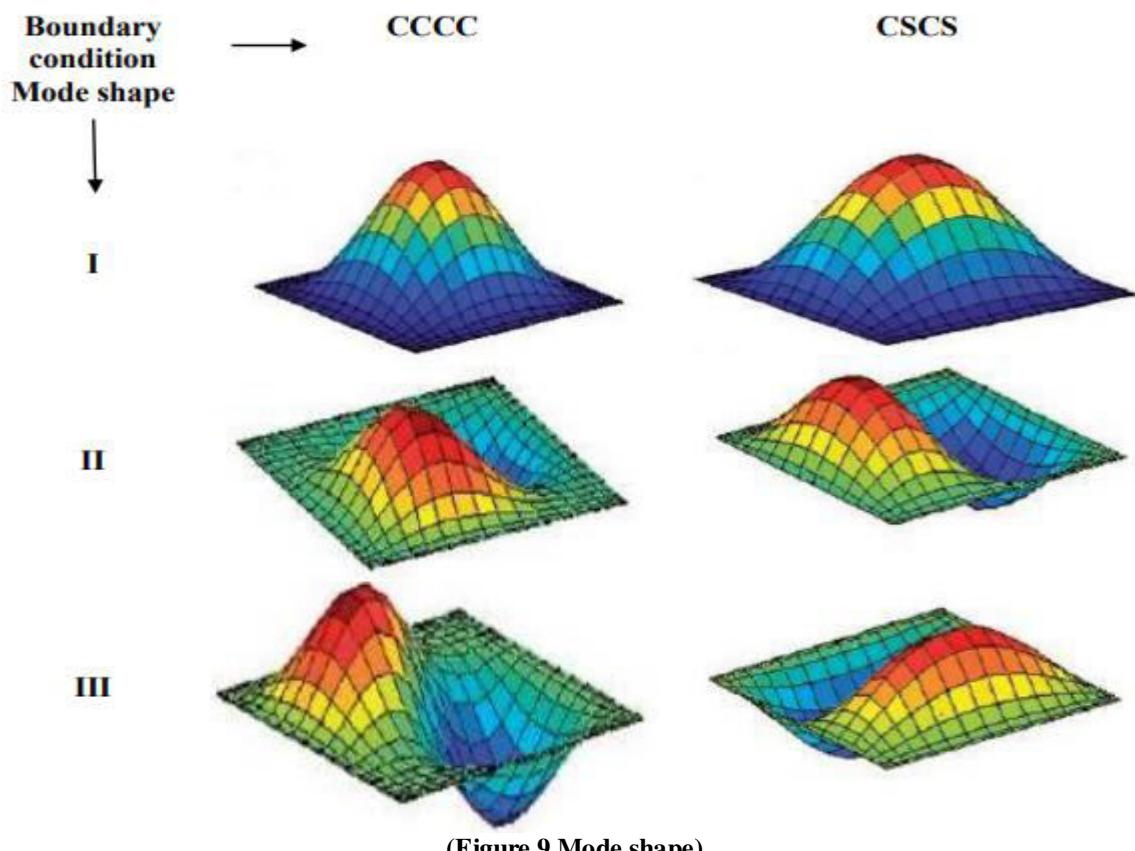
(Figure 6. Frequency parameters  $\Omega$  versus non-homogeneity parameters  $\beta_1$  for a CSCS square plate  $\alpha_1 = \alpha_2 = -0.5$ , for the first, second and third mode respectively)



(Figure 7 frequency parameters  $\Omega$  versus aspect ratio for CCCC plate)



(Figure 8 frequency parameters  $\Omega$  versus aspect ratio for CSCS plate)



(Figure 9 Mode shape)

#### IV. CONCLUSION

The variation of the non-homogeneity of the plate material on the vibration characteristics of the rectangular plates has been analysed using the FDM technique method. For illustration the numerical result for linear variation for the non – homogeneity have been obtained for the first three modes of the vibration for the two different boundary condition CCCC and CSCS.

The following observation has been made:

- (i) With the increase of non-homogeneity parameters of the plate, the non-dimensional frequency parameters increase when the density parameters of that plate material remain constant.
- (ii) When the Young's modulus parameters kept constant, non-dimensional frequency parameters decreases with increasing of the density parameters.
- (iii) For the both boundary condition non-dimensional frequency parameters increase with increase the value of aspects ratio  $\lambda$  when the other parameters of the plate remain constant.
- (iv) When there is no change in the other parameters of the plate the non- dimensional frequency parameters for the PVN is smaller as compared to LVN in nonhomogeneity, when the plate density higher toward the edge  $X = 0$  and  $Y = 0$  as compared to  $X = 1$  and  $Y = 1$ .When the change in order of density of the plate the results become the vice versa.
- (v) When there is no change in the density parameters the value of the non-dimensional frequency parameters is higher for PVN as compared to LVN because, the plate is more stiff towards the edges  $X = 0$  and  $Y = 0$  as compared to the  $X = 1$  and  $Y = 1$  . The results will be vice versa when the order of the stiffness changes from previous case.

#### ACKNOWLEDGMENTS

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