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Some Neutrosophic functions Associated with Neutrosophic Semi-Open Sets

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ABSTRACT :In this present paper, we scrutinize the view for neutrosophic semi-continuous function. Moreover, we study the notion of neutrosophic semi-irresolute functions, neutrosophic semi-open and neutrosophic semi-closed mappings and neutrosophic semi-homeomorphism. These functions are solved using neutrosophic semi-open and neutrosophic semi-closed sets. Further we analyze their properties and relations between them.

KEYWORDS :Neutrosophic semi-continuous, neutrosophic semi-irresolute, neutrosophic semi-open and neutrosophic semi-closed mappings and neutrosophic semi-homeomorphism.

AMS Mathematics Subject Classification :03E72

I. INTRODUCTION

In 1965, the theory of fuzzy set was introduced by Zadeh [10], which have only the degree of membership. Later in 1968, fuzzy topology was defined by Chang [2]. In 1983, Atanassov [1] generalized fuzzy sets as intuitionistic fuzzy sets, which have the degree of membership and degree of non-membership. Both degrees belong to $[0, 1]$ and their sum should not exceed 1. After, Coker [3] introduced intuitionistic fuzzy topological spaces using the concept of intuitionistic fuzzy sets in 1997. In 1999, Smarandache [4] introduced neutrosophic sets with degree of membership, degree of indeterminacy and degree of non-membership. These three degrees belong to $]0, 1^+[$ and their sum lies between 0 to 3^+ . Salama et al. [8] developed neutrosophic topology in 2012. Recently, Iswarya et al. [5,6] deliberated the notion of neutrosophic semi-open and neutrosophic semi-closed sets in 2016, also neutrosophic frontier, neutrosophic semi-frontier in 2017.

The field of the mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. Functions and continuous functions stand among the most important and most research points in the whole of the mathematical science. General topologists have introduced and investigated many different generalizations of continuous functions. Salama et al. [9] studied neutrosophic continuous functions in 2014. In 2018, Mani Parimala et al. [7] introduced and studied the concept of neutrosophic homeomorphism. Here we extend these concept using neutrosophic semi-open sets.

This paper contains five sections. Section 1 contains basic definitions that are used in subsequent sections. Section 2 deals with neutrosophic semi-continuous function. We analyze neutrosophic semi-irresolute function in section 3. In section 4, we talk about neutrosophic semi-open and neutrosophic semi-closed mappings with their characterizations. In fifth section, we describe the concepts of neutrosophic semi-homeomorphism and investigated its relation between neutrosophic semi-continuous functions. Furthermore, several related examples are given.

II. PRELIMINARIES

This part is about some of the vital definitions which are used for subsequent sections.

Definition 1.1. [9] Let (X, τ) and (Y, σ) be two neutrosophic topological spaces and $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic function. Then f is said to be neutrosophic continuous if the inverse image of each neutrosophic open set in (Y, σ) is a neutrosophic open set in (X, τ) .

Definition 1.2. [7] A bijection map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called neutrosophic homeomorphism if f and f^{-1} are neutrosophic continuous functions.

III. NEUTROSOPHIC SEMI-CONTINUOUS FUNCTION

In this section, we introduce the concept of neutrosophic semi-continuous function and some of their properties.

Definition 2.1. Let (X, τ) and (Y, σ) be two neutrosophic topological spaces and $f: (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic function. Then f is said to be a neutrosophic semi-continuous function if the inverse image of each neutrosophic open set in (Y, σ) is a neutrosophic semi-open set in (X, τ) .

Theorem 2.2. Every neutrosophic continuous function is a neutrosophic semi-continuous function. The converse part of above theorem is not true as shown by the following example.

Example 2.3. Let $X = Y = \{a, b, c\}$ with $\tau = \{0_N, A, 1_N\}$ and $\sigma = \{0_N, B, 1_N\}$ are two neutrosophic topological spaces on (X, τ) and (Y, σ) respectively, where $A = \langle (a, 0.3, 0.4, 0.5), (b, 0.5, 0.3, 0.6), (c, 0.8, 0.4, 0.8) \rangle$, $B = \langle (a, 0.3, 0.4, 0.5), (b, 0.8, 0.4, 0.8), (c, 0.5, 0.3, 0.6) \rangle$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = c, f(c) = b$. Clearly f is neutrosophic semi-continuous. Consider $C = \langle (a, 0.3, 0.2, 0.8), (b, 0.6, 0.4, 0.8), (c, 0.4, 0.2, 0.8) \rangle$ is neutrosophic open in (Y, σ) . But $f^{-1}(C) = \langle (a, 0.3, 0.2, 0.8), (b, 0.4, 0.2, 0.8), (c, 0.6, 0.4, 0.8) \rangle$ is not neutrosophic open in (X, τ) . Hence by Definition 1.1, f is not neutrosophic continuous.

Theorem 2.4. A neutrosophic function f is a neutrosophic semi-continuous function if and only if the inverse image of every neutrosophic closed in Y is neutrosophic semi-closed in X .

Proof. Let $f: X \rightarrow Y$ be a neutrosophic semi-continuous function and F be a neutrosophic closed set in Y . Then $C(F)$ is neutrosophic open in Y . Since f is neutrosophic semi-continuous function, $f^{-1}(C(F))$ is neutrosophic semi-open in X . Since $f^{-1}(C(F)) = C(f^{-1}(F))$, $f^{-1}(F)$ is neutrosophic semi-closed in X .

Conversely, assume that the inverse image of every neutrosophic closed in Y is neutrosophic semi-closed in X . Let G be a neutrosophic open set in Y . Then $C(G)$ is neutrosophic closed in Y . By our assumption, $f^{-1}(C(G))$ is neutrosophic semi-closed in X . Thus $f^{-1}(G)$ is neutrosophic semi-open in X . Hence f is a neutrosophic semi-continuous function.

Theorem 2.5. Let $f: X \rightarrow Y$ be a neutrosophic function. Then the following conditions are equivalent :

- (i) f is a neutrosophic semi-continuous function.
- (ii) $f^{-1}(NInt(B)) \subseteq NSInt(f^{-1}(B))$ for each neutrosophic set B in Y .

Proof. (i) \Rightarrow (ii) Let f be a neutrosophic semi-continuous function and B be a neutrosophic set in Y . Then $NInt(B)$ is neutrosophic open in Y . Since f is a neutrosophic semi-continuous function, $f^{-1}(NInt(B))$ is neutrosophic semi-open. By Proposition 3.1.2 (i) [5], $f^{-1}(NInt(B)) = NSInt(f^{-1}(NInt(B))) \subseteq NSInt(f^{-1}(B))$.

(ii) \Rightarrow (i) Let V be a neutrosophic open set in Y . Then by (ii), $f^{-1}(V) = f^{-1}(NInt(V)) \subseteq NSInt(f^{-1}(V))$. Since $NSInt(f^{-1}(V)) \subseteq f^{-1}(V)$, $NSInt(f^{-1}(V)) = f^{-1}(V)$. By Proposition 3.1.2 (i) [5], $f^{-1}(V)$ is neutrosophic semi-open. Therefore by Definition 2.1, f is a neutrosophic semi-continuous function.

Theorem 2.6. Let $f: X \rightarrow Y$ be a neutrosophic function. Then the following are equivalent :

- (i) f is a neutrosophic semi-continuous function.
- (ii) $NSCI(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for every neutrosophic set B in Y .
- (iii) $f(NSCI(A)) \subseteq NCl(f(A))$ for every neutrosophic set A in X .

Proof. (i) \Rightarrow (ii) Let B be a neutrosophic set in Y . Then $NCl(B)$ is neutrosophic closed in Y . Since f is neutrosophic semi-continuous, $f^{-1}(NCl(B))$ is neutrosophic semi-closed in X . Then $NSCI(f^{-1}(NCl(B))) = f^{-1}(NCl(B))$. Now, $NSCI(f^{-1}(B)) \subseteq NSCI(f^{-1}(NCl(B))) = f^{-1}(NCl(B))$.

(ii) \Rightarrow (iii) Let A be a neutrosophic set in X . Then $f(A)$ be a neutrosophic subset in Y . By our assumption, $NSCI(f^{-1}(f(A))) \subseteq f^{-1}(NCl(f(A)))$. Since $NSCI(A) \subseteq NSCI(f^{-1}(f(A)))$, $NSCI(A) \subseteq f^{-1}(NCl(f(A)))$. This implies that $f(NSCI(A)) \subseteq NCl(f(A))$.

(iii) \Rightarrow (i) Let F be a neutrosophic closed set in Y . Then $f^{-1}(F) = f^{-1}(NCl(F))$. By (iii), $f(NSCI(f^{-1}(F))) \subseteq NCl(f(f^{-1}(F))) \subseteq NCl(F) = F$. This implies that $NSCI(f^{-1}(F)) \subseteq f^{-1}(F)$. Since $f^{-1}(F) \subseteq NSCI(f^{-1}(F))$, $NSCI(f^{-1}(F)) = f^{-1}(F)$. Thus By Proposition 3.2.3 (i) [5], $f^{-1}(F)$ is neutrosophic semi-closed. Therefore f is a neutrosophic semi-continuous function.

Remark 2.7. The composition of two neutrosophic semi-continuous function need not be a neutrosophic semi-continuous function which is shown by the following example.

Example 2.8. Let $X = Y = Z = \{a, b\}$ and $\tau = \{0_N, A, B, 1_N\}$, $\sigma = \{0_N, C, D, 1_N\}$, $\eta = \{0_N, E, F, 1_N\}$, $C(\tau) = \{1_N, C(A), C(B), 0_N\}$, $C(\sigma) = \{1_N, C(C), C(D), 0_N\}$ and $C(\eta) = \{1_N, C(E), C(F), 0_N\}$ where $A = \langle (a, 0.5, 0.6, 0.4), (b, 0.8, 0.9, 0.5) \rangle$, $B = \langle (a, 0.3, 0.4, 0.5), (b, 0.4, 0.6, 0.6) \rangle$, $C = \langle (a, 0.4, 0.6, 0.6), (b, 0.3, 0.4, 0.5) \rangle$, $D = \langle (a, 0.5, 0.6, 0.3), (b, 0.4, 0.5, 0.2) \rangle$, $E = \langle (a, 0.4, 0.5, 0.2), (b, 0.5, 0.6, 0.3) \rangle$, $F = \langle (a, 0.3, 0.5, 0.6), (b, 0.4, 0.3, 0.4) \rangle$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$. Let C be a neutrosophic open set in (Y, σ) . Then $f^{-1}(C) = B$ where B is a neutrosophic semi-open set in (X, τ) . Thus f is a neutrosophic semi-continuous function. Define $g: (Y, \sigma) \rightarrow (Z, \eta)$ by $g(a) = b$, $g(b) = a$. Let E be a neutrosophic open set in (Z, η) . Then $g^{-1}(E) = D$ where D is a neutrosophic semi-open set in (Y, σ) . Thus g is neutrosophic semi-continuous. Define $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ by $(g \circ f)(a) = b$, $(g \circ f)(b) = a$. Let E be a neutrosophic open set in Z . Then $(g \circ f)^{-1}(E) = D$. But D is not a neutrosophic semi-open set in (X, τ) . Therefore $g \circ f$ is not a neutrosophic semi-continuous function. Hence f and g are neutrosophic semi-continuous functions but their composition is not a neutrosophic semi-continuous function.

Remark 2.9. Since the composition of two neutrosophic semi-continuous functions is not a neutrosophic semi-continuous function, the collection of neutrosophic semi-continuous functions does not form a group under composition operator.

Theorem 2.10. Let $f: X \rightarrow Y$ be the neutrosophic semi-continuous function. Then $\text{NSFr}(f^{-1}(A)) \subseteq f^{-1}(\text{NFr}(A))$ for any neutrosophic set A in Y .

Proof. Let $f: X \rightarrow Y$ be the neutrosophic semi-continuous function and A be the neutrosophic set in Y . Then $f^{-1}(\text{NCl}(A))$ is a neutrosophic semi-closed set in X . By Definition 3.4.1 [6], $\text{NSFr}(f^{-1}(A)) = \text{NSCl}(f^{-1}(A)) \cap \text{NSCl}(C(f^{-1}(A)))$. Also $\text{NSFr}(f^{-1}(A)) = \text{NSCl}(f^{-1}(A)) \cap \text{NSCl}(f^{-1}(C(A))) \subseteq \text{NSCl}(f^{-1}(\text{NCl}(A))) \cap \text{NSCl}(f^{-1}(\text{NCl}(C(A)))) = f^{-1}(\text{NCl}(A)) \cap f^{-1}(\text{NCl}(C(A))) = f^{-1}(\text{NCl}(A) \cap \text{NCl}(C(A)))$. Using Definition 3.3.2 [6], $\text{NSFr}(f^{-1}(A)) = f^{-1}(\text{NFr}(A))$. Hence $\text{NSFr}(f^{-1}(A)) \subseteq f^{-1}(\text{NFr}(A))$.

IV. NEUTROSOPHIC SEMI-IRRESOLUTE FUNCTION

In this segment, we investigate the concept of neutrosophic semi-irresolute function.

Definition 3.1. A neutrosophic function $f: X \rightarrow Y$ is called a neutrosophic semi-irresolute function if the inverse image of every neutrosophic semi-open set in Y is a neutrosophic semi-open set in X .

Theorem 3.2. A neutrosophic function f is neutrosophic semi-irresolute if and only if the inverse image of every neutrosophic semi-closed set in Y is neutrosophic semi-closed in X .

Proof. Suppose f is neutrosophic semi-irresolute. Let F be a neutrosophic semi-closed set in Y . Then $C(F)$ is a neutrosophic semi-open set in Y . Since f is neutrosophic semi-irresolute, $f^{-1}(C(F))$ is neutrosophic semi-open in X . Since $f^{-1}(C(F)) = C(f^{-1}(F))$, $f^{-1}(F)$ is neutrosophic semi-closed in X .

Conversely, suppose that the inverse image of every neutrosophic semi-closed set in Y is neutrosophic semi-closed in X . Let G be a neutrosophic semi-open set in Y . Then $C(G)$ is neutrosophic semi-closed in Y . By our assumption, $f^{-1}(C(G))$ is neutrosophic semi-closed in X . Thus $f^{-1}(G)$ is neutrosophic semi-open in X . Therefore f is neutrosophic semi-irresolute.

Theorem 3.3. Every neutrosophic semi-irresolute function is a neutrosophic semi-continuous function. The converse of the above theorem is not true as shown by the following example.

Example 3.4. Consider the Example 2.8. Thus f is a neutrosophic semi-continuous function. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$, $f(b) = b$. Let C be neutrosophic semi-open in (Y, σ) . Then $f^{-1}(C) = C$. But C is not a neutrosophic semi-open set in (X, τ) . Thus by Definition 3.1, f is not a neutrosophic semi-irresolute function.

Theorem 3.5. Let $f: X \rightarrow Y$ be a neutrosophic function. Then the following conditions are equivalent :

- (i) f is neutrosophic semi-irresolute.
- (ii) $\text{NSCl}(f^{-1}(B)) \subseteq f^{-1}(\text{NSCl}(B))$ for every neutrosophic set B in Y .



- (iii) $f(\text{NSCI}(A)) \subseteq \text{NSCI}(f(A))$ for every neutrosophic set A in X .
- (iv) $f^{-1}(\text{NSInt}(B)) \subseteq \text{NSInt}(f^{-1}(B))$ for every neutrosophic set B in Y .

Proof.(i) \Rightarrow (ii) Let B be a neutrosophic set in Y . Then $\text{NSCI}(B)$ is neutrosophic semi-closed in Y . Since f is neutrosophic semi-irresolute, $f^{-1}(\text{NSCI}(B))$ is neutrosophic semi-closed in X . Then $\text{NSCI}(f^{-1}(\text{NSCI}(B))) = f^{-1}(\text{NSCI}(B))$. Thus $\text{NSCI}(f^{-1}(B)) \subseteq \text{NSCI}(f^{-1}(\text{NSCI}(B))) = f^{-1}(\text{NSCI}(B))$.

(ii) \Rightarrow (iii) Let A be a neutrosophic set in X . Then $f(A)$ be a neutrosophic subset in Y . By our assumption, $\text{NSCI}(f^{-1}(f(A))) \subseteq f^{-1}(\text{NSCI}(f(A)))$. Since $\text{NSCI}(A) \subseteq \text{NSCI}(f^{-1}(f(A)))$, $\text{NSCI}(A) \subseteq f^{-1}(\text{NSCI}(f(A)))$. This implies that $f(\text{NSCI}(A)) \subseteq \text{NSCI}(f(A))$.

(iii) \Rightarrow (i) Let F be neutrosophic semi-closed in Y . Then $f^{-1}(F) = f^{-1}(\text{NSCI}(F))$. By (iii), $f(\text{NSCI}(f^{-1}(F))) \subseteq \text{NSCI}(f(f^{-1}(F))) \subseteq \text{NSCI}(F) = F$. Thus $(\text{NSCI}(f^{-1}(F))) \subseteq f^{-1}(F)$. Since $f^{-1}(F) \subseteq \text{NSCI}(f^{-1}(F))$, $\text{NSCI}(f^{-1}(F)) = f^{-1}(F)$. This implies that $f^{-1}(F)$ is neutrosophic semi-closed. Therefore f is neutrosophic semi-irresolute.

(i) \Rightarrow (iv) Let f be neutrosophic semi-irresolute and B be a neutrosophic set in Y . Then $\text{NSInt}(B)$ is neutrosophic semi-open in Y . Since f is neutrosophic semi-irresolute, $f^{-1}(\text{NSInt}(B))$ is neutrosophic semi-open in X . This implies that $f^{-1}(\text{NSInt}(B)) = \text{NSInt}(f^{-1}(\text{NSInt}(B))) \subseteq \text{NSInt}(f^{-1}(B))$.

(iv) \Rightarrow (i) Let V be neutrosophic semi-open in Y . Then by (iv), $f^{-1}(V) = f^{-1}(\text{NSInt}(V)) \subseteq \text{NSInt}(f^{-1}(V))$. Since $\text{NSInt}(f^{-1}(V)) \subseteq (f^{-1}(V))$, $\text{NSInt}(f^{-1}(V)) = f^{-1}(V)$. This implies that $f^{-1}(V)$ is neutrosophic semi-open. Thus f is neutrosophic semi-irresolute.

Theorem 3.6. If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are neutrosophic semi-irresolute, then their composition $g \circ f: X \rightarrow Z$ is also neutrosophic semi-irresolute. That is, the composition of two neutrosophic semi-irresolute is neutrosophic semi-irresolute.

Theorem 3.7. Let $\text{NSI}(X, \tau)$ be the set of all neutrosophic semi-irresolute functions defined on (X, τ) . Then $\text{NSI}(X, \tau)$ is a group under usual composition operator.

Proof. Define a binary operation $*$: $\text{NSI}(X, \tau) \times \text{NSI}(X, \tau) \rightarrow \text{NSI}(X, \tau)$ by $f * g = g \circ f$ for all $f, g \in \text{NSI}(X, \tau)$ and \circ is the usual operation of composition maps. Then by Theorem 3.6, $g \circ f \in \text{NSI}(X, \tau)$. Thus \circ is closed under composition. Since $*$ is associative, \circ is associative. Also there exists a identity function $I: \text{NSI}(X, \tau) \rightarrow \text{NSI}(X, \tau)$ such that $f \circ I = I \circ f = f$. This implies that \circ satisfies the identity function under composition. For all $f \in \text{NSI}(X, \tau)$, there exist $f^{-1} \in \text{NSI}(X, \tau)$ such that $f \circ f^{-1} = f^{-1} \circ f = I$. Thus \circ satisfies the inverse function under composition. Hence $(\text{NSI}(X, \tau), \circ)$ is a group under usual composition operator.

Theorem 3.8. Let $f: X \rightarrow Y$ be the neutrosophic semi-irresolute function and A be the neutrosophic set in Y , $\text{NSFr}(f^{-1}(A)) \subseteq f^{-1}(\text{NSFr}(A))$.

Proof. Let $f: X \rightarrow Y$ be the neutrosophic semi-irresolute function and A be the neutrosophic set in Y . Then by Definition 3.4.1 [6], $\text{NSFr}(f^{-1}(A)) = \text{NSCI}(f^{-1}(A)) \cap \text{NSCI}(C(f^{-1}(A)))$. Also $\text{NSFr}(f^{-1}(A)) = \text{NSCI}(f^{-1}(A)) \cap \text{NSCI}(f^{-1}(C(A))) \subseteq \text{NSCI}(f^{-1}(\text{NSCI}(A))) \cap \text{NSCI}(f^{-1}(\text{NSCI}(C(A)))) = f^{-1}(\text{NSCI}(A)) \cap f^{-1}(\text{NSCI}(C(A))) = f^{-1}(\text{NSCI}(A) \cap \text{NSCI}(C(A))) = f^{-1}(\text{NSFr}(A))$. Hence $\text{NSFr}(f^{-1}(A)) \subseteq f^{-1}(\text{NSFr}(A))$.

V. NEUTROSOPHIC SEMI-CLOSED MAP AND NEUTROSOPHIC SEMI-OPEN MAP

In this portion, we analyze the notions of neutrosophic semi-closed and neutrosophic semi-open mappings.

Definition 4.1. A neutrosophic function $f: X \rightarrow Y$ is said to be a neutrosophic semi-closed mapping if image of each neutrosophic closed set in X is a neutrosophic semi-closed set in Y .

Definition 4.2. A neutrosophic function $f: X \rightarrow Y$ is said to be a neutrosophic semi-open mapping if image of each neutrosophic open set in X is a neutrosophic semi-open set in Y .

Theorem 4.3. A neutrosophic function $f: X \rightarrow Y$ is a neutrosophic semi-closed mapping if and only if $\text{NSCI}(f(A)) \subseteq f(\text{NCl}(A))$ for every neutrosophic set A in X .

Proof. Let $f: X \rightarrow Y$ be a neutrosophic semi-closed mapping and A be a neutrosophic set in X . Then $\text{NCl}(A)$ is a neutrosophic closed set in X and $f(\text{NCl}(A))$ is a neutrosophic semi-closed set in Y . Therefore $\text{NSCI}(f(A)) = f(\text{NCl}(A))$. Thus $\text{NSCI}(f(A)) \subseteq \text{NSCI}(f(\text{NCl}(A))) = f(\text{NCl}(A))$. Hence $\text{NSCI}(f(A)) \subseteq f(\text{NCl}(A))$.



Conversely, let A be a neutrosophic closed set in X . Then $NCl(A) = A$. This implies that $f(A) = f(NCl(A))$. By our assumption, $NSCl(f(A)) \subseteq f(A)$. Since $f(A) \subseteq NSCl(f(A))$, $NSCl(f(A)) = f(A)$. Therefore $f(A)$ is neutrosophic semi-closed in Y . Thus f is a neutrosophic semi-closed mapping.

Theorem 4.4. A neutrosophic function $f : X \rightarrow Y$ is a neutrosophic semi-open mapping if and only if $f(NInt(A)) \subseteq NSInt(f(A))$ for every neutrosophic set A in X .

Proof. Let f be a neutrosophic semi-open mapping and A be a neutrosophic set in X . Then $NInt(A)$ is a neutrosophic open set in X . Since f is a neutrosophic semi-open mapping, $f(NInt(A))$ is a neutrosophic semi-open set. Since $NSInt(f(NInt(A))) \subseteq NSInt(f(A))$, $f(NInt(A)) \subseteq NSInt(f(A))$.

Conversely, assume that $f(NInt(A)) \subseteq NSInt(f(A))$ for every neutrosophic set A in X . Let U be a neutrosophic open set in X . Then $NInt(U) = U$ and by assumption, $f(U) \subseteq NSInt(f(U))$. Since $NSInt(f(U)) \subseteq f(U)$, $f(U) = NSInt(f(U))$. This implies that $f(U)$ is neutrosophic semi-open. Hence f is a neutrosophic semi-open mapping.

Theorem 4.5. A neutrosophic function $f : X \rightarrow Y$ is a neutrosophic semi-closed mapping if and only if for each neutrosophic set S in Y and for each neutrosophic open set U in X containing $f^{-1}(S)$, there exists a neutrosophic semi-open set V in Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof. Suppose that f is a neutrosophic semi-closed mapping. Let S be a neutrosophic set in Y and U be a neutrosophic open set in X such that $f^{-1}(S) \subseteq U$. Then $V = C(f(C(U)))$ is a neutrosophic semi-open set containing S such that $f^{-1}(V) \subseteq U$.

Conversely, let S be a neutrosophic closed set in X . Then $f^{-1}(C(f(S))) \subseteq C(S)$ and $C(S)$ is neutrosophic open in X . By our assumption, there exists a neutrosophic semi-open set V in Y such that $C(f(S)) \subseteq V$ and $f^{-1}(V) \subseteq C(S)$. Thus $S \subseteq C(f^{-1}(V))$. Hence $C(V) \subseteq f(S) \subseteq f(C(f^{-1}(V))) \subseteq C(V)$. This implies that $f(S) = C(V)$. Since $C(V)$ is neutrosophic semi-closed, $f(S)$ is neutrosophic semi-closed. Hence f is a neutrosophic semi-closed mapping.

Remark 4.6. The composition of two neutrosophic semi-closed mapping need not be a neutrosophic semi-closed mapping, which is shown by the following example.

Example 4.7. Let $X = Y = Z = \{a, b\}$ and $\tau = \{0_N, A, B, 1_N\}$, $\sigma = \{0_N, C, D, 1_N\}$, $\eta = \{0_N, E, F, 1_N\}$, $C(\tau) = \{1_N, C(A), C(B), 0_N\}$, $C(\sigma) = \{1_N, C(C), C(D), 0_N\}$ and $C(\eta) = \{1_N, C(E), C(F), 0_N\}$ where $A = \langle (a, 0.3, 0.6, 0.5), (b, 0.8, 1, 0.3) \rangle$, $B = \langle (a, 0.1, 0.5, 0.6), (b, 0.5, 0.8, 0.4) \rangle$, $C = \langle (a, 0.5, 0.8, 0.4), (b, 0.1, 0.5, 0.6) \rangle$, $D = \langle (a, 0.3, 0.6, 0.5), (b, 0.1, 0.5, 0.6) \rangle$, $E = \langle (a, 0.1, 0.5, 0.6), (b, 0.3, 0.6, 0.5) \rangle$, $F = \langle (a, 0.3, 0.5, 0.3), (b, 0.4, 0.7, 0.2) \rangle$, $C(A) = \langle (a, 0.5, 0.4, 0.3), (b, 0.3, 0, 0.8) \rangle$, $C(B) = \langle (a, 0.6, 0.5, 0.1), (b, 0.4, 0.2, 0.5) \rangle$, $C(C) = \langle (a, 0.4, 0.2, 0.5), (b, 0.6, 0.5, 0.1) \rangle$, $C(D) = \langle (a, 0.5, 0.4, 0.3), (b, 0.6, 0.5, 0.1) \rangle$, $C(E) = \langle (a, 0.6, 0.5, 0.1), (b, 0.5, 0.4, 0.3) \rangle$, $C(F) = \langle (a, 0.3, 0.5, 0.3), (b, 0.2, 0.3, 0.4) \rangle$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b, f(b) = a$. Let $C(B)$ be neutrosophic closed in (X, τ) . Then $f(C(B)) = C(C)$ where $C(C)$ is neutrosophic semi-closed in (Y, σ) . Thus f is neutrosophic semi-closed mapping. Define $g : (Y, \sigma) \rightarrow (Z, \eta)$ by $g(a) = b, g(b) = a$. Let $C(D)$ be a neutrosophic closed set in (Y, σ) . Then $g(C(D)) = C(E)$ where $C(E)$ is neutrosophic semi-closed in (Z, η) . Thus g is neutrosophic semi-closed. Define $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ by $(g \circ f)(a) = b, (g \circ f)(b) = a$. Let $C(B)$ be neutrosophic closed set in (X, τ) . Then $(g \circ f)(C(B)) = C(C)$. But $C(C)$ is not neutrosophic semi-open in (Z, η) . Therefore $g \circ f$ is not a neutrosophic semi-closed mapping. Hence f and g are neutrosophic semi-closed mappings but their composition is not a neutrosophic semi-closed mapping.

Theorem 4.8. Let $f : X \rightarrow Y$ be a neutrosophic semi-closed mapping. Then

- (i) If A is a neutrosophic closed set in X , then $f|_A : A \rightarrow Y$ is a neutrosophic semi-closed mapping.
- (ii) If $A = f^{-1}(B)$ for some neutrosophic closed set B in Y , then $f|_A : A \rightarrow Y$ is a neutrosophic semi-closed mapping.

Proof.(i) Let B be a neutrosophic closed set in A . Then $B = A \cap F$ for some neutrosophic closed set F in X . Thus B is neutrosophic closed in X . By hypothesis, $f(B)$ is neutrosophic semi-closed in Y . But $f(B) = f|_A(B)$. Hence $f|_A$ is a neutrosophic semi-closed mapping.

(ii) Let D be a neutrosophic closed set in A . Then $D = A \cap H$ for some neutrosophic closed set H in X . Now, $f|_A(D) = f(D) = f(A \cap H) = f(f^{-1}(B) \cap H) = B \cap f(H)$. Since f is neutrosophic semi-closed, $f(H)$ is neutrosophic semi-closed in Y . Hence $f|_A$ is a neutrosophic semi-closed mapping.



Theorem 4.9. Let $f: X \rightarrow Y$ be a bijective mapping. Then the following conditions are equivalent :

- (i) f is a neutrosophic semi-open mapping
- (ii) f is a neutrosophic semi-closed mapping
- (iii) f^{-1} is a neutrosophic semi-continuous function.

Proof.(i) \Rightarrow (ii) Let f be a neutrosophic semi-open mapping. Then the image of $f(V)$ is a neutrosophic semi-open set in Y for every neutrosophic open set V in X . Let V be a neutrosophic closed set in X . Then $C(V)$ is a neutrosophic open set in X . By hypothesis, $f(C(V))$ is a neutrosophic semi-open set in Y . Thus $C(f(C(V)))$ is a neutrosophic semi-closed set in Y . Hence f is neutrosophic semi-closed.

(ii) \Rightarrow (iii) Let V be a neutrosophic closed set in X . Then by hypothesis, $f(V)$ is a neutrosophic semi-closed set in Y . Since $f(V) = (f^{-1})^{-1}(V)$, f^{-1} is a neutrosophic semi-closed set in Y for every neutrosophic closed set in X . Hence f^{-1} is a neutrosophic semi-continuous function.

(iii) \Rightarrow (i) Let V be a neutrosophic open set in X . Then by hypothesis, $(f^{-1})^{-1}(V) = f(V)$ is a neutrosophic semi-open set in Y . Hence f is a neutrosophic semi-open mapping.

VI. NEUTROSOPHIC SEMI-HOMEOMORPHISM

In this section, we talk about neutrosophic semi-homeomorphism. Also we investigate their inter-relations with examples.

Definition 5.1. A bijection map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called neutrosophic semi-homeomorphism if f and f^{-1} are neutrosophic semi-continuous functions.

Proposition 5.2. Every neutrosophic homeomorphism is neutrosophic semi-homeomorphism.

The converse part of the above proposition is not true as shown by the following example.

Example 5.3. Let $X = Y = \{a, b, c\}$ and $\tau = \{0_N, A, B, 1_N\}$ and $\sigma = \{0_N, C, 1_N\}$ are two neutrosophic topological space on (X, τ) and (Y, σ) respectively, where $A = \langle (a, 0.4, 0.3, 0.5), (b, 0.3, 0.6, 0.8), (c, 0.1, 0.6, 0.5) \rangle$, $B = \langle (a, 0.2, 0.2, 0.6), (b, 0.1, 0.4, 0.8), (c, 0.1, 0.3, 0.7) \rangle$ and $C = \langle (a, 0.3, 0.6, 0.8), (b, 0.1, 0.6, 0.5), (c, 0.4, 0.3, 0.5) \rangle$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c, f(b) = a$ and $f(c) = b$. Let C be neutrosophic open set in Y . Then $f^{-1}(C) = A$ where A is neutrosophic semi-open set in X . Thus f is neutrosophic semi-continuous function. Define $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$ by $f^{-1}(a) = b, f^{-1}(b) = c$ and $f^{-1}(c) = a$. Let A be a neutrosophic open set in X . Then $f^{-1}(f^{-1}(A)) = C$ where C is a neutrosophic semi-open set in Y . Thus f^{-1} is a neutrosophic semi-continuous function. Hence f is neutrosophic semi-homeomorphism. Let C be a neutrosophic open set in Y . Then $f^{-1}(C) \neq B$ where B is a neutrosophic open set in X . Thus f is not a neutrosophic continuous function. Therefore by Definition 1.2, f is not neutrosophic homeomorphism. Hence f is neutrosophic semi-homeomorphism but not neutrosophic homeomorphism.

Theorem 5.4. Let $f: X \rightarrow Y$ be a bijective mapping and f be neutrosophic semi-continuous. Then the following conditions are equivalent :

- (i) f is a neutrosophic semi-closed mapping
- (ii) f is a neutrosophic semi-open mapping
- (iii) f is neutrosophic semi-homeomorphism.

Proof.(i) \Rightarrow (ii) Let f be a neutrosophic semi-closed mapping. Then by Theorem 4.9, $f^{-1}: Y \rightarrow X$ is a neutrosophic semi-continuous function. Since every neutrosophic open set is a neutrosophic semi-open set, f is a neutrosophic semi-open mapping.

(ii) \Rightarrow (iii) Let f be a neutrosophic semi-open mapping. Then $f^{-1}: Y \rightarrow X$ is a neutrosophic semi-continuous function. Since f is a neutrosophic semi-continuous function, f is neutrosophic semi-homeomorphism.

(iii) \Rightarrow (i) Let f be neutrosophic semi-homeomorphism. Then f and f^{-1} are neutrosophic semi-continuous functions. Since every neutrosophic closed set in X is neutrosophic semi-closed in Y , f is a neutrosophic semi-closed mapping.

Remark 5.5. The composition of two neutrosophic semi-homeomorphism need not be neutrosophic semi-homeomorphism in general. This can be shown by the following example.

Example 5.6. Let $X = Y = Z = \{a, b\}$ and $\tau = \{0_N, A, B, 1_N\}$, $\sigma = \{0_N, C, D, 1_N\}$, $\eta = \{0_N, E, F, 1_N\}$, $C(\tau) = \{1_N, C(A), C(B), 0_N\}$, $C(\sigma) = \{1_N, C(C), C(D), 0_N\}$ and $C(\eta) = \{1_N, C(E), C(F), 0_N\}$ where $A = \langle (a, 0.3, 0.5, 0.2), (b, 0.3, 0.4, 0.1) \rangle$, $B = \langle (a, 0.1, 0.4, 0.5), (b, 0.2, 0.3, 0.6) \rangle$, $C = \langle (a, 0.3, 0.4, 0.1), (b, 0.3, 0.5, 0.2) \rangle$, $D = \langle (a, 0.1, 0.4, 0.5), (b, 0.2, 0.3, 0.6) \rangle$,

$E = \langle (a, 0.5, 0.6, 0.1), (b, 0.7, 0.4, 0.2) \rangle$, $F = \langle (a, 0.2, 0.3, 0.6), (b, 0.1, 0.4, 0.5) \rangle$,
 $C(A) = \langle (a, 0.2, 0.5, 0.3), (b, 0.1, 0.6, 0.3) \rangle$, $C(B) = \langle (a, 0.5, 0.6, 0.1), (b, 0.6, 0.7, 0.2) \rangle$,
 $C(C) = \langle (a, 0.1, 0.6, 0.3), (b, 0.2, 0.5, 0.3) \rangle$, $C(D) = \langle (a, 0.5, 0.6, 0.1), (b, 0.6, 0.7, 0.2) \rangle$,
 $C(E) = \langle (a, 0.1, 0.4, 0.5), (b, 0.2, 0.6, 0.7) \rangle$, $C(F) = \langle (a, 0.6, 0.7, 0.2), (b, 0.5, 0.6, 0.1) \rangle$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b, f(b) = a$. Let C be neutrosophic open in (Y, σ) . Then $f^{-1}(C) = A$ where A is neutrosophic semi-open in (X, τ) . Thus f is neutrosophic semi-continuous. Define $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$ by $f^{-1}(a) = b, f^{-1}(b) = a$. Let A be a neutrosophic open set in X . Then $f^{-1}(f^{-1}(A)) = C$ where C is a neutrosophic semi-open set in Y . Thus f^{-1} is a neutrosophic semi-continuous function. Hence f is neutrosophic semi-homeomorphism. Define $g: (Y, \sigma) \rightarrow (Z, \eta)$ by $g(a) = b, g(b) = a$. Let F be a neutrosophic open set in (Z, η) . Then $g^{-1}(F) = D$ where D is a neutrosophic semi-open set in (Y, σ) . Thus g is neutrosophic semi-continuous. Define $g^{-1}: (Z, \eta) \rightarrow (Y, \sigma)$ by $g^{-1}(a) = b, g^{-1}(b) = a$. Let D be a neutrosophic open set in Y . Then $g^{-1}(g^{-1}(D)) = F$ where F is neutrosophic semi-open in (Z, η) . Thus g^{-1} is neutrosophic semi-continuous. Hence g is neutrosophic semi-homeomorphism. Define $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ by $(g \circ f)(a) = b, (g \circ f)(b) = a$. Let E be neutrosophic open in (Z, η) . Then $(g \circ f)^{-1}(E) = \langle (a, 0.7, 0.4, 0.2), (b, 0.5, 0.6, 0.1) \rangle$ is not neutrosophic semi-open in X . [Let $\langle (a, 0.7, 0.4, 0.2), (b, 0.5, 0.6, 0.1) \rangle = G$. Then $NInt(G) = B, NCl(NInt(G)) = C(A)$. Thus $G \not\subseteq C(A)$]. Therefore $g \circ f$ is not a neutrosophic semi-continuous function. Thus $g \circ f$ is not neutrosophic semi-homeomorphism. Hence f and g are neutrosophic semi-homeomorphism but their composition is not neutrosophic semi-homeomorphism.

Remark 5.7. Since the composition of two neutrosophic semi-homeomorphisms is not a neutrosophic semi-homeomorphism, the collection of neutrosophic semi-homeomorphism does not form a group under composition operator.

Theorem 5.8. Let $f: X \rightarrow Y$ be a bijective mapping. Then the following conditions are equivalent :

- (i) f is neutrosophic semi-homeomorphism
- (ii) f is neutrosophic semi-continuous and neutrosophic semi-open mapping
- (iii) f and f^{-1} are neutrosophic semi-continuous functions.

Proof. (i) \Rightarrow (ii) Let f be neutrosophic semi-homeomorphism. Then f and f^{-1} are neutrosophic semi-continuous functions. Since every neutrosophic open set in X is a neutrosophic semi-open set in Y , f is a neutrosophic semi-open mapping.

(ii) \Rightarrow (iii) Let f be a neutrosophic semi-open mapping. Then $f(V)$ is a neutrosophic semi-open set in Y for every neutrosophic open set in X . Let $f^{-1}: Y \rightarrow X$. Then by hypothesis, $f^{-1}(V)$ is neutrosophic semi-open in Y for every neutrosophic open in X . Therefore f^{-1} is a neutrosophic semi-continuous function. Hence f and f^{-1} are neutrosophic semi-continuous functions.

(iii) \Rightarrow (i) Let f and f^{-1} be neutrosophic semi-continuous functions. Then by hypothesis, f be a bijective mapping. Hence f is neutrosophic semi-homeomorphism.

Theorem 5.9. Let $f: X \rightarrow Y$ be a bijective mapping. Then the following are equivalent :

- (i) f is neutrosophic semi-continuous and neutrosophic semi-open mapping,
- (ii) f is neutrosophic semi-continuous and neutrosophic semi-closed mapping,
- (iii) f is neutrosophic semi-homeomorphism.

Proof. (i) \Rightarrow (ii) Let f be a neutrosophic semi-continuous function and neutrosophic semi-open mapping. Then by Theorem 4.9, f is a neutrosophic semi-closed mapping.

(ii) \Rightarrow (iii) Let f be a semi-continuous function and neutrosophic semi-open mapping. Again by Theorem 4.9, f^{-1} is neutrosophic semi-continuous function. Hence f is neutrosophic semi-homeomorphism.

(iii) \Rightarrow (i) Let f be neutrosophic semi-homeomorphism. Then f and f^{-1} are bijective mapping and neutrosophic semi-continuous functions. Hence by Theorem 4.9, f is a neutrosophic semi-open mapping.

Theorem 5.10. Let $f: X \rightarrow Y$ be neutrosophic semi-homeomorphism. Then $NSCl(f^{-1}(V)) \subseteq f^{-1}(NCl(V))$ for every neutrosophic set V in Y .

Proof. Let V be a neutrosophic set in Y . Then $NCl(V)$ is neutrosophic closed in Y . Since f is neutrosophic semi-homeomorphism, f is a neutrosophic semi-continuous function. Thus by Theorem 2.4, $f^{-1}(NCl(V))$ is neutrosophic semi-closed in X . Using Proposition 3.2.3 (i) [5], $NSCl(f^{-1}(NCl(V))) = f^{-1}(NCl(V))$. Since $V \subseteq NCl(V)$, $NSCl(f^{-1}(V)) \subseteq NSCl(f^{-1}(NCl(V)))$. This implies that $NSCl(f^{-1}(V)) \subseteq f^{-1}(NCl(V))$.

Theorem 5.11. Let $f: X \rightarrow Y$ be neutrosophic semi-homeomorphism. Then $f^{-1}(NInt(V)) \subseteq NSInt(f^{-1}(V))$ for every neutrosophic set V in Y .



Proof. Let V be a neutrosophic set in Y . Then $NInt(V)$ is neutrosophic open in Y . Since f is neutrosophic semi-homeomorphism, f is a neutrosophic semi-continuous function. Thus by Definition 2.1, $f^{-1}(NInt(V))$ is neutrosophic semi-open in X . Using Proposition 3.1.2 (i) [5], $NSInt(f^{-1}(NInt(V))) = f^{-1}(NInt(V))$. Since $NInt(V) \subseteq V$, $NSInt(f^{-1}(NInt(V))) \subseteq NSInt(f^{-1}(V))$. This implies that $f^{-1}(NInt(V)) \subseteq NSInt(f^{-1}(V))$.

VII. CONCLUSION

In this paper, we introduced neutrosophic semi-continuous function, neutrosophic semi-irresolute function, neutrosophic semi-open and neutrosophic semi-closed mappings. Furthermore, we investigated their properties and relations between them. Also we defined neutrosophic semi-homeomorphism in neutrosophic topological spaces and we analyzed their inter-relations with examples.

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