



e-ISSN: 2319-8753 | p-ISSN: 2347-6710

# IJRSET

International Journal of Innovative Research in  
**SCIENCE | ENGINEERING | TECHNOLOGY**

# INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 11, Issue 6, June 2022

**ISSN** INTERNATIONAL  
STANDARD  
SERIAL  
NUMBER  
INDIA

Impact Factor: 8.118

9940 572 462

6381 907 438

[ijirset@gmail.com](mailto:ijirset@gmail.com)

[www.ijirset.com](http://www.ijirset.com)



# ON SECOND DEGREE EQUATION WITH THREE UNKNOWNNS

$$2(x^2 + y^2) - 3xy = 28z^2$$

S.Vidhyalakshmi<sup>1</sup>, M.A.Gopalan<sup>2</sup>

Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy, Tamil Nadu, India. <sup>1</sup>

Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy, Tamil Nadu, India. <sup>2</sup>

**ABSTRACT:** The cone represented by the ternary quadratic Diophantine equation  $2(x^2 + y^2) - 3xy = 28z^2$  is analyzed for its patterns of non-zero distinct integral solutions.

**KEYWORDS:** Ternary quadratic, cone, integral solutions

## I. INTRODUCTION

The Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-15] for quadratic equations with three unknowns. This communication concerns with yet another interesting equation  $2(x^2 + y^2) - 3xy = 28z^2$  representing homogeneous quadratic with three unknowns for determining its infinitely many non-zero integral points.

## II. METHOD OF ANALYSIS

The ternary quadratic diophantine equation to be solved for its non-zero distinct integral solution is

$$2(x^2 + y^2) - 3xy = 28z^2 \tag{1}$$

Introduction of the linear transformations ( $u \neq v \neq 0$ )

$$x = u + v, y = u - v \tag{2}$$

in (1) leads to

$$u^2 + 7v^2 = 28z^2 \tag{3}$$

We present below different methods of solving (1).

### Method I:

We can write 28 as

$$28 = (i2\sqrt{7})(-i2\sqrt{7}) \tag{4}$$

$$\text{Assume } z = z(a, b) = a^2 + 7b^2 \tag{5}$$

where a and b are non-zero distinct integers.

Using (4) and (5) in (3), we get

$$u^2 + 7v^2 = (i2\sqrt{7})(-i2\sqrt{7})(a + i\sqrt{7}b)^2(a - i\sqrt{7}b)^2$$

Equating the positive and negative factors, the resulting equating are

$$(u + i\sqrt{7}v) = (i2\sqrt{7})(a + i\sqrt{7}b)^2 \tag{6}$$



$$(u - i\sqrt{7}v) = (-i2\sqrt{7})(a - i\sqrt{7}b)^2 \tag{7}$$

Equating the real and imaginary parts either in (6) or (7), we get

$$u = -28ab$$

$$v = 2(a^2 - 7b^2)$$

Substituting the values of u and v in (2) we get,

$$x = x(a,b) = 2a^2 - 14b^2 - 28ab \tag{8}$$

$$y = y(a,b) = -2a^2 + 14b^2 - 28ab \tag{9}$$

which satisfy (1) along with (5)

**NOTE:1**

It is seen that 28 on the R.H.S. of (3) is also represented as follows:

$$28 = \frac{(7 + i3\sqrt{7})(7 - i3\sqrt{7})}{4},$$

$$28 = \frac{(21 + i\sqrt{7})(21 - i\sqrt{7})}{16}$$

Following the above procedure, one obtains two more sets of integer solutions to (1).

**Method II:**

Taking

$$v = 2z - h \tag{10}$$

in (3), the resulting equation is

$$7h^2 - 28hz + u^2 = 0 \tag{11}$$

Treating (11) as a quadratic in h and solving for h, one obtains

$$h = \frac{28z + \sqrt{28z^2 - 28u^2}}{14} \tag{12}$$

$$\left. \begin{aligned} u &= 56rs, h = 4r^2 \\ z &= z(r,s) = r^2 + 28s^2 \end{aligned} \right\} \tag{13}$$

Substituting the values of z and h in (10), the value of v is given by

$$v = 56s^2 - 2r^2 \tag{14}$$

In view of (2), the values of x and y are given by

$$\left. \begin{aligned} x &= x(r,s) = 56rs + 56s^2 - 2r^2, \\ y &= y(r,s) = 56rs - 56s^2 + 2r^2 \end{aligned} \right\} \tag{15}$$

The values of x,y z given by (13) and (15) satisfy (1).

**METHOD III:**

Equation (3) is written as

$$u^2 + 7v^2 = 28z^2 * 1 \tag{16}$$

Write 1 as

$$1 = \frac{(3 + i\sqrt{7})(3 - i\sqrt{7})}{16} \tag{17}$$

Using (4), (5) and (17) in (16) we get,



$$(u + i\sqrt{7}v)(u - i\sqrt{7}v) = \left( \frac{(a + i\sqrt{7}b)^2(a - i\sqrt{7}b)^2(i2\sqrt{7})(-i2\sqrt{7})}{(3 + i\sqrt{7})(3 - i\sqrt{7})} \right) \frac{1}{16}$$

Equating the positive and negative factors, the resulting equations are,

$$(u + i\sqrt{7}v) = (i2\sqrt{7}) \frac{(3 + i\sqrt{7})}{4} (a + i\sqrt{7}b)^2 \tag{18}$$

$$(u - i\sqrt{7}v) = (-i2\sqrt{7}) \frac{(3 - i\sqrt{7})}{4} (a - i\sqrt{7}b)^2 \tag{19}$$

Equating real and imaginary parts in (18), we get

$$u = \frac{-7a^2 + 49b^2 - 42ab}{2},$$

$$v = \frac{3a^2 - 21b^2 - 14ab}{2}$$

Substituting the values of u and v in (2), we get

$$\begin{aligned} x &= -2a^2 + 14b^2 - 28ab, \\ y &= -5a^2 + 35b^2 - 14ab \end{aligned} \tag{20}$$

which along with (5) represent the solutions to (1).

Note: 2

It is seen that 1 on the R.H.S. of (16) is also represented as follows:

$$1 = \frac{(1 + i3\sqrt{7})(1 - i3\sqrt{7})}{64},$$

$$1 = \frac{(3 + i4\sqrt{7})(3 - i4\sqrt{7})}{121},$$

$$1 = \frac{(7r^2 - s^2 + i2rs\sqrt{7})(7r^2 - s^2 - i2rs\sqrt{7})}{(7r^2 + s^2)^2}$$

Following the above procedure, three more sets of integer solutions to (1) are obtained.

**METHOD IV:**

Equation (3) is written as

$$28z^2 - 7v^2 = u^2 * 1 \tag{21}$$

Assume

$$u = 28a^2 - 7b^2 \tag{22}$$

Write 1 as

$$1 = \frac{(2\sqrt{28} + 3\sqrt{7})(2\sqrt{28} - 3\sqrt{7})}{49} \tag{23}$$

Following the analysis as in **METHOD III**, the corresponding integer solutions

to (1) are given by

$$x = x(a, b) = 40a^2 - 4b^2 + 16ab,$$

$$y = y(a, b) = 16a^2 - 10b^2 - 16ab.$$

$$z = z(a, b) = 8a^2 + 2b^2 + 6ab$$

Note: 3

It is seen that 1 on the R.H.S. of (21) is also given by



$$1 = \frac{(4\sqrt{28} + \sqrt{7})(4\sqrt{28} - \sqrt{7})}{441}$$

Following the above procedure, another set of solutions to (1) is obtained.

### III. CONCLUSION

In this paper, we have made an attempt to obtain all integer solutions to (1). As (1) is symmetric in  $x, y, z$ , it is to be noted that, if  $(x, y, z)$  is any positive integer solution to (1), then the triples  $(x, y, -z)$ ,  $(-x, -y, z)$ ,  $(-x, -y, -z)$  also satisfy (1). To conclude, one may search for integer solutions to other choices of homogeneous cones.

### REFERENCES

- [1] L.E. Dickson, History of Theory of Numbers, vol 2, Chelsea publishing company, New York, (1952).
- [2] L.J. Mordell, Diophantine Equations, Academic press, London, (1969).
- [3] R.D. Carmichael, The theory of numbers and Diophantine analysis, New York, Dover, (1959).
- [4] N. Bharathi, S. Vidhyalakshmi, Observation on the Non-Homogeneous Ternary Quadratic Equation  $x^3 - xy + y^3 + 2(x + y) + 4 = 12z^2$ , Journal of mathematics and informatics, vol.10, 2017, 135-140.
- [5] A. Priya, S. Vidhyalakshmi, On the Non-Homogeneous Ternary Quadratic Equation  $2(x^2 + y^2) - 3xy + (x + y) + 1 = Z^2$ , Journal of mathematics and informatics, vol.10, 2017, 49-55.
- [6] M.A. Gopalan, S. Vidhyalakshmi and U.K. Rajalakshmi, On ternary quadratic Diophantine equation  $5(X^2 + Y^2) - 6XY = 196Z^2$ , Journal of mathematics, 3(5) (2017) 1-10.
- [7] M.A. Gopalan, S. Vidhyalakshmi and S. Aarthi Thangam, On ternary quadratic equation  $X(X + Y) = Z + 20$  IJIRSET,6(8) (2017) 15739-15741.
- [8] M.A. Gopalan and Sharadha Kumar, "On the Hyperbola  $2x^2 - 3y^2 = 23$ ", Journal of Mathematics and Informatics, vol-10, Dec(2017), 1-9.
- [9] T.R. Usha Rani and K.Ambika, Observation on the Non-Homogeneous Binary Quadratic Diophantine Equation  $5x^2 - 6y^2 = 5$ , Journal of Mathematics and Informatics, vol-10, Dec (2017), 67-74.
- [10] S. Vidhyalakshmi, A. Sathya, S. Nivetha, "On the pellian like Equation  $5x^2 - 7y^2 = -8$ ", IRJET, volume: 06 Issue: 03, 2019, 979-984.
- [11] T.R. Usha Rani, V. Bahavathi, S. Sridevi, "Observations on the Non-homogeneous binary Quadratic Equation  $8x^2 - 3y^2 = 20$ ", IRJET, volume: 06, Issue: 03, 2019, 2375-2382.
- [12] S. Vidhyalakshmi et al, On Finding Integer Solutions To The Ternary Quadratic Diophantine Equation  $2(x^2 + y^2) - 3xy = 43z^2$ , IJCAR, Vol 10, No 11, 25561-25564, 2021
- [13] S. Vidhyalakshmi and M.A. Gopalan, On Second Degree Equation With Three Unknowns  $2(x^2 + y^2) - 3xy = 32z^2$ , IJRPR, Vol 3, No 6, 1076-1079, 2022
- [14] S. Vidhyalakshmi and M.A. Gopalan, Integral Solutions Of Ternary Quadratic Diophantine Equation  $3(x^2 + y^2) - 5xy = 20z^2$ , IJAREM, Vol 8, No 6, 1-5, 2022
- [15] S. Vidhyalakshmi and M.A. Gopalan, On Second Degree Equation With Three Unknowns  $2(x^2 + y^2) - 3xy = 53z^2$ , IRJMETS, Vol 4, No 6, 1810-1813, 2022



**INNO SPACE**  
SJIF Scientific Journal Impact Factor  
**Impact Factor: 8.118**



**ISSN** INTERNATIONAL  
STANDARD  
SERIAL  
NUMBER  
INDIA



# INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

 **9940 572 462**  **6381 907 438**  **ijirset@gmail.com**



[www.ijirset.com](http://www.ijirset.com)

Scan to save the contact details