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# The Starter of Einstein's Cosmological Constant $\Lambda$ to the Theory of General Relativity

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**Abstract:** Einstein first proposed the cosmological constant, usually symbolized by the Greek letter lambda ( $\Lambda$ ), is a mathematical solution to the Theory of General Relativity. General Relativity predicted that the universe must either expand or contract. In cosmology, Einstein's cosmological constant, is the constant coefficient of a term that Albert Einstein temporarily added to his field equations of general relativity. He later removed it. Much later it was re-energized and reinterpreted as the energy density of space, or vacuum energy, that arises in quantum mechanics. It is closely associated with the concept of dark energy.

## I. INTRODUCTION

Einstein included the cosmological constant as a term in his field equations for general relativity because he was dissatisfied that otherwise his equations did not allow for a static universe: gravity would cause a universe that was initially non-expanding to contract. To counteract this possibility, Einstein added the cosmological constant. However, soon after Einstein developed his static theory, observations by Edwin Hubble indicated that the universe appears to be expanding; this was consistent with a cosmological solution to the original general relativity equations that had been found by the mathematician Friedmann, working on the Einstein equations of general relativity. Einstein reportedly referred to his failure to accept the validation of his equations—when they had predicted the expansion of the universe in theory, before it was demonstrated in observation of the cosmological redshift—as his biggest blunder.

It emerged that adding the cosmological constant to Einstein's equations does not lead to a static universe at equilibrium because the equilibrium is unstable: if the universe expands slightly, then the expansion releases vacuum energy, which causes yet more expansion. Likewise, a universe that contracts slightly will continue contracting.

However, the cosmological constant remained a subject of theoretical and empirical interest. Empirically, the cosmological data of recent decades strongly suggests that our universe has a positive cosmological constant. The explanation of this small but positive value is a remaining theoretical challenge, the so-called cosmological constant problem.

## II. REVIEW OF RELATED LITERATURE

### 2.1 Sequence of Investigations done during 1915–1998

- In 1915, Einstein publishes his equations of general relativity, without a cosmological constant  $\Lambda$ .
- In 1917, Einstein adds the parameter  $\Lambda$  to his equations when he realizes that his theory implies a dynamic universe for which space is function of time. He then gives this constant a value that makes his Universe model remain static and eternal (Einstein static universe).
- In 1922, the Russian physicist Alexander Friedmann mathematically shows that Einstein's equations (whatever  $\Lambda$ ) remain valid in a dynamic universe.



- In 1927, the Belgian astrophysicist Georges Lemaître shows that the Universe is expanding by combining general relativity with astronomical observations, those of Hubble in particular.
- In 1931, Einstein accepts the theory of an expanding universe and proposes, in 1932 with the Dutch physicist and astronomer Willem de Sitter, a model of a continuously expanding Universe with zero cosmological constant (Einstein–de Sitter spacetime).
- In 1998, two teams of astrophysicists, one led by Saul Perlmutter, the other led by Brian Schmidt and Adam Riess, carried out measurements on distant supernovae which showed that the speed of galaxies' recession in relation to the Milky Way increases over time. The universe is in accelerated expansion, which requires having a strictly positive  $\Lambda$ . The universe would contain a mysterious dark energy producing a repulsive force that counterbalances the gravitational braking produced by the matter contained in the universe.

For this work, Perlmutter, Schmidt, and Riess jointly received the Nobel Prize in physics in 2011.

### III. EINSTEIN'S FIELD EQUATION

The cosmological constant  $\Lambda$  appears in the Einstein field equations in the form

$$R_{\mu\theta} - \frac{1}{2}Rg_{\mu\theta} + \Lambda g_{\mu\theta} = \mathcal{K}T_{\mu\theta}$$

where the Ricci tensor  $R_{\mu\nu}$ , Ricci scalar  $R$  and the metric tensor  $g_{\mu\nu}$  describe the structure of spacetime, the stress–energy tensor  $T_{\mu\nu}$  describes the energy density, momentum density and stress at that point in spacetime, and  $\kappa = 8\pi G/c^4$ . The gravitational constant  $G$  and the speed of light  $c$  are universal constants. When  $\Lambda$  is zero, this reduces to the field equation of general relativity usually used in the 20th century. When  $T_{\mu\nu}$  is zero, the field equation describes empty space (a vacuum).

The cosmological constant has the same effect as an intrinsic energy density of the vacuum,  $\rho_{\text{vac}}$  (and an associated pressure). In this context, it is commonly moved to the right-hand side of the equation using  $\Lambda = \kappa\rho_{\text{vac}}$ . It is common to quote values of energy density directly, though still using the name "cosmological constant". The dimension of  $\Lambda$  is generally understood as  $\text{length}^{-2}$ .

Using the values known in 2018 and Planck units for

$\Omega_\Lambda = 0.6889 \pm 0.0056$  and the Hubble constant

$H_0 = 67.66 \pm 0.42 \text{ (km/s)/Mpc} = (2.1927664 \pm 0.0136) \times 10^{-18} \text{ s}^{-1}$ ,

$\Lambda$  has the value of

$$\Lambda = 3 \left( \frac{H_0}{c} \right)^2 \Omega_\Lambda = 1.1056 \times 10^{-52} \text{ m}^{-2} = 2.888 \times 10^{-122} \text{ l}_p^{-2}$$

where  $l_p$  is the Planck length. A positive vacuum energy density resulting from a cosmological constant implies a negative pressure, and vice versa. If the energy density is positive, the associated negative pressure will drive an accelerated expansion of the universe, as observed.

#### 3.1 $\Omega_\Lambda$ (Omega sub-Lambda)

Instead of the cosmological constant itself, cosmologists often refer to the ratio between the energy density due to the cosmological constant and the critical density of the universe, the tipping point for a sufficient density to stop the universe from expanding forever. This ratio is usually denoted by  $\Omega_\Lambda$  and is estimated to be  $0.6889 \pm 0.0056$ , according to results published by the Planck Collaboration in 2018.

In a flat universe,  $\Omega_\Lambda$  is the fraction of the energy of the universe due to the cosmological constant, i.e., what we would intuitively call the fraction of the universe that is made up of dark energy. Note that this value changes over time: The critical density changes with cosmological time but the energy density due to the cosmological constant remains unchanged throughout the history of the universe, because the amount of dark energy increases as the universe grows but the amount of matter does not.

### 3.2 Equation of state

Another ratio that is used by scientists is the equation of state, usually denoted  $w$ , which is the ratio of pressure that dark energy puts on the universe to the energy per unit volume.<sup>[15]</sup> This ratio is  $w = -1$  for the cosmological constant used in the Einstein equations; alternative time-varying forms of vacuum energy such as quintessence generally use a different value. The value  $w = -1.028 \pm 0.032$ , measured by the Planck Collaboration (2018)<sup>[14]</sup> is consistent with  $-1$ , assuming  $w$  does not change over cosmic time.

### 3.3 Positive value

Observations announced in 1998 of distance–redshift relation for Type Ia supernovae indicated that the expansion of the universe is accelerating, if one assumes the cosmological principle. When combined with measurements of the cosmic microwave background radiation these implied a value of  $\Omega_\Lambda \approx 0.7$ , a result which has been supported and refined by more recent measurements. If one assumes the cosmological principle, as in the case for all models that use the Friedmann–Lemaître–Robertson–Walker metric, while there are other possible causes of an accelerating universe, such as quintessence, the cosmological constant is in most respects the simplest solution. Thus, the Lambda-CDM model, the current standard model of cosmology which uses the FLRW metric, includes the cosmological constant, which is measured to be on the order of  $10^{-52} \text{ m}^{-2}$ . It may be expressed as  $10^{-35} \text{ s}^{-2}$  (by multiplication with  $c^2$ , i.e.  $\approx 10^{17} \text{ m} \cdot \text{s}^{-2}$ ) or as  $10^{-122} \ell_p^{-2}$  (where  $\ell_p$  is the Planck length). The value is based on recent measurements of vacuum energy density,  $\rho_{\text{vac}} = 5.96 \times 10^{-27} \text{ kg/m}^3 \cong 5.3566 \times 10^{-10} \text{ J/m}^3 = 3.35 \text{ GeV/m}^3$ . However, due to the Hubble tension and the CMB dipole, recently it has been proposed that the cosmological principle is no longer true in the late universe and that the FLRW metric breaks down, so it is possible that observations usually attributed to an accelerating universe are simply a result of the cosmological principle not applying in the late universe.

As was only recently seen, by works of 't Hooft, Susskind and others, a positive cosmological constant has surprising consequences, such as a finite maximum entropy of the observable universe (see Holographic principle).

## IV. DISCUSSION

### Quantum field theory

A major outstanding problem is that most quantum field theories predict a huge value for the quantum vacuum. A common assumption is that the quantum vacuum is equivalent to the cosmological constant. Although no theory exists that supports this assumption, arguments can be made in its favour.

Such arguments are usually based on dimensional analysis and effective field theory. If the universe is described by an effective local quantum field theory down to the Planck scale, then we would expect a cosmological constant of the order of  $M_{\text{pl}}^2$  (1 in reduced Planck units). As noted above, the measured cosmological constant is smaller than this by a factor of  $\sim 10^{120}$ . This discrepancy has been called "the worst theoretical prediction in the history of physics".

Some supersymmetric theories require a cosmological constant that is exactly zero, which further complicates things. This is the cosmological constant problem, the worst problem of fine-tuning in physics: there is no known natural way to derive the tiny cosmological constant used in cosmology from particle physics.

No vacuum in the string theory landscape is known to support a metastable, positive cosmological constant, and in 2018 a group of four physicists advanced a controversial conjecture which would imply that no such universe exists.

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