

DIFFUSION THERMO EFFECTS OF VISCO-ELASTIC FLUID PAST A POROUS SURFACE IN PRESENCE OF MAGNETIC FIELD AND RADIATION

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Abstract: The two dimensional free convective hydro-magnetic flow of an electrically conducting visco-elastic fluid past a vertical porous surface has been studied in presence of radiation and chemical reaction of first order. Simultaneous effects of heat and mass transfer are also studied in combination of the physics of diffusion thermo (Dufour effect). The suction at the plate is assumed to be constant. A regular perturbation technique has been used to solve the problem. The results are analyzed graphically for various values of visco-elastic parameter along with a combination of other flow parameters involved in the solution.

Keywords: Walters liquid (Model B'), diffusion thermo, Grashof number, viscous drag

I INTRODUCTION

From technological point of view, the visco-elastic MHD convection flow problem are very important in both theoretical and experimental studies as they have overwhelming implications in various fields as petroleum industries, cooling of nuclear reactors, boundary layer control in aerodynamics, crystal growth etc. Again, diffusion rates can be changed tremendously with chemical reactions. A few representative areas of interest in which heat and mass transfer combined along with the chemical reaction play significant role in chemical industries like in food processing and polymer production. The idea of first order chemical reaction where the rate of reaction is directly proportional to the concentration itself has been analyzed by Cussler [3]. Chambre and Young [2] have investigated the flow pattern of a first order chemical reaction in the neighbourhood of a horizontal plate. Gupta et al. [4] analyzed free convection flow past a linearly accelerated vertical plate in the presence of energy dissipation. Kafousias and Raptis [5] have generalized it to study the mass transfer effects subjected to variable suction or injection. Basant kumar Jha and Ravindra Prasad [1] have analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. Effects of mass transfer with a chemical reaction on unsteady flow past an accelerated isothermal vertical plate are investigated by Muthuchumaraswamy et al. [6]. Rajesh et al. [7] have discussed the effects of chemical reaction and radiation effects on MHD flow past an infinite vertical plate with variable temperature. The importance of thermal-diffusion and diffusion-thermo effects for various fluid flows has been studied by Eckert and Drake [8]. Saxena and Dubey [9] have analyzed the flow behavior of unsteady MHD heat and mass transfer free convection flow of a polar fluid past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion. The mechanism of heat and mass transfer effects on MHD free convection flow of a visco-elastic fluid embedded in a porous medium with variable permeability in the presence of radiation and heat source in a slip flow regime has been examined by Saxena and Dubey [10]. Mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux has been studied by Saravana et al [11]. Rathod and Asha [12] have examined the magnetic field effects on two-dimensional viscous incompressible Newtonian fluid by using numerical technique. Kumar et al. [13] have investigated thermal diffusion and radiation effects on unsteady MHD flow through porous medium with variable temperature and mass diffusion in the presence of heat source or sink. Diffusion-thermo and radiation effects on MHD free convective heat and mass transfer flow past an infinite vertical plate in the presence of a chemical reaction of first order have been studied by Raveendra Babu et al. [14].

The aim of the present study is to investigate diffusion-thermo (Dufour number) and radiation effects on MHD free convective heat and mass transfer flow of a visco-elastic fluid past an infinite vertical plate with homogeneous chemical reaction of first order. The solutions for velocity, temperature and concentration fields are obtained by using relevant boundary conditions. The visco-elastic fluid flow is characterized by Walters liquid (Model B').

The constitutive equation for Walters liquid (Model B') is

$$\begin{aligned} \sigma_{ik} &= -p g_{ik} + \sigma'_{ik}, \\ \sigma'^{ik} &= 2\eta_0 e^{ik} - 2k_0 e'^{ik} \end{aligned} \tag{1.1}$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i , v_i is the velocity vector, the contravariant form of e^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e'^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk} \tag{1.2}$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{i,k} + v_{k,i} \tag{1.3}$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau) d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau) d\tau \tag{1.4}$$

$N(\tau)$ being the relaxation spectrum. This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty t^n N(\tau) d\tau, n \geq 2 \tag{1.5}$$

have been neglected.

II. MATHEMATICAL FORMULATION

A study of two dimensional free convective electrically conducting flow of visco-elastic fluid past a vertical porous surface has been investigated in presence of radiation and constant suction. The magnetic field of uniform strength B_0 is applied in the transverse direction to the plate. In this paper, we have studied the simultaneous heat and mass transfer mechanisms of governing fluid flow in connection with chemical reaction and diffusion thermo. Let x' - axis be taken along the plate in the vertically upward direction and y' -axis be taken along the normal to the plate. Then neglecting the induced magnetic field and using Boussinesq approximation, the governing equations of motion are as follows:

$$\frac{\partial v'}{\partial y'} = 0 \tag{2.1}$$

$$\rho v' \frac{\partial u'}{\partial y'} = \rho g \beta (T' - T'_\infty) + \rho g \beta^* (C' - C'_\infty) + \eta_0 \frac{\partial^2 u'}{\partial y'^2} - k_0 v' \frac{\partial^3 u'}{\partial y'^3} - \sigma B_0^2 u' \tag{2.2}$$

$$\rho C_p \left(v' \frac{\partial T'}{\partial y'} \right) = k_1 \left(\frac{\partial^2 T'}{\partial y'^2} \right) - \frac{\partial q_r}{\partial y'} + \left[\rho \frac{D_m K_T}{C_s} \frac{\partial^2 C'}{\partial y'^2} \right] \tag{2.3}$$

$$v' \frac{\partial C'}{\partial y} = D \left(\frac{\partial^2 C'}{\partial y^2} \right) - K_1(C' - C'_{\infty}) \tag{2.4}$$

Subject to the following initial and boundary conditions:

$$\left. \begin{aligned} y' = 0 : u' = U, v' = -v_0 = \text{constant}, T' = T'_w, C' = C'_{\infty} \\ y' \rightarrow \infty : u' \rightarrow 0, T' \rightarrow T'_{\infty}, C' \rightarrow C'_{\infty} \end{aligned} \right\} \tag{2.5}$$

we introduce the following non-dimensional quantities

$$u = \frac{u'}{U}, y = \frac{y'U}{v}, \theta = \frac{T' - T'_{\infty}}{T'_w - T'_{\infty}}, C = \frac{C' - C'_{\infty}}{C'_w - C'_{\infty}}, \lambda = \frac{v_0}{U}, M = \frac{\sigma B_0^2 v}{\rho U^2}, Gr = \frac{vg\beta(T'_w - T'_{\infty})}{U^3}$$

$$Gm = \frac{vg\beta^*(C'_w - C'_{\infty})}{U^3}, Pr = \frac{\eta_0 C_p}{k_1}, K = \frac{vK_1}{U^2}, Sc = \frac{v}{D}, R = \frac{16a^*v^2\sigma T'_{\infty}{}^3}{k_1 U^2}, k = \frac{k_0 U^2}{\rho v^2}, Du = \frac{D_m K_T (C'_w - C'_{\infty})}{C_s C_p v (T'_w - T'_{\infty})}$$

where Gr is the thermal Grashof number transfer, Gm is the solutal Grashof number, Pr is the Prandtl number, M is the Hartmann number, k is the dimensionless visco-elastic parameter, Sc is the Schmidt number and Du is the coefficient of mass diffusivity, R is the radiative parameter, K is the chemical reaction parameter.

The non-dimensional forms of the equations (2.2) to (2.4) are

$$k\lambda \frac{d^3 u}{dy^3} + \frac{d^2 u}{dy^2} + \lambda \frac{du}{dy} + Mu = -(Gr\theta + GmC) \tag{2.6}$$

$$\frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + \lambda \frac{\partial \theta}{\partial y} - \frac{R\theta}{Pr} = -Du \frac{\partial^2 C}{\partial y^2} \tag{2.7}$$

$$\frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + \lambda \frac{\partial C}{\partial y} = KC \tag{2.8}$$

Relevant to the boundary conditions:

$$\left. \begin{aligned} y = 0 : u = 1, \theta = 1, C = 1 \\ y \rightarrow \infty : u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \end{aligned} \right\} \tag{2.9}$$

III. METHOD OF SOLUTION:

The solutions of the equations (2.7) and (2.8) are given by

$$\theta = A_4 e^{-\alpha_4 y} + (1 - A_4) e^{-\alpha_2 y}$$

$$C = e^{-\alpha_2 y}$$

To solve the equation (2.6) we use the perturbation scheme considering the visco-elastic parameter k as the perturbation parameter as $k \ll 1$ in this problem for small shear rate.

Let us assume that the solution of the equation (2.6) subject to the boundary condition is of the form

$$u = u_0 + k u_1 + o(k^2)$$

Substituting this in the equation (2.6) and by equating the coefficients of like terms of k and neglecting the higher order terms we get,

Zeroth order equation

$$u_0'' + \lambda u_0' + M u_0 = -G r \theta - G m C \tag{2.10}$$

First order equation

$$u_1'' + \lambda u_1' + M u_1 = -\lambda u_0''' \tag{2.11}$$

Corresponding to boundary conditions

$$y = 0; \quad u_0 = 1, \quad u_1 = 0$$

$$y \rightarrow \infty; \quad u_0 = 0, \quad u_1 = 0$$

The solutions of the equations (2.10) and (2.11) are given by

$$u_0 = A_8 e^{-\alpha_6 y} + A_9 e^{-\alpha_4 y} + (A_{10} + A_{11}) e^{-\alpha_2 y}$$

$$u_1 = A_{18} e^{-\alpha_4 y} + A_{19} e^{-\alpha_2 y} - (A_{18} + A_{19}) e^{-\alpha_6 y}$$

And thus the velocity profile is given by

$$u = A_8 e^{-\alpha_6 y} + A_9 e^{-\alpha_4 y} + A_{20} e^{-\alpha_2 y} + k(A_{18} e^{-\alpha_4 y} + A_{19} e^{-\alpha_2 y} - A_{21} e^{-\alpha_6 y})$$

The non-dimensional shearing stress at the plate in the direction of free stream is given by

$$\frac{\tau'}{\rho U^2} = \tau = \left(\frac{\partial u}{\partial y} + k \lambda \frac{\partial^2 u}{\partial y^2} \right)_{at \ y=0}$$

The heat flux from the plate to the fluid in terms of Nusselt number is given by

$$Nu = \left(\frac{\partial \theta}{\partial y} \right)_{at \ y=0} = -(\alpha_4 A_4 + \alpha_2 - \alpha_2 A_4)$$

The mass flux at the wall in terms of Sherwood number is given by

$$Sh = \left(\frac{\partial C}{\partial y} \right)_{at \ y=0} = -\alpha_2$$

IV. RESULTS AND DISCUSSION:

A problem of steady two dimensional free convective hydro-magnetic flow of a visco-elastic fluid past a vertical porous surface in presence of radiation, chemical reaction of first order and Dufour effect has been analyzed in this paper. The visco-elastic effect is exhibited through the non-dimensional parameter k . Figures 1 to 5 characterize the velocity profile for various values of visco-elastic parameter along with a combination of other flow parameters present in the problem. The Prandtl number is considered as greater than 1. It physically interprets the higher viscosity of governing fluid flow. The figures enable that the speed of the fluid enhanced in the neighbourhood of the plate but it will slow down as it goes away from the plate in case of externally cooled plate ($Gr > 0$). This physical phenomena of higher viscosity in the neighbourhood of the plate is noticed in both Newtonian fluid ($k = 0$) and non-Newtonian fluid ($k = 0.1, 0.2$). The increasing values of visco-elasticity accelerate the speed of non-Newtonian fluid flow in comparison with the Newtonian fluid flow for externally cooled plate but a reverse behavior is noticed during the fluid flows past an externally heated plate ($Gr < 0$).

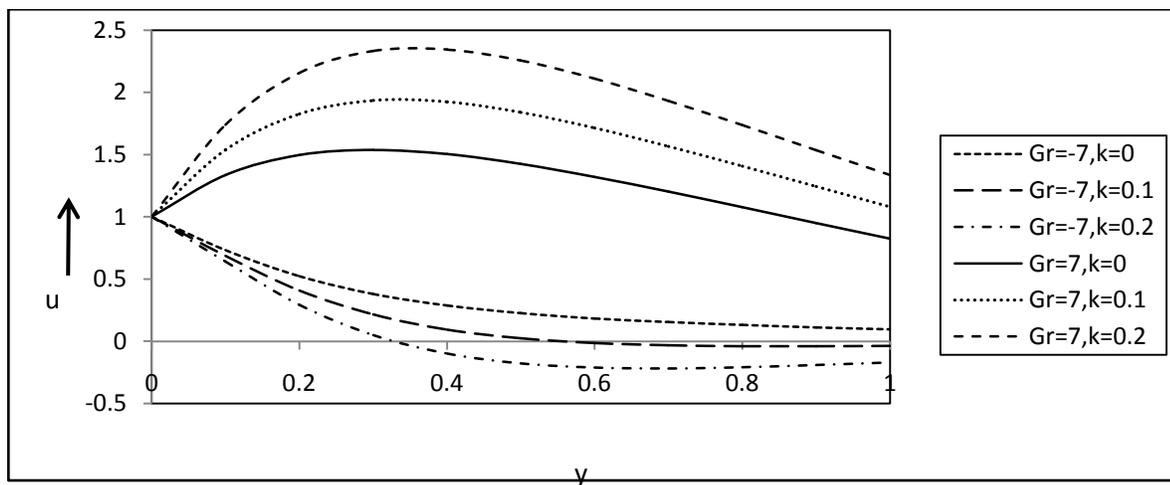


Fig.1: Velocity profile u against y for $Gr=7$ and $Gr=-7$, $M=1$, $Gm=10$, $Sc=5$, $K=1$, $Du=0.5$, $R=10$, $Pr=5$

In figure 1, we have considered positive as well as ~~negative~~ values of thermal Grashof number. $Gr > 0$ signifies that the fluid flow past an externally cooled plate and its negative value indicates that the flow is through an externally heated plate. The differences in flow patterns in both positive and negative values of Grashof number are noticed in the figure 1. In case of positive value of Grashof number, the velocity profile experiences a parabolic profile but for the flow past a heated plate a back flow is noticed for $k \leq 0.1$ as the fluid moves away from the plate.

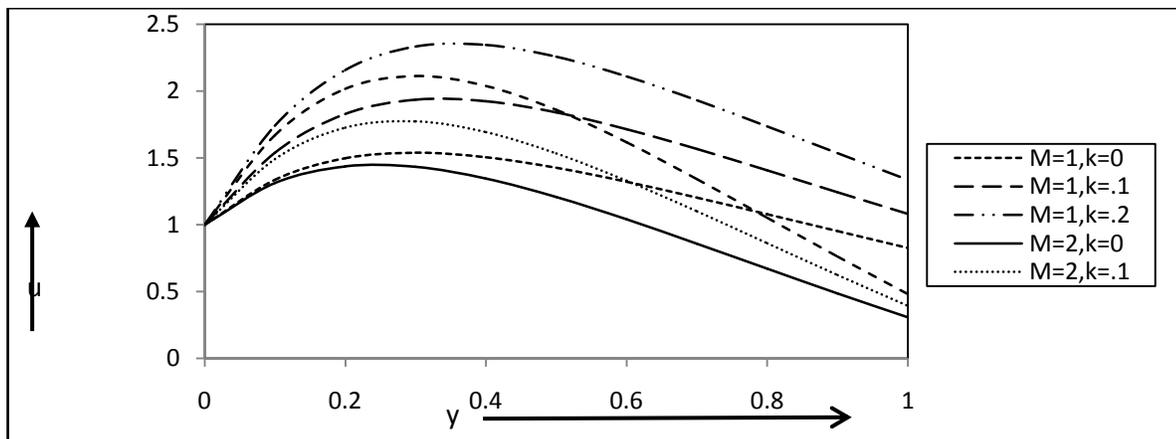


Fig.2: Velocity profile u against y for $M=1$ and $M=2$, $Gr=7$, $Gm=10$, $Sc=5$, $K=1$, $Du=0.5$, $R=10$, $Pr=5$

Figure 2 reveals the effect of Hartmann number on the velocity of the various fluid flow mechanisms. The application of transverse magnetic field creates a Lorentz force which has a retarding effect on the speed of the electrically conducting fluid flow. This diminishing behavior is noticed in both Newtonian as well as non-Newtonian fluid flows for positive values of thermal Grashof number..

Effects of Grashof number for mass transfer and Schmidt number are analyzed graphically in figure 3 and 4 respectively. Increasing values of Schmidt number raises the thickness of the fluid which in turn reduces the speed of both Newtonian and non-Newtonian fluid in combination of other flow parameters. Grashof number studies the nature of free convection and the effect of free convection on mass transfer is characterized through the non-dimensional parameter Gm . For the flow past a cooled plate, the increasing values of Grashof number modifies the speed of visco-elastic fluid flow and Newtonian fluid flows.

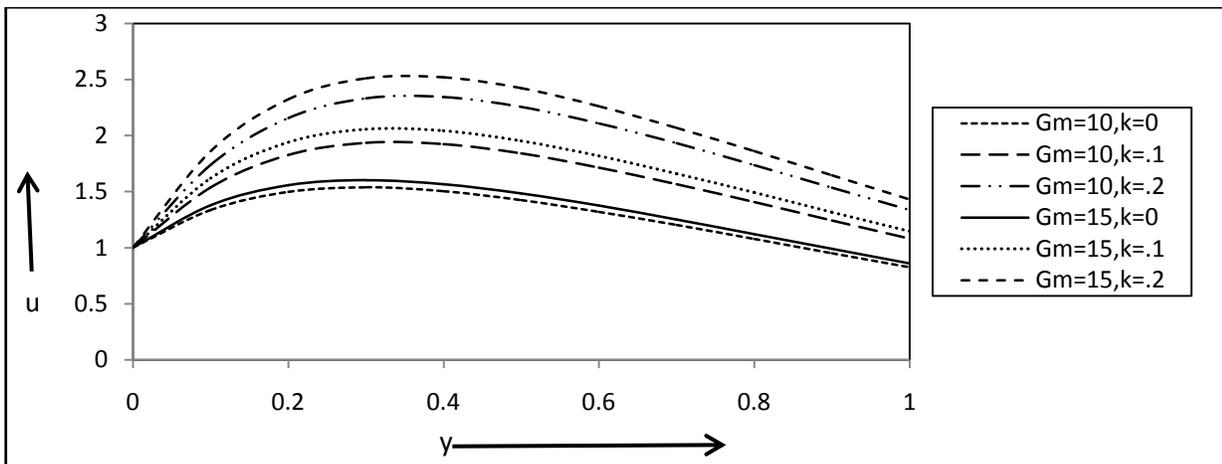


Fig.3: Velocity profile u against y for $Gm=10$ and $Gm=15$, $Gr=7$, $M=1$, $Sc=5$, $K=1$, $Du=0.5$ $R=10$, $Pr=5$

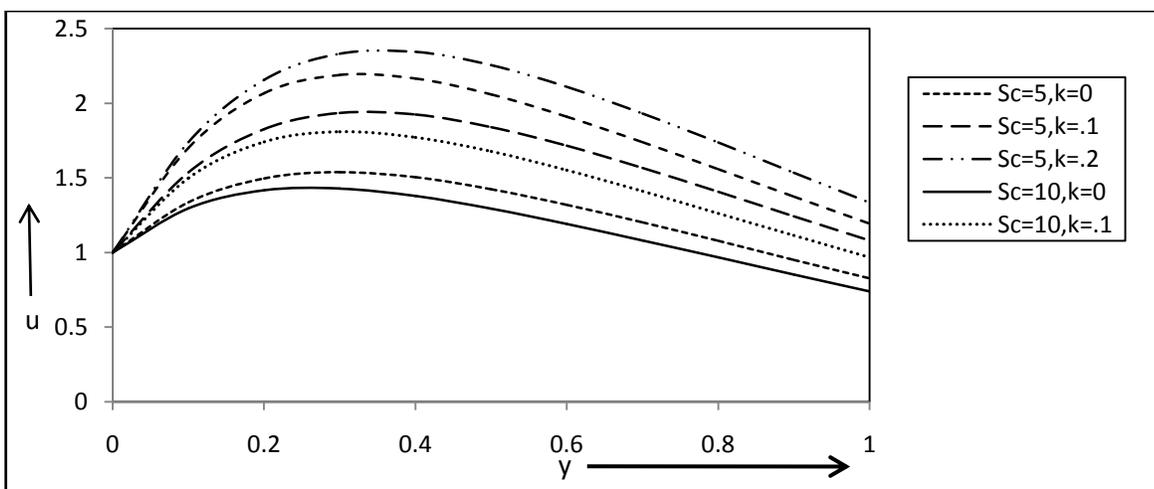


Fig.4: Velocity profile u against y for $Sc=5$ and $Sc=10$, $Gm=10$ $Gr=7$, $M=1$, $K=1$, $Du=0.5$, $R=10$, $Pr=5$

The Dufour effect is energy flux due to mass concentration gradient. It is exhibited through the parameter Du . The enhancement of Dufour number raises the speed of various fluid flows along with the increasing values of visco-elastic parameter (Figure 5).

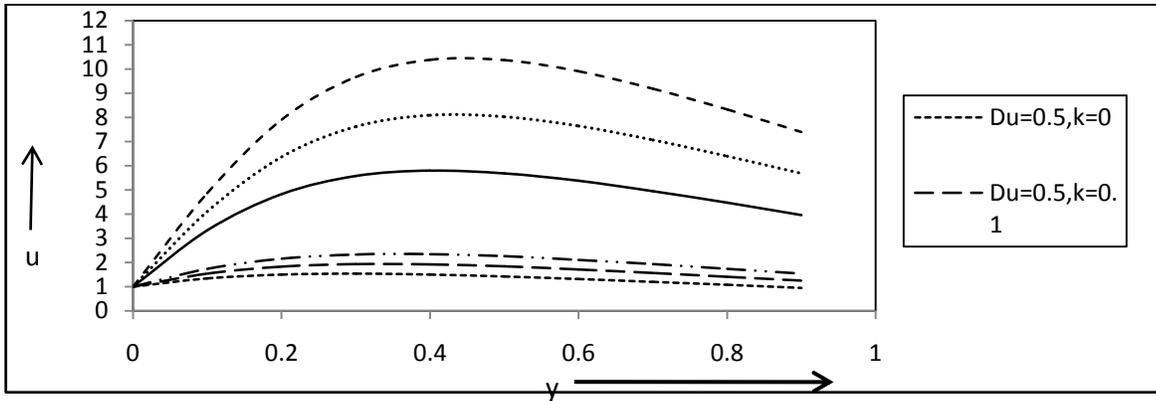


Fig.5: Velocity profile u against y for $Du=0.5$ and $Du=5$, $Sc=5$, $Gm=10$, $Gr=7$, $M=1$, $K=1$, $R=10$, $Pr=5$.

From practical point of view, the importance of shearing stress or viscous drag cannot be neglected. In this discussion, mainly we have concentrated the formation of shearing stress at externally cooled surface. Figures 6 to 9 notify the nature of shearing stress formed at the cooled surface for various values of visco-elastic parameter. The increasing values of visco-elasticity of non-Newtonian fluids make the fluid thicker and hence the drag force will be enhanced in comparison with Newtonian fluid. Figure 6 and 7 represent the behavior of shearing stress experienced by the motion of various fluid flow mechanisms against Prandtl number and Dufour number respectively. It is observed that in both cases, increasing values of Prandtl number and Dufour number lead to raise the magnitude of viscous drag along with the increasing values of visco-elastic parameter. A reverse physical phenomenon is noticed during the growth of Schmidt number (figure 8) for both simple Newtonian fluid and complex visco-elastic fluid. The impact of chemical reaction parameter on the shearing stress is shown in figure 9. It is noticed that the chemical reaction parameter subdues the magnitude of shearing stress formed at the externally cooled surface.

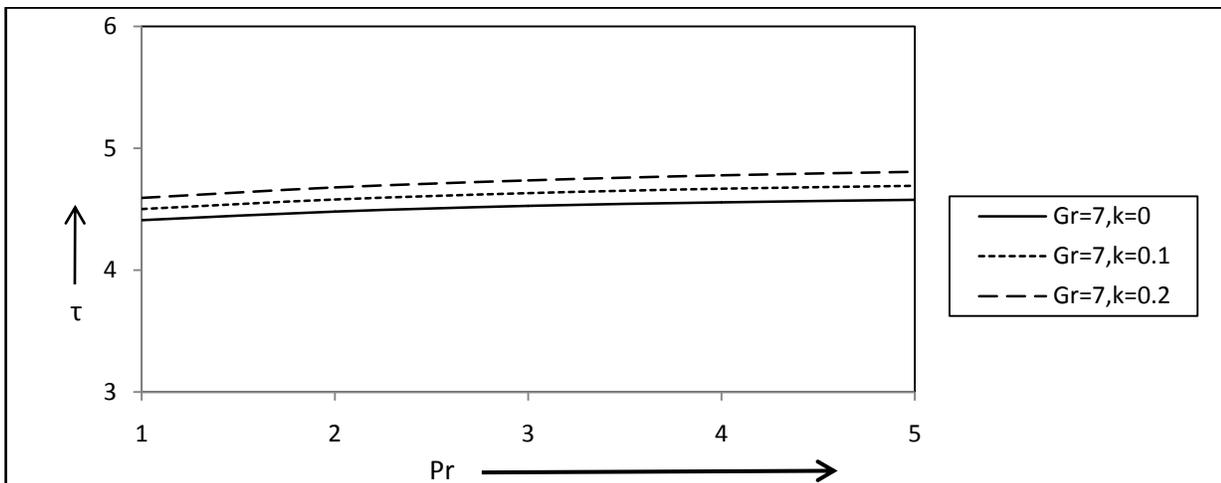


Fig 6: Shearing stress τ against Pr , $M=1$, $Gm=10$, $Sc=5$, $R=10$, $K=5$, $Du=0.5$

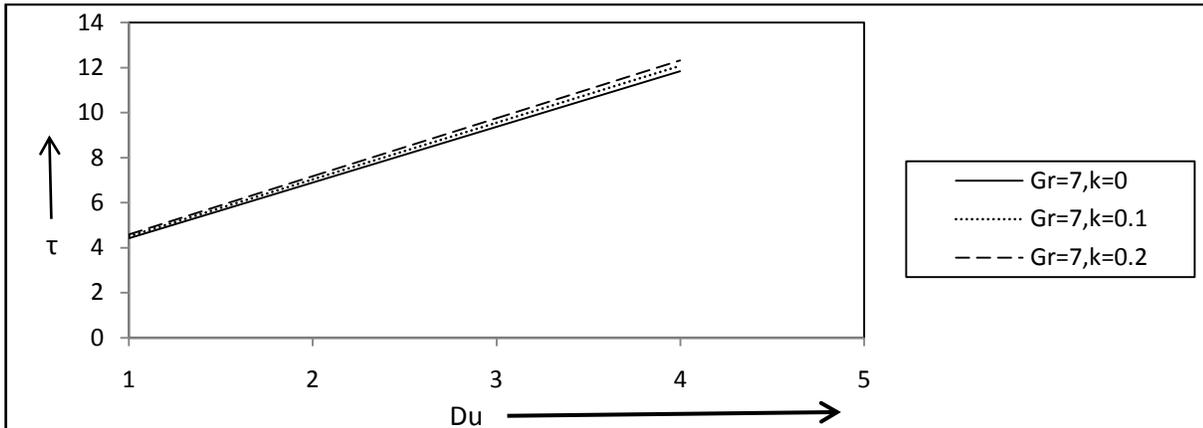


Fig 7: Shearing stress τ against Du for $M=1, Gm=10, Sc=5, R=10, K=5, Pr=5$

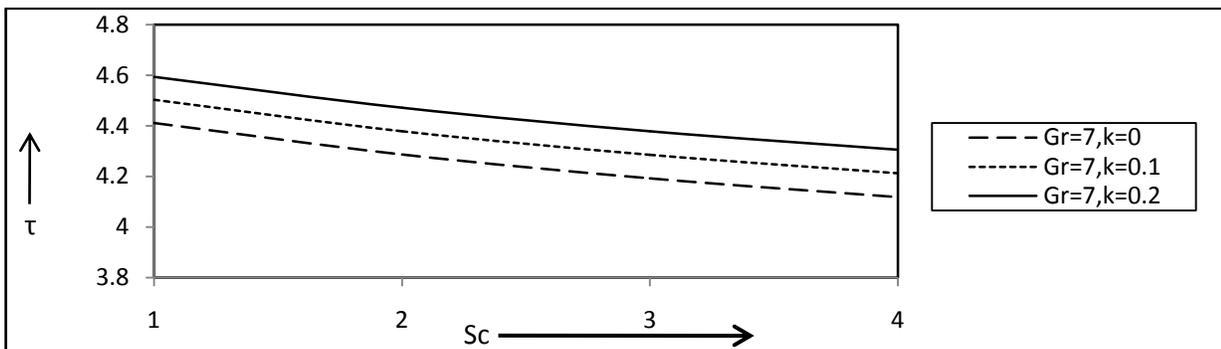


Fig 8: Shearing stress τ against Sc for $M=1, Gm=10, Pr=5, R=10, K=5, Du=0.5$

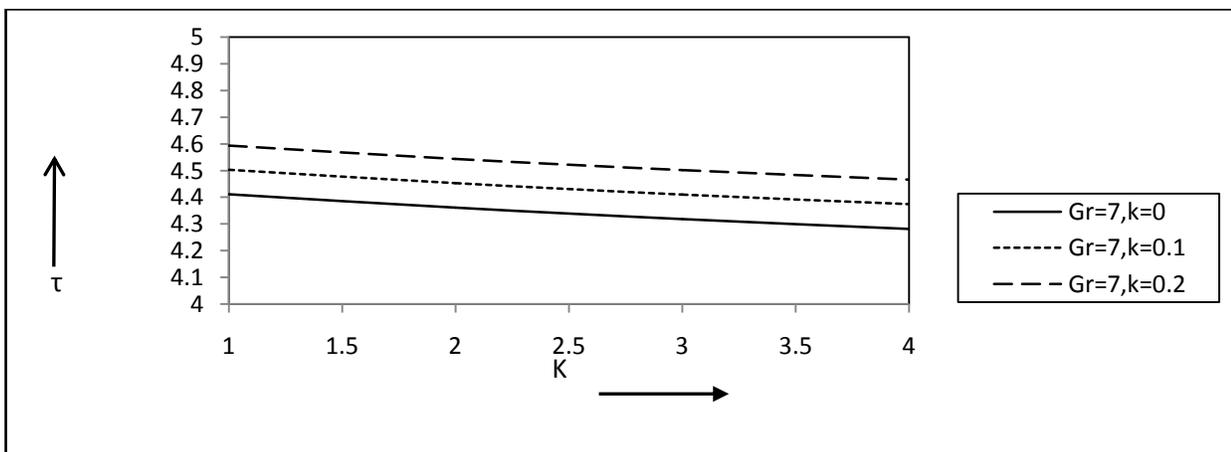


Fig 9: Shearing stress τ against K for $M=1, Gm=10, Sc=5, R=10, Pr=5, Du=0.5$

V. CONCLUSION

An analysis of a homogeneous first order chemical reaction between the fluid and species concentration to study diffusion-thermo(Dufour effect) and radiation effects on steady MHD free convection flow of a visco-elastic, electrically conducting

,radiating fluid past an infinite vertical plate in the presence of transverse applied magnetic field is performed. This study leads to following conclusions:

- The velocity field is considerably affected by the visco-elastic parameter at all points of the fluid flow region in combination of other physical parameters.
- For externally cooled plate, the velocity profile accelerates but the reverse pattern is observed for externally heated plate in both Newtonian and non-Newtonian fluids.
- The effects of magnetic parameter, Grashof number for mass transfer and Schmidt number are significant in the velocity field and the behaviors of the profiles are noticed similar in all the cases for externally cooled plate.
- The speed of the velocity profile increases with the enhancement of the Dufour number in both Newtonian and non-Newtonian cases.
- The enhancement of Prandtl number and Dufour number reveals the rising trend of shearing stress but the reverse tendency is observed for externally cooled surface when Schmidt number increases in both Newtonian and non-Newtonian fluid flow mechanism.
- For externally cooled surface, the chemical reaction parameter subdues the magnitude of shearing stress in both Newtonian and non-Newtonian fluid flow phenomenon.
- The temperature and the mass concentration fields are not significantly affected by the visco-elastic parameter in the fluid flow region.

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