

Trends in Fuzzy Graphs

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Abstract: After introducing and developing fuzzy set theory, a lot of studies have been done in this field and then a result appeared as a Fuzzy Graph (Combination of graph theory and fuzzy set theory). This is now known as Fuzzy graph theory. In this article we review essential works on different types of fuzzy graph and fuzzy hyper graph.

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I. INTRODUCTION

In 1736, Euler first introduced the concept of graph theory. In the history of mathematics, the solution given by Euler of the well known Konigsberg bridge problem is considered to be the first theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operations research, and optimization and computer science.

In 1965, Zadeh published his seminal paper on "Fuzzy sets" which described fuzzy set theory and, consequently, fuzzy logic. The purpose of Zadeh's paper was to develop a theory which could deal with ambiguity and imprecision of certain classes of sets in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction.

Rosenfeld (1975) introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles, connectedness and etc. Zadeh (1987) introduced the concept of fuzzy relations. Bhattacharya (1987) and Bhutani (1989) investigated the concept of fuzzy automorphism graphs. McAlester (1988) presented a generalization of intersection graphs to fuzzy intersection graphs. Mordeson (1993) introduced the concept of fuzzy line graphs and developed its basic properties. The concept of complete fuzzy graph was investigated by Sunitha and Vijayakumar (2002). The concept of domination in fuzzy graphs was investigated by Somasundaram (1998). Ramaswamy and Poornima (2009) introduced the concept of product fuzzy graphs. The first definition of intuitionistic fuzzy relations and intuitionistic fuzzy graphs were introduced by Atanassov (1999), and further studied in [2009]. Parvathi and Thamizhendhi (2010) introduced the concept of domination number in intuitionistic fuzzy graphs. Mahioub Shubatah (2012) introduced the concept of domination in product fuzzy graphs. Vinoth Kumar and Geetha Ramani (2011) introduced the concept of product intuitionistic fuzzy graphs.

A graph is a symmetric binary relation on a nonempty set V . Similarly, a fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. The first definition of a fuzzy graph was by Kaufmann [1] in 1973, based on Zadeh's fuzzy relations [2]. But it was Azriel Rosenfeld [3] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. During the same time R.T.Yeh and S.Y. Bang have also introduced various connectedness concepts in fuzzy graphs.

Zimmermann [4] has discussed some properties of fuzzy graphs. The book [5] by Mordeson and Nair entitled "Fuzzy graphs and Fuzzy hypergraphs" is an excellent source for research in fuzzy graphs and fuzzy hypergraphs.

In this paper we are giving an overview on the fuzzy graph and its various kinds. Here we will only focus on fuzzy graph and its types and not on operations and properties of Fuzzy Graph.

II. PRELIMINARIES

Graph

A graph G is defined as an ordered pair: $G = (V, E)$ where V : Set of Vertices. A vertex is also called a node or element and E : Set of edges. An edge is an unordered pair (x, y) , of vertices in V .

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Fuzzy Graph

A fuzzy graph G is defined as an ordered pair: $G = (V, E)$ where V : Set of Vertices. A vertex is also called a node or element and E : Set of edges. An edge is an element of the fuzzy set $E: X \times Y \rightarrow [0,1]$.

III. TYPE OF FUZZY GRAPHS

1. Regular Fuzzy Graph

A. Nagoor Gani and K.Radha (2008) introduced regular fuzzy graphs on paper "On Regular Fuzzy Graphs". In their paper they have introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. Regular fuzzy graphs and totally regular fuzzy graphs are compared through various examples. A necessary and sufficient condition for equivalence and characterization of regular fuzzy graph was provided. [6]

2. Complementary Fuzzy Graph

Moderson(1994) introduced the concept of complement of fuzzy graphs and M.S.Sunitha and A. Vijayakumar(2001) gave a modified definition of complement of fuzzy graph[7]. R. Sattanathan and S. Lavanya(2009) studied about complementary fuzzy graphs and fuzzy chromatic number. In their paper they find the fuzzy chromatic number of complement of fuzzy graphs and gave the bounds for sum and product of fuzzy chromatic number of fuzzy graph and its complement. [8]

3. Antipodal Fuzzy Graph

A. Nagoor Gani and J.Malarvizhi (2010) defined concept of Antipodal Graph. In crisp graph theory the concept of antipodal graph of a given graph G was introduced by Smith[8]. The condition on the graph G , for $A(G) = G$ and $A(G) = \bar{G}$ are discussed by Aravamudhan and Rajendran[9]. As a fuzzy analog to this, in their paper antipodal fuzzy graph was defined and its nature was discussed.

4. Constant intuitionistic fuzzy graphs

M. G. Karunambigai, R. Parvathi and R. Buvaneshwari (2011) introduced Constant intuitionistic fuzzy graphs. In their work, Constant Intuitionistic Fuzzy Graphs (IFGs), and totally constant IFGs were introduced. A necessary and sufficient condition for equivalence was provided. A characterization of constant IFGs on a cycle was also given. Some properties of constant IFGs with suitable illustrations were also discussed.[10]

5. Fuzzy Graph Structures

T. Dinesh and T.V. Ramakrishnan (2011) introduced the concept of a fuzzy graph structure based on the concept of graph structure. A new concept, namely, graph structure has been introduced by E. Sampathkumar which, in particular, is a generalization of the notions like graphs, signed graphs and edge-coloured graphs with the colourings. According to him, $G = (V, R_1, R_2, \dots, R_k)$ is a graph structure if V is a nonempty set and R_1, R_2, \dots, R_k are relations on V which are mutually disjoint such that each $R_i, i = 1, 2, 3, \dots, k$, is symmetric and irreflexive. This is the motivation for the study of fuzzy graph structures. New concepts like π -edge, π -cycle, π -tree, π -forest, fuzzy π -cycle, fuzzy π -tree, fuzzy π -forest, π -connectedness etc. are introduced and studied.[11]

6. Product Intuitionistic Fuzzy Graph

The first definition of intuitionistic fuzzy graphs was proposed by Atanassov [12]. Dr. V. Ramaswamy and Poornima .B introduce the concept of product fuzzy graph. N. Vinoth Kumar and G.Geetha Ramani(2011) develop the concept of Product Intuitionistic fuzzy graphs of intuitionistic fuzzy graphs. Further investigate properties Product Intuitionistic fuzzy graphs.

7. Bipolar Fuzzy Hypergraphs

In 1994 , Zhang [13] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose range of membership degree is $[-1,1]$. In bipolar fuzzy set, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree $(0,1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree $[-1,0)$ of an element indicates the element somewhat satisfies the implicit counter property.

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S.Samanta and M .Pal(2012) define some basic concepts of bipolar fuzzy hypergraphs, cut level bipolar fuzzy hypergraphs, dual bipolar fuzzy hypergraphs and bipolar fuzzy transversal. Also some basic theorems related to the stated graphs have been presented.[14]

8. Irregular Fuzzy Graphs

A. Nagoor Gani and S. R. Latha (2012) defined Neighbourly total irregular fuzzy graphs, highly irregular fuzzy graphs and highly total irregular fuzzy graphs were introduced. A comparative study between neighbourly irregular and highly irregular fuzzy graphs was made. Also some results on neighbourly irregular fuzzy graphs were studied.[15]

9. Irregular Bipolar Fuzzy Graphs

S.Samanta and M .Pal (2012) define irregular bipolar fuzzy graphs and its various classifications. Size of regular bipolar fuzzy graphs was derived. The relation between highly and neighbourly irregular bipolar fuzzy graphs were established. Some basic theorems related to the stated graphs have also been presented.[16]

10. Fuzzy Labeling Graph

A.Nagoor Gani and D. Rajalaxmi (a) Subahashini (2012) introduced a new concept of fuzzy labeling. A graph is said to be a fuzzy labeling graph if it has fuzzy labeling. Fuzzy sub graph, union, fuzzy bridges, fuzzy end nodes, fuzzy cut nodes and weakest arc of fuzzy labeling graphs have been discussed. And number of weakest arc, fuzzy bridge, cut node and end node of a fuzzy labeling cycle has been found. It is proved that $\Delta(G^{\circ})$ is a fuzzy cut node and $\delta(G^{\circ})$ is a fuzzy end node of fuzzy labeling graph. Also it was proved that If G° is a connected fuzzy labeling graph then there exists a strong path between any pair of nodes.[17]

11. Strong Intuitionistic Fuzzy Graphs

Muhammad Akram and Bijan Davvaz (2012) introduce the notion of strong intuitionistic fuzzy graphs and investigate some of their properties. They discuss some propositions of self complementary and self weak complementary strong intuitionistic fuzzy graphs. They introduced the concept of intuitionistic fuzzy line graphs.[18]

12. Interval-valued Fuzzy Graph

Muhammad Akram and Wieslaw A. Dudek (2012) define the operations of Cartesian product, composition, union and join on interval-valued fuzzy graphs and investigate some properties. They gave concept of isomorphism (resp. weak isomorphism) between interval-valued fuzzy graphs. They also introduced the notion of interval-valued fuzzy complete graphs and present some properties of self complementary and self weak complementary interval-valued fuzzy complete graphs.[19]

13. Fuzzy Middle Graph

Veena Mathad and B. Sharada (2012) introduced the concept of fuzzy middle graph and its properties.[20]

14. Complete interval-valued Fuzzy Graph

Hossein Rashmanlou and Young Bae Jun (2013) In their paper, they provided three new operations on interval-valued fuzzy graphs; namely direct product, semi strong product and strong product. They gave sufficient conditions for each one of them to be complete.[21]

15. Balanced Intuitionistic Fuzzy Graph

Al-Hawary [1] introduced the concept of balanced fuzzy graphs and studied some operations of fuzzy graphs. Atanassov [6] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs(IFGs). Parvathy and Karunambigai[8] introduced the concept of IFG elaborately and analysed its components. Articles[24, 25, 26] motivated us to analyze balanced IFGs and their properties. Their paper deals with the significant properties of balanced IFG. The basic definition and theorems needed are discussed in section 2.The necessary condition for an IFG to be a Balanced IFG if the graph G is complete, strong, regular and self complementary IFG were discussed. They also discussed some properties of complementary and self complementary balanced IFGs. Also deals with direct product, semi strong product and strong product of intuitionistic fuzzy graphs and their properties with suitable illustrations were given.[22]

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16. Fuzzy Dual Graph

Nuha Abdul-Jabbar, Jehan H. Naom, Eman H. Ouda (Dec 2009) introduced Fuzzy dual graph. In their paper the definition of fuzzy dual graphs was considered with the following properties were obtained, which was the dual of the dual of fuzzy graph is the fuzzy graph itself, and the dual of fuzzy bipartite graph is Eulerian fuzzy graph.[23]

IV. CONCLUSION

In this paper we are willing to summarize types of fuzzy graph, fuzzy graph structure and also type of intuitionistic fuzzy graph along with persons related to respective topic. We hope this paper will help the researchers to look the field of Fuzzy Graph at a glance.

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