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# Effect of stiffeners in Lean Duplex Stainless Steel (LDSS) hollow circular stub columns under pure axial compression

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**ABSTRACT:** The present study is an attempt to study the effect of various geometrical stiffeners parameters in the critical/ultimate buckling load capacity of circular Lean Duplex Stainless Steel (LDSS) stub column loaded axially using finite element (FE) analysis, through Abaqus (2009). The effect of increase in number of stiffeners (Ns), thickness of stiffener and width of stiffener were studied on the critical buckling load capacity. Based on the FE analyses, an increase of buckling load capacity has been observed with increasing number of stiffeners. The increase of buckling load followed a nearly linear trend. The buckling load for the stiffened column in comparison to the unstiffened column, increases from ~ 120 % to 180 %, when N<sub>s</sub> (number of stiffeners) is increased by 500% from  $N_s = 4$ . An increase of buckling load capacity has also been observed for both increasing in thickness and width of the stiffeners.

Keywords: Buckling load, Hollow stub columns; Stiffener, Lean Duplex Stainless Steel, Finite element analysis, Abaqus

## I. INTRODUCTION

In view of certain drawbacks of carbon steel viz., low corrosion resistance and higher material cost, there has been a continuous look out of various engineering materials such as stainless steel to provide better material characteristics and mechanical properties to suit the demands of various construction applications. Some of the distinct advantages of stainless steel include relatively high corrosion resistance (in moderate to highly aggressive environments), high ductility, high strength, impact resistance, smooth and uniform surface, aesthetic appearance, and ease of maintenance and construction. These benefits have encouraged a moderately increase in the use of stainless steel in construction industry in the recent years. Amongst the various grades of stainless steel, austenitic grade are generally popular in the construction industry, however, with increasing nickel prices (nickel content of  $\sim 8\%$  -11% in austenitic stainless steel) there is an escalation in the demand for newer form of duplex stainless steel like Lean Duplex Stainless Steel (LDSS) with low nickel content of ~ 1.5%, such as grade EN 1.4162 (e.g. [1-3]). LDSS grade EN 1.4162 in particular, provides around twice the mechanical strength of conventional austenitic and ferric stainless steel, making a potential candidate for use in construction industry, as main structural members. Owing to economical incentive without compromising the strength requirements has catapult a significant growth and development of LDSS over the last two decades. The growth for LDSS development has been fuelled by soaring raw material costs, such as nickel, along with increasing demand for improved corrosion resistance and strength, enabling a reduction in section sizes leading to higher strength to weight ratios.

Thin-walled or shell structures in the form of tubular structural members have recently become popular in the construction of buildings, bridges, towers etc., although its use as basic structural components in aircraft structures, nautical hulls, automobiles etc. have been there for a long time. However, as these tubular structures are light and thin, they are susceptible to deformation, strength, buckling, and vibration and noise problems. One common and cost-effective way to improve the structure over these



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problems is to add stiffeners, which can substantially improve the bending rigidity of the basic shell structures by increasing their geometrical moment of inertia. In the literature, researchers have generally focused on buckling of hollow columns under various loading conditions [4-7], or on columns with various cross-sections [8-13]. Based on the work by Liu and Shimoda [14] on parameter-free optimum design method of stiffeners on thin-walled structures, it has been shown that shape of stiffener could influence the rigidity of thin wall structures. Chen et.al., [15] studied the stub column tests of thin-walled complex section with intermediate stiffeners and reported that the design strengths calculated using direct strength method based on the buckling stresses obtained from finite element analysis results generally agree with the test results well. Dabaon et. al., (2009) [16] presented a comparative experimental study between stiffened and unstiffened stainless steel hollow tubular stub columns and observed that the design rules specified in European specifications and ASCE standard generally overestimate the column strengths of stainless steel square and rectangular hollow section stub columns fabricated by cold- forming and welding. Tafreshi [17] studied the buckling and post-buckling behaviour of composite cylindrical shells under internal pressure and axial compression using ABAQUS and showed that the response of a compression loaded cylinder with a cutout is influenced by the internal pressure, cutout area and orientation. However, it may be noted that, to the best of authors' knowledge, no systematic study on the effect of stiffeners on load carrying capacity of LDSS hollow stub circular columns have been reported. Hence, considering the advantages of LDSS as mentioned above and scarcity of work reported on buckling capacity of stiffened circular LDSS columns, a systematic study of the effects of number of stiffeners (Ns), stiffener thickness (t) and width of stiffener (b) and diameter of tube (D) on buckling capacity of hollow stub columns (see Fig. 1) has been attempted in the present research effort.



Fig 1: Schematic diagram of the LDSS hollow stub column/tube (a) isometric view, (b) sectional view (w = width of stiffener, D = diameter of column, L = length of column, t = thickness of stiffener)

#### II. FINITE ELEMENT MODELLING

#### 2.1 General

ABAQUS v6.9 EF1 [18], a general purpose commercial FE software have been used to determine load carrying capacity of LDSS stiffened circular hollow columns subjected to pure axial compression by performing study on effects of various parameters of stiffeners i.e. number of stiffeners(Ns), width of stiffener (w) and thickness of stiffener(t). The experimental data on FE results of square hollow column (SHC) from the literature have been used to validate approach on FE analysis. FE modeling details are shown in the subsequent sub-sections.

2.2 Geometry and Boundary conditions

Modeling of typical geometric LDSS SHC with square cross-section which was published in literature [19-21] has been used in this paper. The bottom part of column is fixed while the top part which is loaded is allowed to undergo free vertical translation along the column length direction. The boundary conditions were accomplished using two

reference points (RP-1 and RP-2) that were tied to the column ends via nodes-to-node tie constraints available in Abaqus [18]. Couple constraint has been used to couple reference point to respective surface of column so that the condition given at reference point is applied throughout the surface. A centrally concentrated normal load was applied statically at the reference node (RP-1) (see: fig 2) using displacement control, thus applying uniform pressure at the top edges of the column. Stiffeners are tied



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to circular hollow column along the length direction of column using surface to surface tie constraint. The stiffened hollow column section considered has an inner diameter of 200 mm and thickness of 3.75 mm. The column lengths (L) were set equal to three times the diameter (3 x 200 mm = 600 mm), to avoid the effects of flexural buckling and end conditions, as well as to satisfy the condition for stub column. Number of stiffeners (Ns) was varied by changing from 0 to 30 (Ns = 0 indicates column without stiffener). Width of stiffener (w) and stiffener thickness (t) were varied by changing w/D from 0.1 to 0.4, and changing t from 3.25mm to 4 mm, respectively

## 2.3 Finite element mesh

Shell elements were employed to discretise the models. Four-noded doubly curved shell element with reduced integration (S4R) [18] with six degrees of freedom per node and known to provide accurate solutions to most applications (for both thin and thick shell problems) have been utilized to discretise the models in this study. The aspect ratio of the element is kept at ~ 1.0 in all the FE



Fig 2: (a) FE mesh (b) boundary conditions for LDSS circular HC

models (Fig. 2). The number of S4R elements used in the analyses for various models ranges from ~ 7000 to 37000. Linear elastic eigen value buckling analysis was then used to check mesh convergence by monitoring the first eigen buckling mode with mesh refinement.

2.4 Local geometric imperfection

For extracting both local and global the buckling mode shapes, linear elastic eigen value buckling analyses were initially performed on fixed-ended ends using subspace iteration method. The first (i.e. lowest) local was then utilised as initial geometric imperfection patterns to perturb the geometry of the stub column/column. The perturbation of the geometry of the columns was based on the local geometric properties proposed by [22] for stub columns, by scaling with local imperfection amplitude [22] given by Eq. (1).

$$w_0 = 0.023 - t$$
 (1)

where, is the 0.2% proof stress, t is the thickness, and is the elastic critical buckling stress (Eq. (2)).

(2)

In Eq. (2),  $\lambda$  is the eigen value obtained from the results of FE analysis, E<sub>0</sub> is the initial Young's modulus of elasticity,  $\Delta I$  is the initial displacement value provided at the movable end, and L is the length of the column.

2.5 Material modeling

In the present study, the material properties given by [19] (Table 1) were used in deriving the stress-strain curve of LDSS material through a material model suggested by Gardner and Ashraf [23] (modified version of the original Ramberg–Osgood [24, 25]). Poisson's ratio was taken as 0.3. The minimum specified material properties of LDSS Grade EN 1.4162 according to EN 10088-4 [3] are 0.2% proof stress ( $\sigma_{0.2}$ ) of 530 MPa and ultimate stress ( $\sigma_u$ ) of X00-900 MPa.

## S4R elements

Compressive flat material properties (Theofanous and Gardner [19]).

Cross- section	E (Mpa)	σ <sub>0.2</sub> (Mpa)	σ <sub>1.0</sub> (Mpa)	CompoundR-O coefficients	
				n	$n'_{0.2, 1.0}$
80x80x4- SC2	197200	657	770	4.7	2.6



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Fig 3: Compressive flat material properties (Theofanous and Gardner [19]).

 Table 2 Stub column

 dimensions (Theofanous and Gardner [19]).

Specimen	L	B	H	t	r <sub>i</sub>	w <sub>0</sub>
	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)
80x80x4- SC2	332.2	80	80	3.81	3.6	Eq. (1)

 $L = Length, \ B = Width, \ H = Depth, \ t = thickness, \ r_i = internal \ corner \ radius, \ w_0 = local \ geometric \ imperfection$ 

The stress ( $\sigma$ ) - strain ( $\epsilon$ ) curve of LDSS (Fig. 3) is divided into two parts viz., (a) from  $\sigma_{0.2}$  to  $\sigma_{1.0}$ , where Gardener and Asraf model [23] is used and (b) up to 0.2% proof stress ( $\sigma_{0.2}$ ), where Ramberg-Osgood model [24,25] is used to define the curve. In the absence of necking phenomena in compression, Gardner and Ashraf [23] proposed a model (Eq. 3) which applies for stresses greater than  $\sigma_{0.2}$ .

(3)

here  $\epsilon_{t0.2}$  and  $\epsilon_{t1.0}$  are total strains at  $\sigma_{0.2}$  and  $\sigma_{1.0}$ , respectively; and is the strain hardening exponent (Table 1). Eq. 3 has been reported to give excellent

agreement with measured stress-strain curves in both tension and compression.

2.6 Verification of finite element model

The results from several experiments conducted by Theofanous and Gardner [19] on LDSS stub SHC (Square Hollow Column) was employed here to verify the developed FE modeling approach. Table 1 and 2 show the LDSS material properties and column dimensions for flat portions of the tested specimen as reported by Theofanous and Gardner [19]. In the present FE study, same compressive flat material properties (Table 1) has been used for both the flat and corner portions of the column even though the experimental column was formed using cold-form technique resulting in different properties between flat and corner portions of the column. Hence the strength enhancement in the corner regions by cold

Fig. 3 shows the stress-strain plot of the LDSS material using the material properties from Table 1 with Gardner and Ashraf model as mentioned in Section 2.5. The material model shown in Fig. 3 is used as input parameters to Abaqus [18], by converting into true stress ( $\sigma_{true}$ ) and true plastic strains ( ) using the following Eqs (4) and (5).

$$\sigma_{true} = \sigma_{nom} (1 + \varepsilon_{nom}) \tag{4}$$

$$\varepsilon^{pl}_{true} = \ln(1 + \varepsilon_{nom}) - \frac{\sigma_{true}}{E_o}$$
(5)

where  $\sigma_{nom}$  and  $\varepsilon_{nom}$  are engineering stress and strain respectively.

Local (Eq. (1)) imperfection magnitudes were seeded in the FE models. Modified Riks method [18], (a variation of the classical arc-length method [26-28]), was employed for the non-linear analyses to capture the full (i.e. both pre and post ultimate load) load-deformation response. Fig. 3 shows a comparison of experimental [19] and FE plot of load-axial displacement of hollow stub column (SHC 80x80x4-SC2). The location of ultimate load ( $P_u$ ) and deformation at  $P_u$  are also shown in Fig. 4 and good agreement was achieved between experimental and numerical results.



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It can be seen that the present modeling approach is able to achieve good agreement with the experimental values for LDSS SHC. This modeling approach is then used for parametric study of perforated LDSS circular columns.

## III. PARAMETRIC STUDY OF LDSS STIFFENED HOLLOW COLUMNS

In this study, the number of stiffeners (Ns), thickness of stiffener (t), and width of stiffener (w) has been adopted as the key parameters. To investigate the effects of these parameters on critical buckling load, various stiffened hollow columns by adopting different thicknesses viz., t = 3.25mm, 3.5mm, 3.75mm and 4mm. It may be mentioned that these thicknesses lie in the Class 4 classification (i.e. slender cross-section) of EN 1993-1-4 [29].

## 3.1 Variation of critical buckling load

Variation of load (P) vs axial deformation ( $\delta$ ) are shown in Fig. 5(a) for number of stiffeners, Ns = 0, 4 and 30, for w/D = 0.1, t = 3.75mm. Ns = 0 indicates unstiffened column. It can be seen from Fig. 5(a) that ultimate buckling load (P<sub>u</sub>) and  $\delta_u$  (deformation at P<sub>u</sub>) increases

with increasing values of Ns. In Fig 5 (b) the variations of normalized load (P/A<sub>g</sub>f<sub>y</sub>, where A<sub>g</sub> = gross cross-sectional area and f<sub>y</sub> = yield stress) with  $\delta$  is plotted. It can be seen from Fig. 5 (b) that the ultimate load is greater than the yield load (A<sub>g</sub>f<sub>y</sub>), indicating strain hardening beyond the yield, followed by local buckling of the column, even for column without stiffeners. It can also be seen that addition of stiffeners has improved the strain hardening effect. The increase in load carrying capacity may be associated with the increase cross-sectional area, together with the reduction of local buckling of the column due to the presence of stiffeners. The effect Ns, t and w on the normalised buckling strength  $\Gamma$  (= P<sub>u</sub>/P<sub>u(us)</sub> x 100%), where P<sub>u(us)</sub> are the buckling strength unstiffened columns respectively, are presented in the following sub-sections.





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Fig. 5: (a) Variation of normalised load with axial displacement (w/D=0.1, t=3.75mm) (b) Variation of load with axial displacement (w/D=0.1, t=3.75mm)

3.1.1 Effect of number of stiffeners (Ns)

Fig. 6 shows the variation of  $\Gamma$  with Ns for various values of t = 3.75 – 4.00 mm (w/D = 0.1). It can be observed from Fig. 6 that with increase in number of stiffeners, the load carrying capacity of the column increases, for all the thickness of the stiffeners. For the ranges of t considered, an increase of ultimate load for the stiffened column in comparison to the unstiffened column, increases from ~ 120 % to 180 %, when Ns is increased by 500% from Ns = 4. However, the effect of t is not very obvious for low value of Ns (i.e. Ns  $\leq \sim 10$ ), whilst for Ns  $\geq \sim 10$ ,  $\Gamma$ showed higher values for increasing values of t. This may be related to the increasing restrain of the local buckling of the column due to decrease in the separation distance between adjacent stiffeners (as a result of increasing Ns).



Fig. 6: Variation of  $\Gamma$  with Ns (w/D=0.1)

#### 3.1.2 Effect of thickness (t)

Variation of  $\Gamma$  with t for w/D = 0.1 is shown in Fig. 7 for various number of stiffeners. It can be seen from Fig. 7 that  $\Gamma$  increases in a nearly linear trend with increasing t for all values of Ns. It can be observed from Fig. 7 that the increase in buckling load when t is increased by ~20% from t = 3.25 mm to 4 mm are ~ 3.3 %, 3.7 %, 4.1 %, 7.4 %, 8.8%, 11.6 %, 23.3 % respectively, for Ns = 4, 6, 8, 10, 14, 18, 24 respectively. Thus, it can be seen that thickness has lesser effect at smaller values of Ns on the buckling capacity as compared to larger number of stiffeners. Again, this can be due to increase in spacing between adjacent stiffeners, leading to lesser control on the buckling behavior of the column.

## 3.1.3 Effect of width of stiffener (w)

Fig. 8 shows the variation of  $\Gamma$  with w/D (corresponding to w = 20 mm, 40 mm, 60 mm and 80 mm; D = 200 mm) for Ns = 6, 12, 14, 18, 20, 30 and t = 3.75 mm. It can be observed from Fig. 8 that the increase in buckling load are ~ 10.9%, 38 %, 57.9 %, 73.2 %, 83.1 %, 124.9 %, for Ns = 6, 12, 14, 18, 20, 30 respectively when w/D is increased from 0.1 to 0.4 i.e. 300 % increase in w. The lower influence of w at lower number of stiffener (N<sub>s</sub> = ~ 6) may be because of the increase in the separation distance between adjacent stiffeners (as a result of decreasing Ns), thereby having relatively control on the buckling capacity of the column. A high value of N<sub>s</sub> helps in better redistribution and mobilization of the applied load, with local buckling on the column being forced to confine between transverse spacing between stiffeners.



Fig.7: Variation of  $\Gamma$  with t (w/D=0.1)

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Fig. 8: Variation of  $\Gamma$  with w/D (t = 3.75 mm)

## IV. CONCLUSIONS

This paper presents a systematic study of the effects of stiffeners viz., spacing, size and thickness, on buckling load capacity of LDSS hollow stub columns subjected to axial load using Abaqus (2010). Based on the study following conclusions are summarized:

- 1. An increase of buckling load capacity has been observed with increasing number of stiffeners. The increase of buckling load followed a nearly linear trend. The buckling load for the stiffened column in comparison to the unstiffened column, increases from  $\sim 120$ % to 180%, when  $N_s$  is increased by 500% from  $N_s=4.$
- The increase in buckling load when thickness of the stiffener is increased by ~20% from 3.25 mm to 4mm is found to be ~ 3.3%, 3.7%, 4.1%, 7.4%, 8.8%, 11.6%, 23.3% respectively for Ns = 4, 6, 8, 10, 14, 18, 24 respectively.
- Increase in the width of the stiffeners also improved the load carrying capacity of the column. It is observed that when w/D is increased from 0.1 to 0.4 i.e. 300 % increase in w, the increase in buckling load capacity has been found to be ~ 10.9%, 38 %, 57.9 %, 73.2 %, 83.1 %, 124.9 %, for Ns = 6, 12, 14, 18, 20, 30 respectively.

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