

Some Application of Mathematical Analysis Using Integers

M. Somasundari¹, K. Janaki²

^{1,2}Department of Mathematics, Bharath University, Chennai, Tamil Nadu, India

ABSTRACT: In this paper, we study some application of mathematical analysis using integers. We define some applications on prime, Mersenne prime and Fermat prime.

KEYWORDS: Integers, prime, Mersenne prime, Fermat prime.

I. INTRODUCTION

This concepts related in some way to real numbers, so we begin our study of analysis with a discussion of the real-number system. Several methods are used to introduce real numbers. One method starts with positive integers 1,2,3,... as undefined concepts and uses them to build a largest system, the prime, Mersenne prime, Fermat prime & Fibonacci numbers. The set of all points between a and b is called an *interval*[10]. Sometimes it is important to distinguish between intervals which include their endpoints and intervals which do not. The notation $\{x: x \text{ satisfies } P\}$ will be used to designate the set of all real numbers x satisfy property p . Assume $a < b$. the *open interval* (a,b) is defined to be the set $(a,b) = \{x: a < x < b\}$. The *closed interval* $[a,b]$ is the set $\{x: a \leq x \leq b\}$. This section describes the integers, a special subset of \mathbf{R} . Before we define the integers it is convenient to introduce first the notion of an *inductive set*[9].

Definition 1.1. A set of real numbers is called an inductive set if it has the following two properties:

- The number 1 is in the set.
- For every x in the set, the number $x+1$ is also in the set.

For example, \mathbf{R} is an inductive set. So is the set \mathbf{R}^+ .

Definition 1.2. A *Prime Number* can be divided evenly only by 1, or itself. And it must be a whole number greater than 1. Example: 5 can only be divided evenly by 1 or 5, so it is a *prime* number[8]. But 6 can be divided evenly by 1, 2, 3 and 6 so it is *not a prime* number (it is a composite number).

Definition 1.3. *Mersenne prime* is a *prime* number of the form $M_n = 2^n - 1$. This is to say that it is a **prime** number which is one less than a power of two[7].

Definition 1.4. A *Fermat prime* is a *Fermat* number that is *prime*[6]. *Fermat primes* are therefore near-square *primes*.

Definition 1.5. The *Fibonacci sequence* is a set of *numbers* that starts with a one or a zero, followed by a one, and proceeds based on the rule that each *number* (called a *Fibonacci number*) is equal to the sum of the preceding two *numbers*[5].

II. MAIN RESULTS

In this section, We prove some applications on prime, Mersenne prime and Fermat prime using integers[4].

Theorem 2.1. there is no largest prime.

Proof. Suppose p is the largest prime. Then $p! + 1$ is NOT a prime. So,

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there exists a prime q such that

$$q \mid (p! + 1) \Rightarrow q \mid 1$$

which is impossible. So, there is no largest prime [3].

Theorem 2.2. If n is a positive integer then the algebraic identity is

$$a^n - b^n = (a - b) \sum_{k=0}^{n-1} a^k b^{n-1-k}$$

Proof. It suffices to show that

$$t^n - 1 = (t - 1) \sum_{k=0}^{n-1} t^k$$

Consider the right hand side, we have

$$\begin{aligned} (t-1) \sum_{k=0}^{n-1} t^k &= \sum_{k=0}^{n-1} t^{k+1} - \sum_{k=0}^{n-1} t^k \\ &= \sum_{k=1}^n t^k - \sum_{k=0}^{n-1} t^k \\ &= t^n - 1. \end{aligned}$$

Theorem 2.3. If $2^n - 1$ is a prime, prove that n is prime. A prime of the form $2^p - 1$, where p is prime, is called a Mersenne prime [2].

Proof. If n is not a prime, then say $n = ab$, where $a > 1$ and $b > 1$. So, we have

$$2^{ab} - 1 = (2^a - 1) \sum_{k=0}^{b-1} (2^a)^k$$

which is not a prime by **Theorem 2.2**. So, n must be a prime.

Remark 2.3. The study of Mersenne prime is important. It is related with so called Perfect number.

Theorem 2.4. If $2^{n+1} + 1$ is a prime, prove that n is a power of 2 [2]. A prime of the form $2^{2^m} + 1$ is called a Fermat prime. Hint. Use **Theorem 2.2**.

Proof. If n is not a power of 2, say $n = ab$, where b is an odd integer. So,

$$2^{a+1} \mid 2^{ab+1}$$

and $2^{a+1} < 2^{ab+1}$. It implies that $2^{a+1} \mid 2^{n+1}$ is not a prime. So, n must be a power of 2.

Remark 2.4. In the proof, we use the identity [1]

$$x^{2n-1} + 1 = (x+1) \sum_{k=0}^{2n-2} (-1)^k x^k$$

Proof. Consider

$$\begin{aligned} (x+1) \sum_{k=0}^{2n-2} (-1)^k x^k &= \sum_{k=0}^{2n-2} (-1)^k x^{k+1} + \sum_{k=0}^{2n-2} (-1)^k x^k \\ &= \sum_{k=0}^{2n-1} (-1)^{k+1} x^k + \sum_{k=0}^{2n-2} (-1)^k x^k \\ &= x^{2n-1} + 1. \end{aligned}$$

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