

Localized Dromion Solution Using Painleve Truncation Approach

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ABSTRACT: Using the methodology of Painleve Truncation Approach (PTA), an infinite integrable non linear dynamical system named long dispersive wave equation (2LDW) has been solved and localized dromion solution is obtained from generalized periodic solution.

KEYWORDS: Dromions, velocity potentials, localized solutions.

I. INTRODUCTION

The identification of dromions [1-6] has triggered a renewed interest in the study of integrable models in (2+1) dimensions. Dromions arise essentially by virtue of coupling the field variable to a mean field / potential, thereby preventing wave collapse in (2+1) dimensions and they can, in general undergo inelastic collision unlike one dimensional solitons. The identification of a large number of arbitrary functions in the solutions of (2+1) dimensional integrable models has only added to the richness in the structure of them and hence construction of localized excitations in (2+1) dimensions continues to be a challenging and rewarding contemporary problem. In this equation we developed a very simple and straight forward procedure to generate a rather extended class of generic solutions of physical interest. In this 2LDW equation we utilize the local Laurent expansion of the general solution and truncate it at the constant level term (painleve truncation approach) and obtain solutions in terms of arbitrary functions. Through this procedure we generate various periodic and exponentially localized solutions. The novelty here is that the solution is generated through a simple procedure. But the solution obtained is rich in structure because of the arbitrary functions [9-14].

II. DROMIONS

Dromions which originate at the crosspoint of the intersection of two nonparallel ghost solitons decay exponentially in all directions and are driven by lower dimensional boundaries or velocity potentials (arbitrary functions) in the system. It must be mentioned that the presence of these so-called boundaries enriches the structure of (2+1) dimensional integrable models leading to the formation of localized solutions like dromions

III. PAINLEVE TRUNCATION APPROACH

Painleve truncation approach is a methodology in which we are going to prove that the (2+1) dimensional long dispersive wave equation is a completely integrable infinite dimensional nonlinear dynamical system.

The recent interest in the investigation of integrable models in (2+1) dimensions can be attributed to the identification of dromions in the Davey-Stewartson I equation [3]. These dromions which originate at the cross point of the intersection of two nonparallel ghost solitons decay exponentially in all directions and are driven by lower dimensional boundaries or velocity potentials (arbitrary functions) in the system. It must be mentioned that the

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presence of these so called boundaries enriches the structure of (2+1) dimensional integrable models leading to the formation of localized solutions like dromions, positions, etc. [5]. Thus a judicious harnessing of these lower dimensional arbitrary functions [9-14] of space and time may give rise to new spatio-temporal patterns in higher dimensions besides throwing more light on their integrability.

In this way, a simple procedure is used to solve the (2+1) dimensional system namely Painleve Truncation Approach (PTA) [5]. This approach converts the given evolution equation into a multi linear equation in terms of non characteristic manifold. This multi linear equation can be solved in terms of lower dimensional arbitrary functions of space and time. Using this approach one can generate various generalized periodic and localized solutions. Through PTA (2+1) dimensional equation can be trilinearized. A four parameter special class of solutions involving three arbitrary functions in the variables naturally arises for this **trilinear** equation. Thus, there exists a possibility of obtaining more general solutions.[16,17]

Thus, there exists a possibility of obtaining more general solutions.
After solving the physical system such as 2LDW equation,

Considering the long dispersive wave equation (2LDW) [15,16]

$$\lambda_{qt} + q_{xx} - 2_{qv} = 0 \tag{1a}$$

$$\lambda_{rt} - r_{xx} + 2_{rv} = 0 \tag{1b}$$

$$(qr)_x = V_\eta \tag{1c}$$

the trilinear equation obtained is as follows.

$$V_{1\eta} = \Phi_{x\eta}^2 + (\Phi_x \Phi_{x\eta\eta} / \Phi_\eta) - (\Phi_x \Phi_{x\eta} \Phi_{\eta\eta} / \Phi_\eta^2) - (\Phi_{xx\eta} \Phi_{x\eta}^2 / \Phi_\eta) + \Phi_x \Phi_{x\eta\eta} \Phi_\eta - \Phi_x \Phi_{x\eta} \Phi_{\eta\eta} - \Phi_\eta^2 \Phi_{xx\eta} = 0 \tag{2}$$

IV.NOVEL EXACT SOLUTIONS OF 2LDW EQUATION

Now we can make use of the truncated Laurent expansion solution to obtain exact solutions of the 2LDW equation (1) for the variables q (x, η, t) and V (x, η, t). Taking into account our notation in equation (1). From the results, we find that the solution of the original variable takes the form[18,19]

$$q = [F_1(\eta) F_2(t) (\Phi_{(1x)}^{\frac{1}{2}}) / \Phi_1(x, t) + \Phi_{21}(\eta) + \Phi_{22}(t)] * e^{\int -1/2\lambda(\Phi_1 t + \Phi_{22} t) / \Phi_{1x} dx} \tag{3}$$

The form of V (x, η, t) remains the same as given in truncated Laurent expansion with the above general form of the solutions, we now identify interesting classes of exact solutions to the original equation including periodic and localized solutions by giving

specific forms for the three arbitrary functions $\Phi_1(x, t)$ $\Phi_{21}(\eta)$ $\Phi_{22}(t)$

A. Periodic solution and localized dromion solution

Let us now choose the arbitrary functions Φ_1 and $F_{1\eta}$ and F_{2t} to be Jacobian elliptic functions namely sn or cn functions. The motivation behind this choice of arbitrary functions stems from the fact that the limiting forms of these functions happen to be localized functions. Hence a choice of sn or cn functions can yield periodic solutions

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which are more general than exponentially localized solutions (dromions). We choose for example

$$\Phi_1(x, t) = n_1 + \text{sn}(ax + b\eta + c; m_1), \tag{4a}$$

$$\Phi_{21}(\eta) = \text{sn}(d\eta; m_2), \tag{4b}$$

$$\Phi_{22}(t) = \text{sn}(lt; m_3), \tag{4c}$$

$$F_1(\eta) = \text{cn}(g\eta; m_4) \tag{4d}$$

$$F_2(t) = \text{cn}(ht; m_5) \tag{4e}$$

$$q(x, \eta, t) =$$

$$\frac{[\text{cn}(g\eta; m_4)\text{cn}(ht; m_5)][\sqrt{\text{acn}(u_1; m_1)\text{dn}(u_1; m_1)}]}{(n_1 + \text{sn}(u_1; m_1) + \text{sn}(d\eta; m_2) + \text{sn}(lt; m_3))} \tag{4f}$$

where $u_1 = (ax + b\eta + c)$.

In (4f) the quantities m_1 and m_2 are the modulus parameters of the respective Jacobian elliptic functions while a, b and c are arbitrary constants. The corresponding expression for $|s(x, \eta, t)|^2$ takes the form

$$|q|^2 = \frac{[\text{cn}(g\eta; m_4)\text{cn}(ht; m_5)]^2 [\text{acn}(u_1; m_1)\text{dn}(u_1; m_1)]}{|4 + \text{sn}(d; m_2) + \text{sn}(lt; m_3)|^2} \tag{4g}$$

B. (1,1)dromion solution

As a limiting case of the periodic solution (27a) when m_1, m_2, m_3, m_4 and m_5 tends to 1, the above solution degenerates into an exponentially localized solution (dromion). Noting that

$$\text{cn}(u; 1) = \text{dn}(u; 1) = \text{sech}(u) \text{ and } \text{sn}(u; 1) = \tanh(u),$$

the limiting forms corresponding to (1,1) dromion takes the expressions

$$|q|^2 = \frac{[\text{asech}(g\eta)\text{sech}(ht)\text{sech}^2(ax + b\eta + c)]}{|n_1 + \tanh(ax + by + c) + \tanh(d\eta) + \tanh(lt)|^2}$$

A schematic form of (1,1) dromion for the parametric choice $a=2, b=3, c=1$ is shown in figure 4.3.2(b). Again note that the dromion travels with a unit velocity in a diagonal direction in the x - y plane. [20,21]

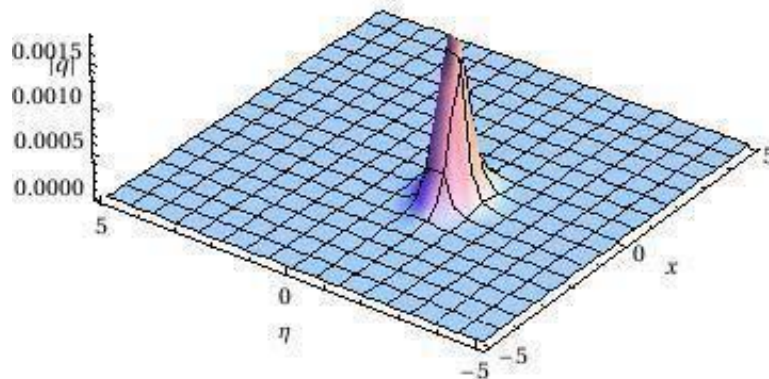


Figure 4.3.2 (b) - Localized Dromion Solution

V.CONCLUSION

In this paper, I has taken the (2+1) dimensional long dispersive wave equation which is solved already in terms of the arbitrary functions using the **Painleve Truncation Approach (PTA)** method. More general elliptic function solution such as limiting form of (1,1) **dromion** is generated from the trilinear equation. It will enrich the existence of lower dimensional arbitrary functions, which will help us further to construct further higher dromion solutions.

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