

Joint Channel and Carrier Frequency Offset Estimation in Decode and Forward Multi-Relay Cooperative Networks using the PX-EM Algorithm

D. Dhivya Bharathi¹, S. Sundaravadivelu²

Department of ECE, SSN College of Engineering, Chennai, India^{1,2}

Abstract—Cooperative networks employing multiple relays subject to multiple carrier frequency offsets (MCFOs), which lead to time varying channels and error in detection. This paper deals with joint estimation of channel and MCFOs in flat-fading decode and forward (DF) relaying cooperative networks using parameter expanded expectation maximization (PX-EM) algorithm. The proposed algorithm is to increase the convergence speed of existing expectation conditional maximization (ECM) and space alternating generalized expectation maximization (SAGE) algorithms. After estimation maximum likelihood (ML) decoding is done at the destination in the presence of MCFOs. Simulation results show that the number of iterations of proposed algorithm is lesser than the existing algorithms and also achieves the Cramer-Rao Bound Lower (CRLB) and finally ML decoding improves the performance of the system.

Index Terms—Cooperative networks, flat-fading channel, channel estimation, multiple carrier frequency offsets (MCFO) estimation, parameter expanded expectation maximization (PX-EM), maximum likelihood (ML) decoding, Cramer-Rao Lower Bound (CRLB).

I. INTRODUCTION

Demand for higher data rates and reliability increases in wireless communication which is affected by the effects of multipath fading, shadowing and path loss. These effects result in random variation in channel quality and also degrade the performance of the system. To overcome these effects transmit and receive diversity techniques are employed. Among various diversity techniques spatial diversity is more advantageous as it can be achieved without the channel knowledge. Multiple-Input-Multiple-Output (MIMO) systems are developed which achieves spatial diversity by employing multiple

transmit and receive antennas in single transceiver. Deploying multiple antennas in same device increases the cost and size of the device. To overcome this virtual MIMO technique is developed in which, multiple antennas are distributed among the devices in the network. Signal from the source is broadcasted to the multiple relays and processed in the relays and then is retransmitted to the destination. This technique is called cooperative relay communication [1]-[5].

Multiple relay nodes with its own local oscillator lead to MCFOs. MCFOs in the system result in phase error, inter carrier interference, effective power decrease and amplitude reduction.

CRLB which is used to assess the performance of the estimator and different algorithms for estimating MCFOs in MIMO are addressed in [7]-[10]. [6], [11] and [12] addresses CRLB and ML estimation algorithm for decode and forward (DF) and amplify and forward (AF) cooperative network. Iterative algorithms based on ECM and SAGE are developed in [10] for jointly estimating frequency offset and channel in distributed MIMO systems. Correlation based frequency offset estimation in MIMO is addressed in [9]. [11] deals with jointly estimating frequency offset and channel in AF cooperative OFDM systems. Various methods for compensating the effects of frequency offsets are addressed in [13]-[15]. [13] uses distributed linear convolutional space time codes to compensate the effects of MCFOs. This method increases the overhead and computational complexity. [18] uses ML decoding at the destination in the presence of impairments, which improves the performance of the system.

This paper addresses the joint estimation of MCFOs and flat fading channel gains in DF multi-relay cooperative networks. We assume perfect timing synchronization and jointly estimate MCFOs and channel gains at the destination. New algorithm based on PX-EM is derived. Proposed algorithm increases the speed of convergence and achieves the CRLB. ML decoding the

received signal in the presence of MCFOs improves the performance of the system. ML decoding compensates the effect of MCFOs in flat fading channel.

The organization of the paper is as follows:

- Section II presents the system model, training and data transmission method and structure of source transmitter, relay transceiver and destination receiver.
- Section III presents the overview of existing ECM and SAGE algorithm.
- Section IV describes the proposed PX-EM algorithm.
- Section V describes the ML decoding which is done at the destination in the presence of MCFO.
- Section VI presents the simulation results.

Notation: Superscripts $(\cdot)^*$, $(\cdot)^H$ and $(\cdot)^T$ denote the conjugate, the conjugate transpose and the transpose operators, respectively. Bold face small letters, e.g., x , are used for vectors, bold face capital alphabets, e.g., X , are used for matrix representation. \odot stands for Schur (element-wise) product, $|\cdot|$ is the modulus operator and $\|x\|$ represents the norm of a vector x . $E\{\cdot\}$ denotes the expected value of the corresponding sequence. $\Re\{\cdot\}$ and $\Im\{\cdot\}$ take the real and imaginary parts of a complex quantity.

II. SYSTEM MODEL

Consider a half-duplex space division multiple access (SDMA) single input single output (SISO) cooperative communication system with one source node, S, K relays, R_1, \dots, R_K , and a single destination node, D, as shown in Fig. 1.

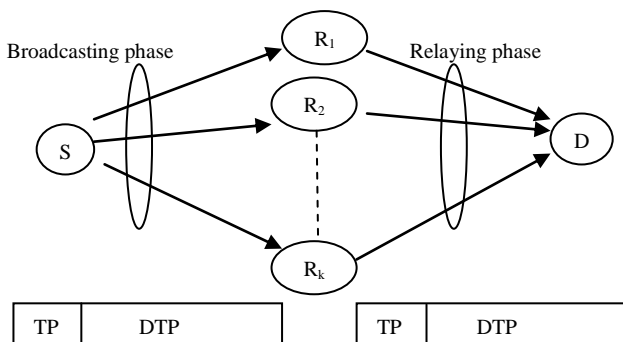


Fig. 1. System model for the cooperative network

Quasi-static flat fading channels are considered. Signal is transmitted from source node to destination node in two phases, broadcasting phase and relaying phase. In

broadcasting phase, signal from source is broadcasted to the relays and in relaying phase, signal from all relays are transmitted to the destination. Transmission of signals from source to relays to destination consists of training period (TP) and data transmission period (DTP). During TP training signal (TS) is transmitted and during DTP actual data to be transmitted is transmitted.

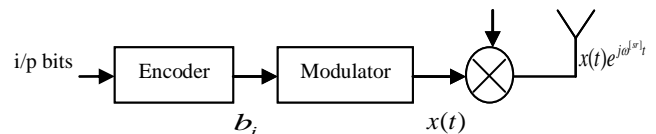


Fig. 2. Block Diagram for Source Transmitter

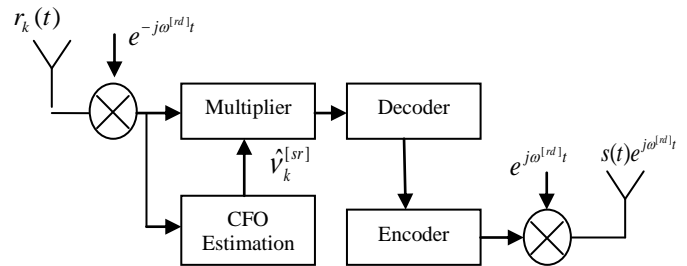


Fig. 3. Block Diagram for DF Relay Transceiver

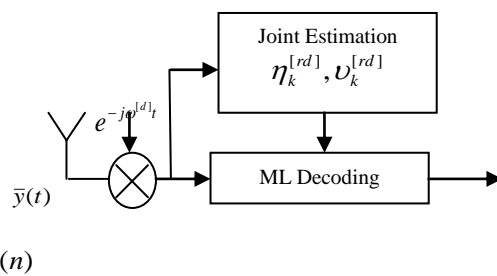


Fig. 4. Block Diagram for Destination Receiver

The block diagram for source transmitter, DF relay receiver and destination receiver are represented in Fig. 2, Fig. 3, and Fig. 4, respectively. The training and data transmission methods are given by following subsections.

1) Training Period: Consider, number of symbols in a frame as L and number of samples in each symbol as L. During TP, TS is up-converted by the frequency $\omega^{[sr]}$, and then broadcasted to all the relays. At the relays, the received signal is down-converted by the frequency $\omega_k^{[r]}$. The received signal is given by,

$$r_k(i) = \rho_k \sum_{n=0}^{L-1} t^{[s]}(n) e^{j2\pi v_k^{[sr]} n} + u_k(i) \quad (1)$$

where, $v_k^{[sr]}$ is the carrier frequency offset between S and R_k , ρ_k denotes the unknown channel gain from S to R_k and is assumed to be distributed as $CN(0, \sigma_p^2)$. $u_k(i)$ is the additive white Gaussian noise (AWGN) from S to R_k with variance σ_{uk}^2 . At the relay, $v_k^{[sr]}$ is estimated using the methods used in SISO. DF relays forward only the perfectly decoded signal to the destination and hence perfect synchronization is assumed at the relays. The output of the multiplier at relay is given by,

$$s_k(t) = t_k^{[r]}(t) \rho_k e^{j2\pi v_k^{[sr]} t} \sum_{n=0}^{L-1} t^{[s]}(n) + t_k^{[r]}(t) u_k(t) \quad (2)$$

where, $\hat{v}_k^{[sr]}$ is the estimated CFO at the relay. The signal received at the destination during TP is given by

$$y = [\Lambda_1 t_1^{[r]}, \dots, \Lambda_K t_K^{[r]}] \eta + w \quad (3)$$

where,

- $y = [y(0), \dots, y(L-1)]^T$ is the received signal vector at D.
- $\Lambda_k = [e^{j2\pi \hat{v}_k^{[rd]}(0)}, \dots, e^{j2\pi \hat{v}_k^{[rd]}(L-1)}]$
- $t_k^{[r]} = [t_k^{[r]}(0), \dots, t_k^{[r]}(L-1)]^T$
- $\eta = [\eta_1, \dots, \eta_K]^T$ is the unknown channel gain from R to D
- $w = [w(0), \dots, w(L-1)]^T$ is the AWGN at D
- $v_k^{[rd]}$ is the CFO from R to D
- Number of samples, $i = \{0, 1, \dots, L-1\}$
- Number of symbols, $n = \{0, 1, \dots, L-1\}$

2) Data Transmission Period: During DTP, the signal received at the destination is given by

$$y = \sum_{k=1}^K \eta_k \Lambda_k s + w \quad (4)$$

III. OVERVIEW OF ECM AND SAGE ALGORITHM

The ML estimate for channel and CFO estimation is given by [16],

$$\hat{v} = \arg \max_{\hat{v}} y^H \Omega (\Omega^H \Omega)^{-1} \Omega^H y \quad (5)$$

$$\hat{\eta} = (\Omega^H \Omega)^{-1} \Omega^H y \quad (6)$$

ML estimation increases the computational complexity and hence we use expectation maximization (EM) algorithm which iteratively maximizes ML estimate. In EM algorithm, the observed data y , is the incomplete data set and we define the complete or hidden data set x , from which the parameter to be estimated is defined in parameter space θ , and the estimate is given by,

$$\hat{\theta} = \arg \max_{\theta} f(y | \theta) \quad (7)$$

Estimation is done in two steps, E (expectation)-step and M (maximization)-step. E-step is given by,

$$Q(\theta | \hat{\theta}^{[m]}) = E\{\log f(x | \theta) | y, \hat{\theta}^{[m]}\} \quad (8)$$

M-step is to obtain the estimate of the parameter to maximize the above obtained function and is given by,

$$\hat{\theta}^{[m+1]} = \arg \max_{\theta} Q(\theta | \hat{\theta}^{[m]}) \quad (9)$$

It is evident from [19,10] that the M-step of EM is complicated and also the rate of convergence of EM algorithm is slow. So ECM and SAGE algorithms are used for increasing the speed of convergence where the parameter space is divided into blocks $\theta = [\theta_1^T, \theta_2^T, \dots, \theta_i^T]^T$ and the complete data space is defined as, $z = [z_1^T, \dots, z_k^T]^T$.

1) ECM: In ECM, the parameter space is divided into smaller blocks so that the complicated M-step is avoided. Common hidden data set is defined and each parameter block is updated. The E-step and M-step are given as follows,

$$\text{E-step: } Q(\theta | \hat{\theta}^{[m]}) = E\{\log f(z_k) | y_k, \hat{\theta}^{[m]}\} \quad (10)$$

$$\text{M-step: } \hat{\theta}^{[m+1]} = \arg \max_{\theta} Q(\theta | \hat{\theta}^{[m]}) \quad (11)$$

Following the derivation of ECM in [8] and [18], the summary for estimating the channel and MCFO is given in Table. I.

2) SAGE: In SAGE, parameter space is divided into smaller blocks and hidden space is defined for each smaller block and parameter in each block is updated while keeping the remaining blocks fixed with latest updated values. The E-step and M-step are given as follows,

$$\text{E-step: } Q(\theta_i | \hat{\theta}^{[m]}) = E\{\log f(z_k) | y_k, \hat{\theta}^{[m]}\} \quad (12)$$

$$\text{M-step: } \hat{\theta}_i^{[m+1]} = \arg \max_{\theta_i} Q(\theta_i | \hat{\theta}_i^{[m]}) \quad (13)$$

Following the derivation of SAGE in [8] and [18], the summary for estimating the channel and MCFO is given in Table. II.

IV. PROPOSED ALGORITHM

The proposed PX-EM algorithm is to increase the rate of convergence of EM algorithms by expanding the complete data set and then estimate the parameter from the expanded complete data set. For expanding the complete data set, known value a_0 is added to the parameter to be estimated. PX-EM

TABLE I

Summary of ECM algorithm for estimating MCFO and channel gain

1. Initialization

Obtain $\hat{\nu}_k^{[0]}, \hat{\eta}_k^{[0]}$ for $k = 1, 2, \dots, K$

2. ECM

for $m = 0, 1, \dots$

for $k = 1, 2, \dots, K$

Compute

$$\hat{z}_k^{[m]} = \hat{\eta}_k^{[m]} \hat{\Lambda}_k^{[m]} t_k^{[r]} + \beta_k (y - \sum_{k=1}^K \hat{\eta}_k^{[m]} \hat{\Lambda}_k^{[m]} t_k^{[r]})$$

for $k = 1, 2, \dots, K$

$$\hat{\nu}_k^{[m+1]} = \hat{\nu}_k^{[m]} - \frac{\sum_{i=0}^{L-1} i \Im \{ (\hat{z}_k^{[m]}(i))^* t_k^{[r]}(i) \hat{\eta}_k^{[m]} e^{j2\pi i \hat{\nu}_k^{[m]}} \}}{\sum_{i=0}^{L-1} i^2 \Re \{ (\hat{z}_k^{[m]}(i))^* t_k^{[r]}(i) \hat{\eta}_k^{[m]} e^{j2\pi i \hat{\nu}_k^{[m]}} \}}$$

$$\hat{\eta}_k^{[m+1]} = \frac{1}{\sum_{i=0}^{L-1} |t_k^{[r]}(i)|^2} \sum_{i=1}^{L-1} \frac{\hat{z}_k^{[m]}(i) t_k^{*[r]}(i)}{e^{j2\pi i \hat{\nu}_k^{[m+1]}}}$$

MCFOs and channel parameters, this paper deals with expanding complete data set of those two algorithms. Generally, E-step and M-step for PX-EM is given by,

E-step:

$$Q((\theta + a_0) | \hat{\theta}^{[m]}) = E\{\log f(x | (\theta + a_0) | y, \hat{\theta}^{[m]})\} \quad (14)$$

$$M\text{-step: } \hat{\theta}^{[m+1]} = \arg \max_{\theta} (Q((\theta + a_0) | \hat{\theta}^{[m]}) - a_0) \quad (15)$$

Where, a_0 is the known value added to the parameter to be estimated.

1) PX-ECM: For estimating MCFO and channel gains the E-step is given by,

$$\hat{z}_k^{[m]} = (\hat{\eta}_k^{[m]} + a_0) \hat{\Lambda}_k^{[m]} \Lambda_k^{[a_0]} t_k^{[r]} + \beta_k (y - \sum_{k=1}^K (\hat{\eta}_k^{[m]} + a_0) \hat{\Lambda}_k^{[m]} \Lambda_k^{[a_0]} t_k^{[r]})$$

$$\text{where, } \Lambda_k^{[a_0]} = [e^{j2\pi a_0(0)}, \dots, e^{j2\pi a_0(L-1)}] \quad (16)$$

$$a_0 = \frac{y(0)}{y(L-1)} \quad (18)$$

Summary for estimating the channel and MCFO is given in Table. III.

TABLE II

Summary of SAGE for estimating MCFO and channel gain

1. Initialization

Obtain $\hat{\nu}_k^{[0]}, \hat{\eta}_k^{[0]}$ for $k = 1, 2, \dots, K$

2. ECM

for $m = 0, 1, \dots$

for $k = 1, 2, \dots, K$

Compute

$$\hat{z}_k^{[m]} = y - \sum_{k=1}^K \hat{\eta}_k^{[m]} \hat{\Lambda}_k^{[m]} t_k^{[r]}$$

for $k = 1, 2, \dots, K$

$$\hat{\nu}_k^{[m+1]} = \hat{\nu}_k^{[m]} - \frac{\sum_{i=0}^{L-1} i \Im \{ (\hat{z}_k^{[m]}(i))^* t_k^{[r]}(i) \hat{\eta}_k^{[m]} e^{j2\pi i \hat{\nu}_k^{[m]}} \}}{\sum_{i=0}^{L-1} i^2 \Re \{ (\hat{z}_k^{[m]}(i))^* t_k^{[r]}(i) \hat{\eta}_k^{[m]} e^{j2\pi i \hat{\nu}_k^{[m]}} \}}$$

$$\hat{\eta}_k^{[m+1]} = \frac{1}{\sum_{i=0}^{L-1} |t_k^{[r]}(i)|^2} \sum_{i=1}^{L-1} \frac{\hat{z}_k^{[m]}(i) t_k^{*[r]}(i)}{e^{j2\pi i \hat{\nu}_k^{[m+1]}}}$$

TABLE III

Summary of PX-ECM algorithm for estimating MCFO and channel gain

1. Initialization

Obtain $\hat{\nu}_k^{[0]}, \hat{\eta}_k^{[0]}$ for $k = 1, 2, \dots, K$

2. ECM

for $m = 0, 1, \dots$

for $k = 1, 2, \dots, K$

Compute

$$\hat{z}_k^{[m]} = (\hat{\eta}_k^{[m]} + a_0) \hat{\Lambda}_k^{[m]} \Lambda_k^{[a_0]} t_k^{[r]} + \beta_k (y - \sum_{k=1}^K (\hat{\eta}_k^{[m]} + a_0) \hat{\Lambda}_k^{[m]} \Lambda_k^{[a_0]} t_k^{[r]})$$

for $k = 1, 2, \dots, K$

$$\hat{\nu}_k^{[m+1]} = \hat{\nu}_k^{[m]} - \frac{\sum_{i=0}^{L-1} i \Im \{ (\hat{z}_k^{[m]}(i))^* t_k^{[r]}(i) (\hat{\eta}_k^{[m]} + a_0) e^{j2\pi i \hat{\nu}_k^{[m]} + a_0} \}}{\sum_{i=0}^{L-1} i^2 \Re \{ (\hat{z}_k^{[m]}(i))^* t_k^{[r]}(i) (\hat{\eta}_k^{[m]} + a_0) e^{j2\pi i \hat{\nu}_k^{[m]} + a_0} \}}$$

can be applied to any variants of EM algorithm. ECM and SAGE are commonly used in literature for estimating

$$\hat{\eta}_k^{[m+1]} = \frac{1}{\sum_{i=0}^{L-1} |t_k^{[r]}(i)|^2} \sum_{i=1}^{L-1} \frac{\hat{z}_k^{[m]}(i) t_k^{*[r]}(i)}{e^{j2\pi\hat{v}_k^{[m+1]} + a_0}}$$

if $\hat{v}_k^{[m+1]} = \hat{v}_k^{[m]}$, $\hat{\eta}_k^{[m+1]} = \hat{\eta}_k^{[m]}$
 $\hat{v}_k = \hat{v}_k^{[m+1]} - a_0$, $\hat{\eta}_k = \hat{\eta}_k^{[m+1]} - a_0$

Summary of the ML decoding algorithm is given in Table.V. following [18],

TABLE V
Summary of ML decoding algorithm

for $i = 0, 1, \dots, L-1$

$$d_p(i) = \sum_{k=1}^K \hat{\eta}_k e^{j2\pi\hat{v}_k} c_p(i)$$

$$e_p(i) = |y(i) - d_p(i)|^2$$

$$\varepsilon_p = \arg \min e_p(i)$$

$$\hat{s}(i) = a_{\varepsilon_p}(0)$$

2) PX-SAGE: For estimating MCFO and channel gains the E-step is given by,

$$\hat{z}_k^{[m]} = y - \sum_{k=1}^K (\hat{\eta}_k^{[m]} + a_0) \hat{\Lambda}_k^{[m]} \Lambda_k^{[a_0]} t_k^{[r]} \quad (17)$$

Summary for estimating the channel and MCFO is given in Table. IV.

TABLE IV
Summary of PX-SAGE algorithm for estimating MCFO and channel gain

1. Initialization
Obtain $\hat{v}_k^{[0]}$, $\hat{\eta}_k^{[0]}$ for $k = 1, 2, \dots, K$
2. ECM
for $m = 0, 1, \dots$
for $k = 1, 2, \dots, K$
Compute

$$\hat{z}_k^{[m]} = y - \sum_{k=1}^K (\hat{\eta}_k^{[m]} + a_0) \hat{\Lambda}_k^{[m]} \Lambda_k^{[a_0]} t_k^{[r]}$$
for $k = 1, 2, \dots, K$

$$\hat{v}_k^{[m+1]} = \hat{v}_k^{[m]} - \frac{\sum_{i=0}^{L-1} i \Im\{(\hat{z}_k^{[m]}(i) t_k^{[r]}(i) (\hat{\eta}_k^{[m]} + a_0) e^{j2\pi\hat{v}_k^{[m]} + a_0}\}}{\sum_{i=0}^{L-1} i^2 \Re\{(\hat{z}_k^{[m]}(i) t_k^{[r]}(i) (\hat{\eta}_k^{[m]} + a_0) e^{j2\pi\hat{v}_k^{[m]} + a_0}\}}$$

$$\hat{\eta}_k^{[m+1]} = \frac{1}{\sum_{i=0}^{L-1} |t_k^{[r]}(i)|^2} \sum_{i=1}^{L-1} \frac{\hat{z}_k^{[m]}(i) t_k^{*[r]}(i)}{e^{j2\pi\hat{v}_k^{[m+1]} + a_0}}$$
if $\hat{v}_k^{[m+1]} = \hat{v}_k^{[m]}$, $\hat{\eta}_k^{[m+1]} = \hat{\eta}_k^{[m]}$
 $\hat{v}_k = \hat{v}_k^{[m+1]} - a_0$, $\hat{\eta}_k = \hat{\eta}_k^{[m+1]} - a_0$

VI. SIMULATION RESULTS

Simulations are carried out using MATLAB software. Performances are evaluated in terms of mean square error (MSE) and bit error rate (BER). Quadrature phase shift keying (QPSK) modulation is used. Length of TS, L=64 and number of relays K=4 is considered. Carrier frequency offsets are assumed to be uniformly distributed over the range (-0.25, 0.25). Channel gains specified for simulation are $\eta = [0.7820 + j0.6233, 0.9474 - j0.3203, -0.2413 + j0.724j, 0.5141 - j0.893j]$ similar to [9]-[11].

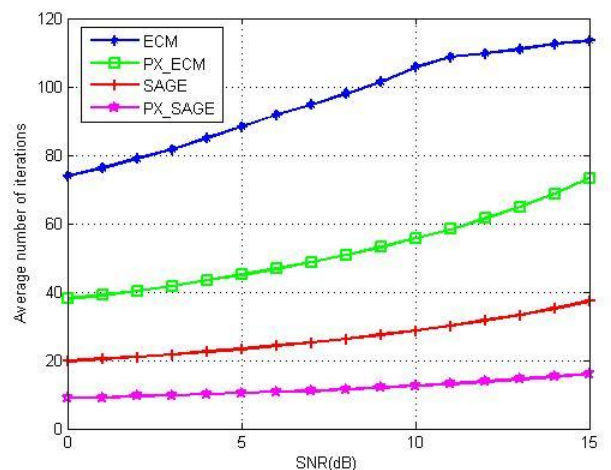


Fig. 5. Average number of iterations for ECM, PX-ECM, SAGE, PX-SAGE.

V. ML DECODING

ML decoding reduces the probability of decoding the incorrect code word. Here the signal received during DTP in the presence of impairments is ML decoded to compensate the impacts of channel and MCFOs.

Fig. 5 shows the average number of iterations required for convergence of existing ECM and SAGE algorithm and is compared with the proposed PX-ECM and PX-SAGE algorithms. The average number of iterations

increases as the SNR value increases. It is evident that the proposed algorithms converge faster than the existing algorithms.

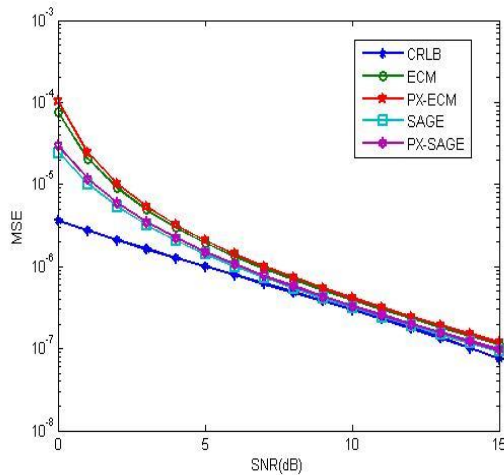


Fig. 6. Comparison of MSE of frequency offsets of ECM, PX-ECM, SAGE and PX-SAGE

Performance of estimators is given in terms of MSE as MSE is directly proportional to the variance of the estimator. MSE performance of carrier frequency offsets of ECM, PX-ECM, SAGE and PX-SAGE are simulated and compared with CRLB in Fig. 6. Derivation of CRLB is referred from [7]. It is evident that the proposed algorithms reach the CRLB. Similarly MSE of channel gains are also simulated and the result is shown in Fig. 7.

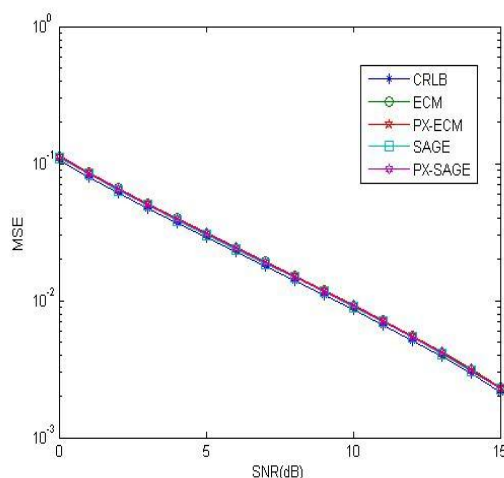


Fig. 7. Comparison of MSE of channel gains of ECM, PX-ECM, SAGE and PX-SAGE

After estimating the offsets during TP, the signal received during the DTP in the presence of impairments is ML decoded. It is evident from the Fig.8, that the BER of ML decoded signal is much better than the signal with offsets. At SNR of 15dB BER of signal with offsets is greater than 10^{-1} . For ML decoded signal BER at SNR of 15dB is 10^{-3} nearly.

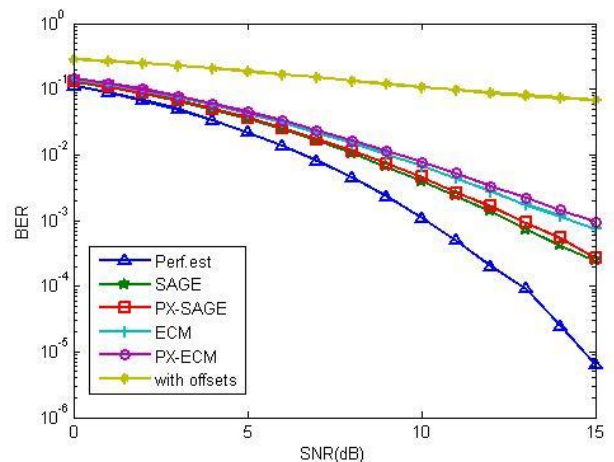


Fig. 8. Comparison of BER performance of ECM, PX-ECM, SAGE and PX-SAGE

VII. CONCLUSION

This paper deals with jointly estimating the carrier frequency offsets and channel gains in the DF multi-relay cooperative networks. Training and data transmission methods affected by offsets are presented. Proposed PX-EM algorithm converges faster than the existing algorithm. Also the proposed algorithm reaches the CRLB. Finally ML decoding the signal during DTP in the presence of impairments reduces the BER. MSE and BER performances are shown in simulation results.

REFERENCES

- [1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperative diversity - part I: system description; part II: implementation and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1948, Nov. 2003.
- [2] J. Yindi and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 3, no. 6, pp. 473–476, Apr. 2007.
- [3] Y. Zheng, H. Mehrpouyan, and S. D. Blostein, "Application of phase shift in coherent multi-relay MIMO communications," in *Proc IEEE Inter. Conf. on Commun. (ICC)*, Germany, 2009.
- [4] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.
- [5] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage

- behavior," IEEE Trans. Inform. Theory, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [6] —, "Bounds and algorithms for multiple frequency offset estimation in cooperative networks," IEEE Trans. Wireless Commun., vol. 10, no. 4, pp.1300–1311, Apr. 2011.
- [7] O. Besson and P. Stoica, "On parameter estimation of MIMO flat-fading channels with frequency offsets," IEEE Trans. Signal Processing, vol. 51, no. 3, pp. 602–613, Mar. 2003.
- [8] T. Pham, A. Nallanathan, and Y. Liang, "Joint channel and frequency offset estimation in distributed MIMO flat-fading channels," IEEE Trans. Wireless Commun., vol. 7, no. 2, pp. 648–656, Feb. 2008.
- [9] Y. Yao and T. Ng, "Correlation-based frequency offset estimation in MIMO system," in Proc. IEEE Vehicular Tech. Conf. (VTC), U.S.A, Oct. 2003.
- [10] Z. Lu, J. Li, L. Zhao, and J. Pang, "Iterative parameter estimation in MIMO flat-fading channels with frequency offsets," in Proc IEEE Inter. Conf. on Advanced Inform. Network. and Appl. (AINA), Austria, Apr. 2006.
- [11] Q. Jiang, K. Zhang, J. Liu, and G. Shen, "Joint carrier frequency offset and channel estimation for AF cooperative OFDM systems," Springer Journal Wireless Personal Commun., vol. 55, pp. 1–27, Oct. 2010.
- [12] F. Gao, T. Cui, and A. Nallanathan, "On channel estimation and optimal training design for amplify-and-forward relay networks," IEEE Trans. Wireless Commun., vol. 7, no. 5, pp. 1907–1916, May 2008.
- [13] H. Wang, X.-G. Xia, and Q. Yin, "Computationally efficient equalization for asynchronous cooperative communications with multiple frequency offsets," IEEE Trans. Wireless Commun., vol. 8, no. 2, pp. 648–655, Feb. 2009.
- [14] F. Tian, X.-G. Xia, and P. C. Ching, "Signal detection in a cooperative communication system with multiple CFOs by exploiting the properties of space-frequency codes," in Proc. IEEE Inter. Conf. on Commun. (ICC), China, Jun. 2008.
- [15] D. Veronesi and D. L. Goeckel, "Multiple frequency offset compensation in cooperative wireless systems," in Proc. IEEE Global Commun. Conf (GLOBECOM), U.S.A, Dec. 2006.
- [16] Y. Tian, X. Lei, Y. Xiao, and S. Li, "ML synchronization algorithm and estimation bounds for cooperative systems," in Proc IEEE Pacific Asia Conf. on Circuits, Commun. and Systems (PACCS), China, Aug. 2010.
- [17] T. A. Fesler and A. O. Hero, "Space-alternating generalized expectation maximization algorithm," IEEE Trans. Signal Processing, vol. 42, no. 10, pp. 2664–2677, Oct. 1994.
- [18] Ali A. Nasir, Hani Mehrpouyan, Steven D. Blostein, Salman Durrani, and Rodney, "Timing and Carrier Synchronization with Channel Estimation in Multi-Relay Cooperative Networks," IEEE Trans. Signal Processing, 2011.