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# Hermitian Precoding For Distributed MIMO Systems with Imperfect Channel State Information

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ABSTRACT— A distributive multiple-input multiple output (MIMO) system is taken, in which multiple transmitters co-operatively serve a common receiver under the assumption of imperfect and individual channel state information. Due to practical constraint, in this paper, the individual channel state information is assumed to be imperfect. It leads to performance degradation when compared to a channel with perfect channel knowledge. It is usually very costly to acquire full channel state information at the transmitter (CSIT). In such a case, especially for large-scale antenna systems, it is assumed to have individual CSIT (I-CSIT), i.e., each transmitter has perfect CSI of its own link but only slow fading factors of the others. A linear Hermitian precoding is used for transforming the equivalent channel, including a physical channel and a precoder, into a Hermitian matrix form. The performance analysis and optimization of this Hermitian precoding scheme for an imperfect channel is presented in this paper.

**KEYWORDS**— Distributed MIMO systems, Imperfect channel state information, Large scale antenna system, Cooperative cellular systems, Asymptotic performance, Hermitian precoding.

## I. INTRODUCTION

In a distributive MIMO system, multiple transmitters at different places cooperatively send common message to

a single receiver. Transmitter and receiver are equipped with multiple antennas. In a co-operative cellular system, several adjacent base stations simultaneously serve a mobile terminal in the downlink transmission [1]. The

wireless channels capacity is reduced by an imperfect knowledge of the channel state information (CSI) at the receiver [2],[3]. The transmitter optimization is considered for a point-to-point communication system with multiple base-stations cooperating to transmit to a single user [6]. In distributive multiple-input multipleoutput (MIMO) systems, with no Channel State Information at the transmitter (CSIT) and imperfect CSI at the receiver (CSIR), capacity degradations due to channel estimation errors may significantly compromise performance, and set severe limits to the capacity growing with the increasing signal-to-noise ratio (SNR) and number of transmit and receive antennas. Block diagonalization (BD) is a linear precoding technique that eliminates the inter-user interference in downlink multiuser MIMO systems [7]-[9]. A MIMO channel with a set of transmit power constraints corresponding to individual BSs in the multi-cell system is considered [10]. Assumed to have no channel state information at the transmitter, distributed space-time coding is being proposed for efficient and fast transmission [12]–[14]. In full CSIT every transmitter perfectly knows all the channel state information (CSI). But in practice it is too expensive to acquire full CSIT. In this paper, individual and imperfect CSIT (I-CSIT) is studied, in which each transmitter has CSI of its own link but only the slow fading factors of the others. The performance of the I-CSIT is very close to the system capacity with full CSIT as referred in [1].

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An important aspect to be considered in this paper is imperfect channel state information at the base station which has a significant effect on precoding performance. The impact of CSI accuracy on the precoding performance both analytically and numerically via the average achievable rate is analyzed.

The rest of the paper is organized as follows. In Section 2, the system model with Channel state information and different power constraints are presented. The hermitian precoding scheme is presented in Section 3 and its performance is analyzed and optimized in Section 4. Simulation results and conclusions are provided in Sections 5 and 6, respectively.

The notation <sup>2</sup> is a shorthand of  $AA^{H}$ ; and represents the Euclidean norm of vector a.

## II. SYSTEM MODEL

## A. System Model

Consider a distributed MIMO system, in which *K* transmitters cooperatively transmit common messages to a single receiver. A multi-user communication system operating in the downlink is considered. Each transmitter has *N* antennas and the receiver has *M* antennas. We assume i.i.d. Rayleigh flat fading channels  $\mathbf{H}_k$ , k = 1, ... K, from the base station to the individual users. The k<sup>th</sup> user receives

$$r=$$
 (1)

where  $\mathbf{r}$  is an *M*-by-1 received signal vector,  $\mathbf{x}_k$  is an *N*-by-1 signal vector sent by the transmitter k,  $\mathbf{H}_k$  is an *M*-by-*N* channel transfer matrix for link between the transmitter k and the receiver, and  $\mathbf{n} = CN(0, \sigma^2 \mathbf{I})$  is an additive white Gaussian noise vector. It is assumed that each transmitter k has an individual power constraint of

E 
$$P_k$$
  $k=1,\ldots,K$ , (2)

where  $P_k$  is the maximum transmission power of transmitter *k*. Assuming that the receiver perfectly knows  $\{H_k/k = 1, \ldots, K\}$ .

Assuming that all channels between the transmitters and the receivers are Rayleigh fading, so that the entries of the *M*-by-*N* channel matrix  $H_k$  are i.i.d. drawn from CN(0, 1).  $H_kH_k^{H}$  is a unitarily invariant and central *Wishart* matrix.  $H_kH_k^{H}$  is decomposed as

$$\boldsymbol{H}_{k}\boldsymbol{H}_{k}^{\mathrm{H}} = \boldsymbol{U}_{k}\boldsymbol{D}_{k}\boldsymbol{D}_{k}^{\mathrm{H}}\boldsymbol{U}_{k}^{\mathrm{H}}$$
(3a)

with  $U_k$  is a *Haar* matrix independent of the *M*by-*N* diagonal matrix  $D_k$ . For a diagonal matrix of size *M*-by-*N*, the non-zero entries are located at the  $(i, i)^{\text{th}}$ position with  $i = 1, ..., \min\{M,N\}$ . A random square matrix *U*, is a *Haar* matrix if it is uniformly distributed on the set of all the unitary matrices of same size as *U*. The singular value decomposition (SVD) of  $H_k$  can be given as

$$H_k = U_k D_k V_k^{H}$$
(3b)

## B. Channel State Information

Assuming that the base station has imperfect CSI and is given below:

$$H_k = k + E_k.$$

where  $_{k}$  is the estimated individual channel state information(I-CSIT) and  $\mathbf{E}_{k}$  is the channel estimated error matrix whose elements are where is the channel estimation error variance.

Transmitter 1



Fig.1. System model for a distributed perfect MIMO channel.

The elements of the CSI matrix  $_k$  are hence i.i.d. CN(0, 1 - ). The perfect CSI at the base station corresponds to  $\rightarrow 0$ .  $_k$  behave like an interference channel. The singular value decomposition(SVD) of  $_k$  can be given as,

$$_{k}=U_{k}(D+)_{k}V_{k}^{H}$$
(4b)

## III. TRANSMISSION STRATEGY WITH I-CSIT

Linear precoding is used to shape the channel input covariance matrix, in order to achieve MIMO capacity [15]. The conventional SVD water filling (SVD-WF) approach [15] can be seen as a special case of distributed linear precoding technique.

### A. Modeling of the Transmit Signal

Assuming that the transmitters share the same data to be transmitted. The transmitters use the same or different codebooks in channel coding. The transmitted signals from different transmitters may be either correlated or uncorrelated.  $a_0$  is used to represent the correlated signal

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component which is shared by all the transmitters, and  $a_k$  is used to represent the uncorrelated signal component in each transmitter k (for k = 1, ..., K), where  $\{a_k\}$  are M-by-1 random vectors with the entries independently drawn from CN(0, 1). By definition,  $E[a_k a_k^H] = I$  and  $E[a_k a_j^H] = 0$ , k, j = 0, 1, ..., K, k j. The transmitted signal of transmitter k can be given as

$$\begin{array}{ll} \boldsymbol{x}_k = \boldsymbol{F}_k \boldsymbol{a}_0 + \boldsymbol{G}_k \boldsymbol{a}_k, & k=1,\ldots,K, \\ (5) \end{array}$$

where  $F_k$  and  $G_k$  are *N*-by-*M* precoding matrices of transmitter *k* designed to exploit the available CSI.With(5), the received signal in (1) is rewritten as

$$r = \sum_{k=1}^{K} \hat{H}_k (F_k a_0 + (6))$$

Further in this paper  $\{F_k\}$  and  $\{G_k\}$  is designed to enhance the system performance.

#### B. Distributive Precoder Design

Consider a transmitter k, which has individual CSIT (I-CSIT) that is, if it knows only  $_k$ . A precoding design is *distributive* if it has individual CSIT only, which shows that both  $F_k$  and  $G_k$  in (6) are determined by  $_k$ , i.e.,

$$\boldsymbol{F}_k = f_k ( \ k)$$
 and  $\boldsymbol{G}_k = g_k ( \ k), \qquad k = 1, \dots K,$ 
(7)

where  $f_k(\cdot)$  and  $g_k(\cdot)$ ,  $k = 1, \ldots, K$ , are precoding functions to be optimized. The distributive precoding functions must be optimized  $\{f_k(\cdot)\}$  and  $\{g_k(\cdot)\}$ ,  $k = 1, \ldots, K$ , so that the average achievable rate of the system is maximized in (6) under the individual transmitter power constraints in (2). This problem can be formulated a,

$$\max \quad E[\operatorname{logdet}(I \qquad \frac{1}{\sigma^2}(| H_k f_k)] + \sum_{k=1}^{\kappa} |H_k f_k \rangle ]$$
(8a)

s.t. 
$$\operatorname{tr} \{ 2^{2} + / 2^{2} \} \leq (8b)$$

where the expectation is taken over the joint distribution of  $\{k\}$ , and the power constraint is for every realization of  $\{k\}$ .

## IV. HERMITIAN PRECODING

In this section, a distributive Hermitian precoding technique for the system in (6) is discussed and its local optimality is proved. Further the power control problem for the proposed Hermitian precoder is studied and its asymptotic performance is analyzed for large-scale MIMO systems.

#### A. Basic Precoder Structure

Let  $U_k$  and  $V_k$  be given by the SVD of in (4b). As  $U_k$  and  $V_k$  are invertible, the precoders in (7) can be Written as follows, k = 1, ..., K.

$$F_{k} = f_{k}( {}_{k}) = V_{k}W_{k}U_{k}^{\mathrm{H}}$$

$$G_{k} = g_{k}( {}_{k}) = V_{k}\Sigma_{k}U_{k}^{\mathrm{H}}$$

$$(9a)$$

$$(9b)$$

the sizes of  $W_k$  and  $\Sigma_k$  are both *N*-by-*M* and they depend on  $_k$  due to the I-CSIT assumption. The optimal  $\Sigma_k$ ' is given by  $\Sigma_k = \Sigma_k Q_k$ , where  $\Sigma_k$ ' is a real diagonal matrix and  $Q_k$  is a unitary matrix and  $Q_k$  has no impact on the achievable rate in (8a), it is always assumed that  $\{W_k\}$ and  $\{\Sigma_k\}$  are real diagonal matrices and their optimization techniques are discussed.

## *B. Power Allocation: Determining* $\{\Sigma_k\}$

Due to the symmetry of the transmitters, the optimization problem with k = 1 is performed. Specifically, we assume that  $f_k'(k') = V_k' W_k' U_k^{H'}$  and  $g_k'(k') = V_k' \Sigma_k' U_k^{H}$ , k' = 1. Then the objective function is given by:

E [logdet  

$$(I + \frac{1}{\sigma^2} (|(D + E)_1 W_1 + \sum_{k=2}^{K} U_1^H U_k (D + E)_K W_k U_1 U_k^{h^2} + / \frac{2}{\gamma^2} + \frac{\sum_{k=2}^{K} U_1^H U_k (D + E_1^{-2})}{(10a)}$$

<sup>=</sup>E

$$\begin{array}{ccccc} \text{[logdet}(I + & + & \sum_{k=2}^{K} U_{k}(D^{2} + \\ +/ & /^{2} + & \sum_{k=1}^{K} |U_{k}(D^{2})|/ & ] \\ & & (10b) \end{array}$$

where (10) follows fact that  $\mathbf{U}_{k}(\mathbf{D}+\mathbf{E})_{k}\mathbf{W}_{k}\mathbf{U}_{k}^{H}$  and  $U_{k}(\mathbf{D}+\mathbf{E})_{k}\boldsymbol{\Sigma}_{k}U_{k}^{H}$  are Hermitian matrices and that  $\{U_{k}\}$  is a *Haar* matrix independent of  $\{W_{k}\}$  and  $\{\Sigma_{k}\}$ .  $\{U_{1}^{H}U_{k}(\mathbf{D}+\mathbf{E})_{k}W_{k}U_{k}^{H}U_{1}\}, \{U_{1}^{H}U_{k}(\mathbf{D}+\mathbf{E})_{k}\boldsymbol{\Sigma}_{k}U^{H}U_{1}\}$  have the same distribution as  $\{U_{k}(\mathbf{D}+\mathbf{E})_{k}W_{k}U_{k}^{H}\}$  and  $\{U_{k}(\mathbf{D}+\mathbf{E})_{k}\boldsymbol{\Sigma}_{k}U_{k}^{H}\}$ 

#### C. Power Allocation: Determining $\{W_k\}$ :

Optimization problem of  $W_1$  For transmitter 1(i.e., k = 1) is given by,

x 
$$\sum_{i=1}^{M} \log \left(1 + \frac{1}{\sigma^2} (d_i^2 w_i^2 + 2\mu)\right)$$

Ma

The maximum in (11) is achieved when all  $\{w_i\}$  have the same sign of  $\mu$ , which shows that the optimal  $W_1$  must be either positive semi-definite or negative semi-definite. Noting the symmetry, it is assumed that  $W_1$  is positive semi-definite. (11) is solved using the Karush-Kuhn-Tucker (KKT) conditions [15].

The associated Lagrangian is

$$L(wl, ..., wm, \lambda) = \sum_{i=1}^{M} \log (1 + \frac{1}{\sigma^2} (d_i^2 w_i^2 + 2\mu d_i w_i v) - \lambda (\sum_{i=1}^{M} w_i^2 - P_1)$$

(12)

The corresponding KKT conditions are given by,

$$\lambda d_i^2 w_i^3 + 2\lambda \mu d_i w_i^2 + (\lambda \sigma^2 + \lambda v - d_i^2) - \mu = 0$$
(13a)

$$\lambda$$
()=0 and  $\lambda \ge 0$   
(13b)

) and 
$$w_i \geq 0$$
 (13c)

The above KKT conditions are easy to solve since, for any given  $\lambda$ , (13a) is a univariate cubic equation of  $w_i$ with at most three different solutions.  $\lambda$  can be found by a bisection method.  $\lambda$  is a monotonically decreasing function with respect to each  $w_i$ . The KKT conditions in (13) can be numerically solved, which yields the optimal  $W_1$  to (11).

## D. Intuitions and Discussions

Given  $_{k}=U_{k}D_{k}V_{k}^{H}$ , then precoding strategy is,

$$F_{k} = f_{k}(k) = V_{k}W_{k}U_{k}^{H}$$

$$G_{k} = g_{k}(k) = 0$$
(14)
$$G_{k} = g_{k}(k) = 0$$

Then the transmitted and received signals in (6) and (7) can be written as

$$\mathbf{x}_{k} = \mathbf{F}_{k} \mathbf{a}_{0} = \mathbf{V}_{k} \mathbf{W}_{k} \mathbf{U} \mathbf{k}^{\mathrm{H}} = \mathbf{a}_{0},$$
(15a)

$$r= \qquad \qquad \sum a_0 + n = Aa_0 + n,$$
(15b)

where A= with  $A_k \equiv U_k D_k W_k U_k^{\text{H}}$ . Since  $A_k$  is a positive semi-definite Hermitian matrix, (14) is referred as a *Hermitian precoding* scheme.

## E. Asymptotic Performance in Large-Scale MIMO Systems

Consider system in which N is sufficient large but M remains small, thus N > M. This arises in practice when multiple base stations jointly serve a common mobile terminal. The latter typically has a limited physical size and thus a small M. For simplicity, we

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assume in this subsection ,that all transmitters have the same power constraint *P* (i.e.  $P_k = P_k = 1, \ldots, K$ ). Let  $R_H$  be the achievable rate of the Hermitian precoding scheme. Denote by  $I_{m \times n}$  an *m*-by-*n* matrix with the only nonzero elements being 1s located at (i,i)th position for  $i = 1, \ldots, \min\{m, n\}$ .

The hermitian precoder  $\mathbf{F}_k$  in (14) reduces to

$$\boldsymbol{F}_{k} = \boldsymbol{V}_{k} \boldsymbol{W}_{k} \boldsymbol{U}_{k}^{H} =$$
(16)

Substituting  $\mathbf{D}_{k}$ = and  $\mathbf{F}_{k}$  in (16) into (8a). The achievable rate of the hermitian precoder given by

$$R_H = M \log \left( 1 + \right) = M \log \left( \right)$$
 (17)

#### V. NUMERICAL RESULTS

In this section, numerical results used to demonstrate the performance of the proposed Hermitian precoding technique in large scale distributed MIMO channels. For comparison, we list below a variety of alternative choices of  $F_k$  and  $G_k$ . Full CSIT capacity with total power constraint is also included as an upper bound. For simplicity of discussion, we assume N = M.



Fig. 2. Performance comparison among different precoding schemes with M = N = K = 2 in Rayleigh-fading distributed MIMO Gaussian channels SNR =  $(P1 + P2)/\sigma^2$  with P1 = P2.

(i) Full CSIT capacity with total power constraint (Full CSIT): The input covariance matrix is determined by standard SVD water-filling method [16] over

$$=$$
 [ $H_1$ , $H_2$ , ..., $H_K$ ].

(ii) Hermitian Precoding with Optimized Power Allocation (HP-OPA):

$$\boldsymbol{F}_{k} = \boldsymbol{V}_{k} \boldsymbol{W}_{k} \boldsymbol{U}_{k}^{H}$$
 and  $\boldsymbol{G}_{k} = \boldsymbol{0}$ .

Here  $W_k$  is obtained using the technique proposed in Section IV.

(iii) No-CSIT with Independent Signaling (No-CSIT-IS):

$$F_k=0$$
 and  $G_k=$ 

Fig. 2 show the performance of a Hermitian precoder. In this figure, equal power constraints is assumed for all transmitters (i.e.,  $P_k = P$ ,  $\forall k$ ) due to the symmetry property among them. The performance of various precoding schemes are compared with K = M = N = 2. Fig. 3 shows the comparison of performance of hermitian precoder with and without perfect channel knowledge (with K = M = N = 2). It is seen that HP-OPA performs very close to the upper bound obtained by assuming full CSIT. This implies that the potential performance loss due to the I-CSIT assumption is marginal. Note that the performance of. No-CSIT-IS is relatively poor in both Figs.2 and 4.



Fig. 3. Performance comparison of hermitian precoding with perfect and imperfect channel with individual channel state information. M = N = K = 2 in Rayleigh-fading distributed MIMO Gaussian channels SNR =  $(P1 + P2)/\sigma^2$  with P1 = P2.



Fig. 4. Performance comparison of hermitian precoding with perfect and imperfect channel with individual channel state information and perfect channel with no CSIT M = N = K = 2 in Rayleigh-fading distributed MIMO Gaussian channels SNR =  $(P1 + P2)/\sigma^2$  with P1 = P2.

Fig.4. shows that there is a performance degradation for an imperfect channel with individual channel state information when compared a perfect channel but it is enhanced when compared with a perfect channel with no -CSIT(channel state information)

## VI. CONCLUSION

A Hermitian precoding technique for distributed MIMO channels with imperfect and individual CSIT is analyzed and its local optimality is proved. Numerical results show that the proposed scheme with imperfect I-CSIT has degraded performance compared to the channel capacity with perfect CSIT. Also the imperfect channel knowledge has enhanced system performance compared with no CSIT case. The proposed technique can be used to reduce the overhead related to acquiring CSIT in distributed MIMO channels, especially for large-scale antenna systems.

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