

# Effect of ion temperature on dust ion acoustic solitary waves in a warm unmagnetized plasma with electron inertia

R.Das

Department of Mathematics, Arya Vidyapeeth College, Guwahati –781016, Assam, India

**Abstract:** The existence of Korteweg-de Vries (KdV) solitons has been established in an unmagnetized warm dusty plasma consisting of positive ion, electrons and static negatively charged dust particles. Depending upon  $\sigma = \frac{T_i}{T_e}$  (= ion to electron temperature), only fast compressive and rarefactive ion-acoustic solitons are found to exist. The amplitude of the rarefactive fast ion acoustic solitons exists only for smaller values of  $\alpha \left( = \frac{n_{e0}}{n_{i0}} = 1 - \frac{Z_d n_{d0}}{n_{i0}} \right) < 1$  ( $Z_d$  is the number of elementary charges residing on the dust grain) and its amplitude increases as  $\alpha (< 1)$  increases. Furthermore, they are found to exist only in the lower regime of  $\alpha (< 1)$ . On the other hand, compressive fast ion acoustic solitons exist only in the upper regime of  $\alpha (< 1)$ .

**Keywords:** Electron inertia, Soliton, Solitary wave.

## I. INTRODUCTION

The field of dusty plasmas is a multi-disciplinary research topic of recent interest because of its new aspects in space and modern astrophysics, semiconductor technology, fusion devices, plasma chemistry, crystal physics, biophysics, etc. [1]. Dusty plasmas are present in planetary rings, comet tails, and interplanetary and interstellar clouds [1, 2, 3, 4, 5, 6]. Further, dusty plasma can be produced in laboratories by Modified Q-machine, dc discharges, rf discharges etc [1]. Dusty plasma plays an important role in plasma crystals [8], coating and etching of thin films [9] etc. The study of waves and instabilities has always been important in plasma physics. The presence of dust particles in a plasma generates many new problems to investigate [10, 11, 12]. By means of reductive perturbation method, Rao et al. [13] have first reported the existence of the dust acoustic waves for low frequency in unmagnetized dust plasma and verified in a laboratory experiment by Barken et al. [14]. D' Angelo [15], Nakamura et al. [16] and Duan et al. [17] have studied the dust ion acoustic waves. Bharuthram and Shukla [18] have studied the dust ion-acoustic solitary waves in an unmagnetized dusty plasma consisting of isothermal electrons, cold ions, in both static and mobile dust particles. Roychoudhury and Mukharjee [19] have reported that the finite dust temperature restricts the region for the existence of nonlinear solitary waves. Shukla and Silin [20] have reported about the existence of dust ion acoustic waves for higher frequency. Duan [21] has investigated the dust acoustic solitary waves in hot dust plasma. Duan et al. [22] have investigated dusty plasma with variable dust charge. El-Labany and El-Taibany [23] have investigated the effects of variable dust charge, dust temperature and an arbitrary streaming ion beam on small amplitude dust acoustic waves. In their investigation they found that both compressive and rarefactive solitons as well as double layers exist. El-Labany and El-Taibany [24] have investigated the effects of variable dust charge, dust temperature, and trapped electrons on small amplitude dust acoustic waves. They found that both compressive and rarefactive solitons as well as double layers exist depending on the nonisothermality parameter. Ghosh [25] has investigated the role of negative ions in dusty plasma with variable dust charge. Zhang and Xue [26] have studied the effects of the dust charge variation and non-thermal ions on the dust acoustic solitary structure in magnetized dusty plasmas. El-Labany et al. [27] have studied the propagation of nonlinear dust acoustic waves in the dusty plasma consisting of a mixture of various charged dust

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

particles, isothermal electrons and two-temperature isothermal ions. Mamun and Shukla [28] have investigated the cylindrical and spherical dust ion acoustic solitary waves in an unmagnetized dusty plasma consisting of inertial ions, isothermal electrons, and stationary dust particles using reductive perturbation method. Mamun and Shukla [29] have also investigated the condition for the existence of positive and negative potential dust ion acoustic solitary waves. Sayed and Mamun [30] have investigated the effect of ion-fluid temperature on dust ion acoustic solitary waves structures in a dusty plasma containing adiabatic ion-fluid, Boltzmann electrons, and static dust particles. Baluku et al. [31] have investigated dust ion acoustic solitary waves in an unmagnetized dusty plasma consisting of cold dust particles and kappa distributed electrons using both small and arbitrary amplitude techniques. Das and Das [32] have investigated the dust ion-acoustic solitary structures in an unmagnetized non-thermal plasma consisting of negatively charged dust grains, adiabatic positive ions, and non-thermal electrons. Very recently Das and Karmakar [33] have established the existence of modified Korteweg-de Vries (KdV) solitons in presence electron inertia in an interesting situation for different values of electrons' drift velocity ( $v_e'$ ) in a dusty plasma.

In this paper, we have investigated the effect of ion temperature on dust of ion-acoustic solitons in an unmagnetized warm dusty plasma consisting of positive ion, electrons and static negatively charged dust particles. Depending upon various ion temperatures ( $\sigma$ ), both compressive and rarefactive ion-acoustic solitons are found to exist.

## II. BASIC EQUATION AND DERIVATION OF KdV EQUATION

We consider a one dimensional, unmagnetized collision less dusty plasma with positive ion, electrons and static negatively charged dust particles. The dynamics of dust ion acoustic waves is governed by the following equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \tag{1}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{\sigma}{n_i} \frac{\partial p_i}{\partial x} + \frac{\partial \phi}{\partial x} = 0 \tag{2}$$

$$\left( \frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x} \right) p_i + 3p_i \frac{\partial v_i}{\partial x} = 0 \tag{3}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0 \tag{4}$$

$$\left( \frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x} \right) v_e = \frac{1}{Q} \left( \frac{\partial \phi}{\partial x} - \frac{1}{n_e} \frac{\partial n_e}{\partial x} \right) \tag{5}$$

Again for charge imbalances these equations are to be combined by the Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} = \alpha n_e - n_i + 1 - \alpha \tag{6}$$

where,  $i$  and  $e$  stand for positive ion and electron respectively,  $Q = \frac{m_e}{m_i}$  (= electron to ion mass ratio),  $\sigma = \frac{T_i}{T_e}$  (= ion

to electron temperature) and  $\alpha = \frac{n_{e0}}{n_{i0}} = 1 - \frac{Z_d n_{d0}}{n_{i0}}$  ( $Z_d$  is the number of elementary charges residing on the dust grain).

We have normalized densities  $n_i$  and  $n_e$  by the unperturbed densities  $n_{i0}$ , pressure  $p_i$  by the characteristic ion

pressure  $k_b n_{e0} T_i$ , time  $t$  by the inverse of the characteristic ion plasma frequency i.e.,  $\omega_{pi}^{-1} = \left( \frac{m_i}{4\pi n_{i0} e^2} \right)^{\frac{1}{2}}$ , distance  $x$

**International Journal of Innovative Research in Science,  
Engineering and Technology**

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

by the electron Debye length  $\lambda_{De} = \left( \frac{k_b T_e}{4\pi n_{i0} e^2} \right)^{\frac{1}{2}}$ , velocities by the ion-acoustic speed  $C_s = \left( \frac{k_b T_e}{m_i} \right)^{\frac{1}{2}}$ , and the potential  $\phi$  by  $\frac{k_b T_e}{e}$ ;  $k_b$  is the Boltzmann constant.

To derive the KdV equation from the set (1) – (6), we use the stretched variables

$$\xi = \varepsilon^{\frac{1}{2}}(x - Ut), \quad \tau = \varepsilon^{\frac{3}{2}}t \tag{7}$$

with phase velocity  $U$  of the wave and using the following expansions of the flow variables in terms of the smallness parameter  $\varepsilon$ :

$$\begin{aligned} n_i &= 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \dots, \\ n_e &= 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \dots, \\ v_i &= \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \dots, \\ v_e &= \varepsilon v_{e1} + \varepsilon^2 v_{e2} + \dots, \\ p_i &= 1 + \varepsilon p_{i1} + \varepsilon^2 p_{i2} + \dots, \\ \phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \dots \end{aligned} \tag{8}$$

With the use of the transformation (7) and the expansion (8) in the normalized set of equations (1) – (6), we get the expression for the phase velocity and the KdV equation as follows-

$$U^2 = \frac{1 + 3\alpha\sigma}{Q + \alpha} \tag{9}$$

and

$$\frac{\partial \phi_1}{\partial \tau} + p \phi_1 \frac{\partial \phi_1}{\partial \xi} + q \frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{10}$$

where  $p = \frac{A}{2B}$  and  $q = \frac{1}{2B}$

with  $A = \frac{3(U^2 + \sigma)}{(U^2 - 3\sigma)^3} - \frac{\alpha(3QU^2 - 1)}{(1 - QU^2)^3}$

and  $B = \frac{U}{(U^2 - 3\sigma)^2} + \frac{\alpha QU}{(1 - QU^2)^2}$

From the expressions of  $A$  and  $B$ , we observe that the nonlinear dust ion-acoustic solitons, in this model of plasma, exist when  $U^2 \neq 3\sigma, \frac{1}{Q}$ .

III. SOLITARY WAVE SOLUTION

To find a stationary solution of the K dV equation (10), we use the transformation  $\chi = \eta - V\tau$ . Using the boundary conditions  $\phi_1 = \frac{\partial \phi_1}{\partial \eta} = \frac{\partial^2 \phi_1}{\partial \eta^2} = 0$  as  $|\eta| \rightarrow \infty$ , the KdV equation (10) can be integrated to give  $\phi_1 = \frac{3V}{p} \operatorname{sech}^2 \left( \frac{1}{2} \sqrt{\frac{V}{q}} \chi \right)$ , where  $V$  is the velocity with which the solitary waves travel to the right.

**International Journal of Innovative Research in Science,  
Engineering and Technology**

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

Thus, the wave amplitude of the soliton is given by  $\phi_0 = \frac{3V}{p}$  and the corresponding width by  $\Delta = 2\sqrt{\frac{q}{V}}$ .

IV. RESULTS AND DISCUSSION

In presence of ion temperature in this model of plasma only fast compressive and rarefactive solitons are found to exist. The amplitude of the rarefactive fast ion acoustic solitons exist only for smaller values of  $\alpha(<1)$ [Fig. 1 a] and its amplitude increases as  $\alpha(<1)$  increases. Furthermore, they are found to exist only in the lower regime of  $\alpha(<1)$ . On the other hand, compressive fast ion acoustic solitons exist [Fig. 1 a] only in the upper regime of  $\alpha(<1)$  for fixed  $V=0.05$  and  $Q=0.00054$  for different values of  $\sigma = 0.1(1), 0.3(2), 0.5(3)$ . The character of the fast rarefactive solitons changes to fast compressive solitons after certain  $\alpha^*(<1)$  characterizing an unaccountable regime. The corresponding widths [Fig. 1 b] of the fast (compressive and rarefactive) ion acoustic solitons decrease uniformly for the same set of values. The amplitudes [Fig. 2(a)] of the fast compressive solitons decreases nonlinearly with the increase in  $\sigma$  for different values of  $\alpha = 0.7(1), 0.8(2), 0.9(3)$ . The same conclusion can be made from figure 2 b in case of width. Of course, the decrease in this case is observed to be linear. Figure 3(a) shows that the amplitude of the fast compressive soliton remains almost same for smaller values of  $\alpha$  but increases numerically for higher value of  $\alpha = 0.15(3)$  with the increase in  $\sigma$ . It further shows that amplitude increases numerically with the increase in  $\alpha = 0.05(1), 0.10(2), 0.15(3)$  for fixed  $V=0.10$  and  $Q=0.00054$ . The corresponding widths [Fig. 3(b)] of the fast compressive solitons remains almost constant with the increase of  $\sigma$ . The figure further reveals that the magnitude of the width decreases with the increase in  $\alpha = 0.05(1), 0.10(2), 0.15(3)$ . The phase velocity [Fig. 4] of the fast compressive solitons increases steadily in magnitude with the increase of  $\sigma$ , whereas phase velocity decreases in magnitude with the increase of  $\alpha = 0.05(1), 0.10(2), 0.15(3)$ . The phase velocity [Fig. 5] of fast compressive solitons increases steadily with the increase of  $\sigma$ . Moreover, the phase velocity decreases in magnitude with the increase of  $\alpha = 0.20(1), 0.25(2), 0.35(3)$ . Comparison of figures 4 and 5 further shows that the magnitude of the phase velocity is more for smaller values of  $\alpha$ . Difference in the magnitude of the amplitude [Fig. 6] of the fast rarefactive solitons is imperceptible for smaller values and can be visualized only for the higher values of  $V$ . At the same time, it is seen that the amplitude increases in magnitude with the increase of  $\sigma = 0.25(1), 0.35(2), 0.45(3)$ . Fig. 7(a) exhibits the uniform increase in amplitudes of compressive fast ion-acoustic solitons at the increase of wave speed  $V$ . To the contrary, the width of the fast ion-acoustic solitons rapidly decreases [Fig. 7(b)] for small  $V$  but uniformly decreases for relatively higher  $V$  for fixed  $\alpha = 0.40$  and  $Q = 0.00054$  for different values of  $\sigma = 0.25(1), 0.35(2), 0.45(3)$ .

**International Journal of Innovative Research in Science,  
Engineering and Technology**

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

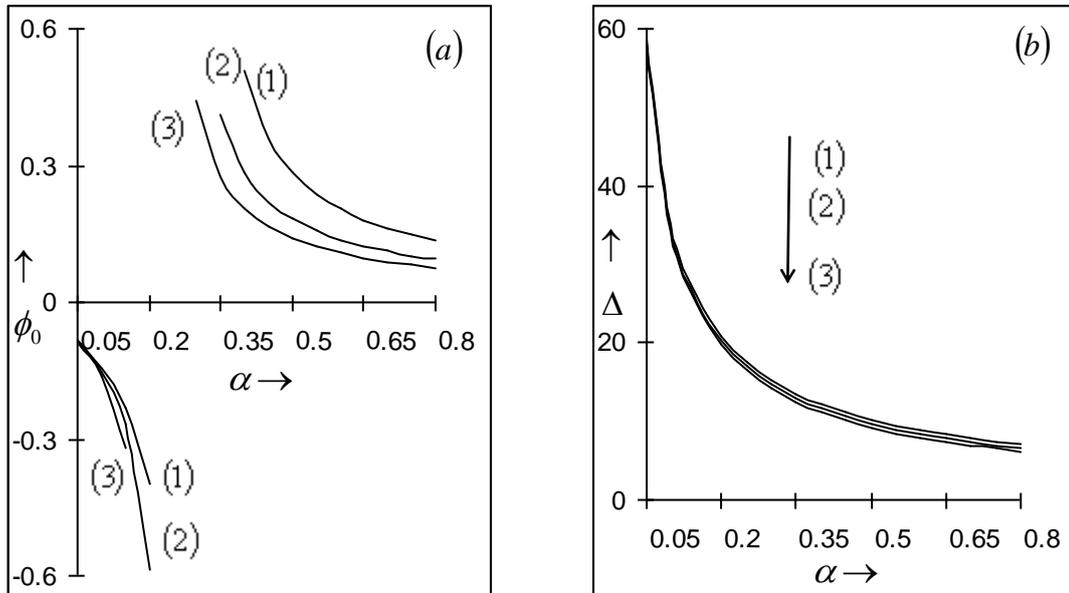


FIG.1. Amplitudes (a) and widths (b) of compressive and rarefactive solitons versus  $\alpha$  for fixed  $V = 0.05$  and  $Q = 0.00054$  for different values of  $\sigma = 0.1(1), 0.3(2), 0.5(3)$ .

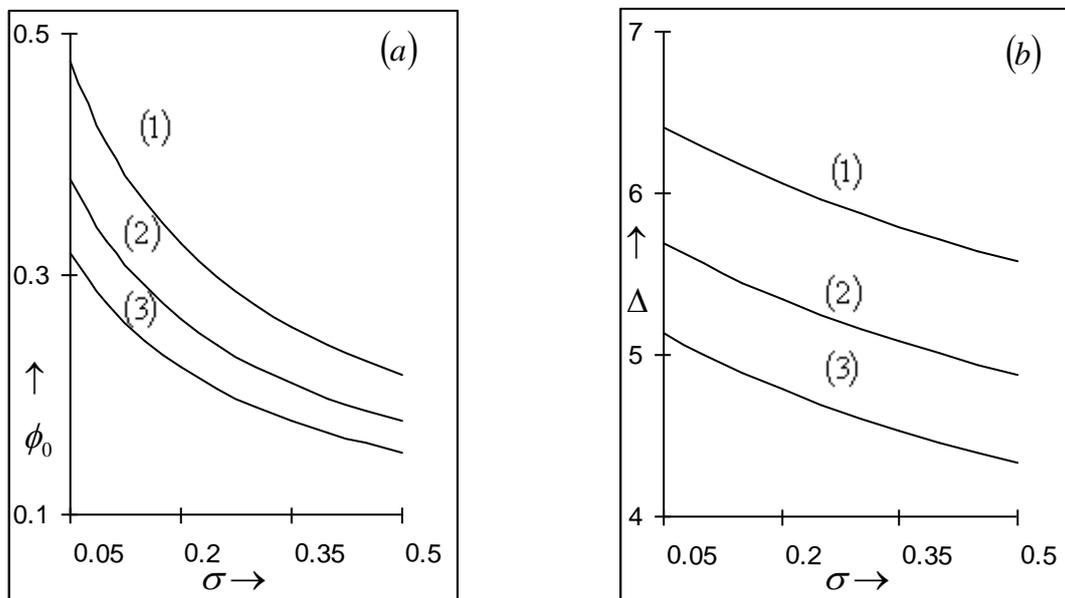


FIG.2. Amplitudes (a) and widths (b) of compressive solitons versus  $\sigma$  for fixed  $V = 0.10$  and  $Q = 0.00054$  for different values of  $\alpha = 0.7(1), 0.8(2), 0.9(3)$ .

**International Journal of Innovative Research in Science,  
Engineering and Technology**

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

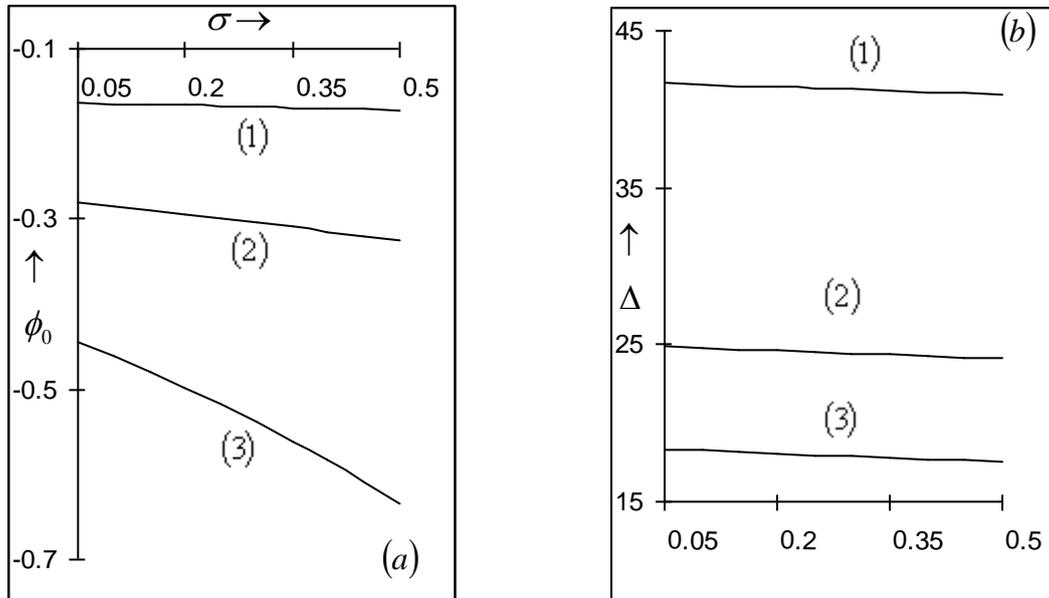


FIG.3. Amplitudes (a) and widths (b) of rarefactive soliton versus  $\sigma$  for fixed  $V = 0.10$  and  $Q = 0.00054$  for different values of  $\alpha = 0.05(1), 0.10(2), 0.15(3)$ .

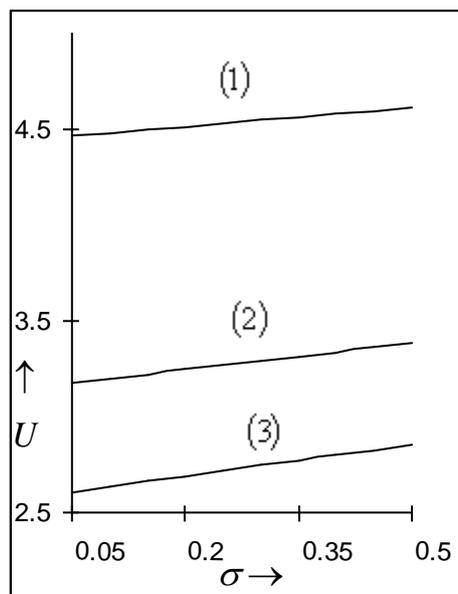


FIG.4. Phase velocity of compressive soliton versus  $\sigma$  for fixed  $V = 0.10$  and  $Q = 0.00054$  for different values of  $\alpha = 0.05(1), 0.10(2), 0.15(3)$ .

**International Journal of Innovative Research in Science,  
Engineering and Technology**

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

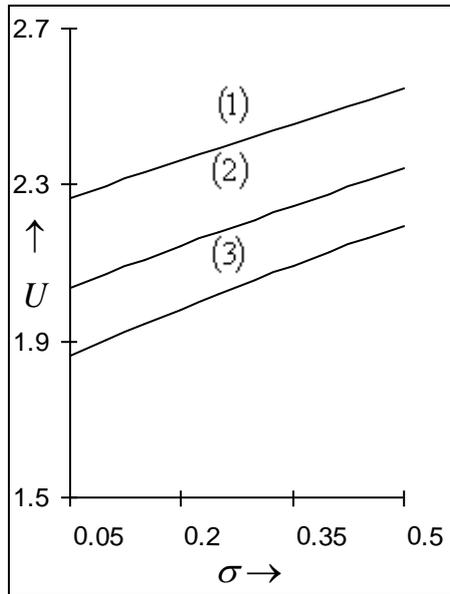


FIG.5. Phase velocity of soliton versus  $\sigma$  for fixed  $V=0.10$  and  $Q=0.00054$  for different values of  $\alpha = 0.20(1), 0.25(2), 0.30(3)$ .

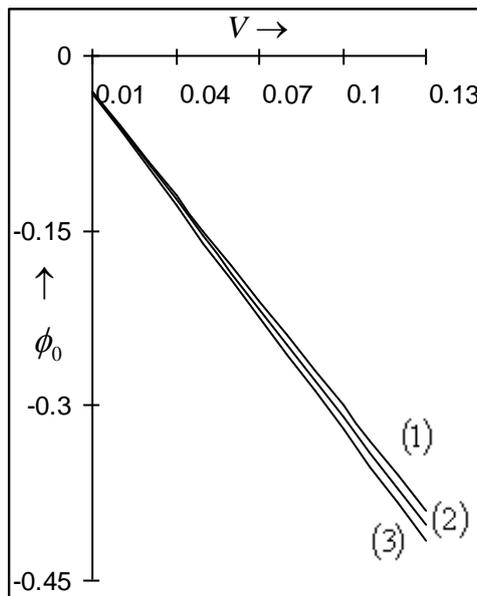


FIG.6. Amplitudes of rarefactive soliton versus  $V$  for fixed  $\alpha=0.10$  and  $Q=0.00054$  for different values of  $\sigma = 0.25(1), 0.35(2), 0.45(3)$ .

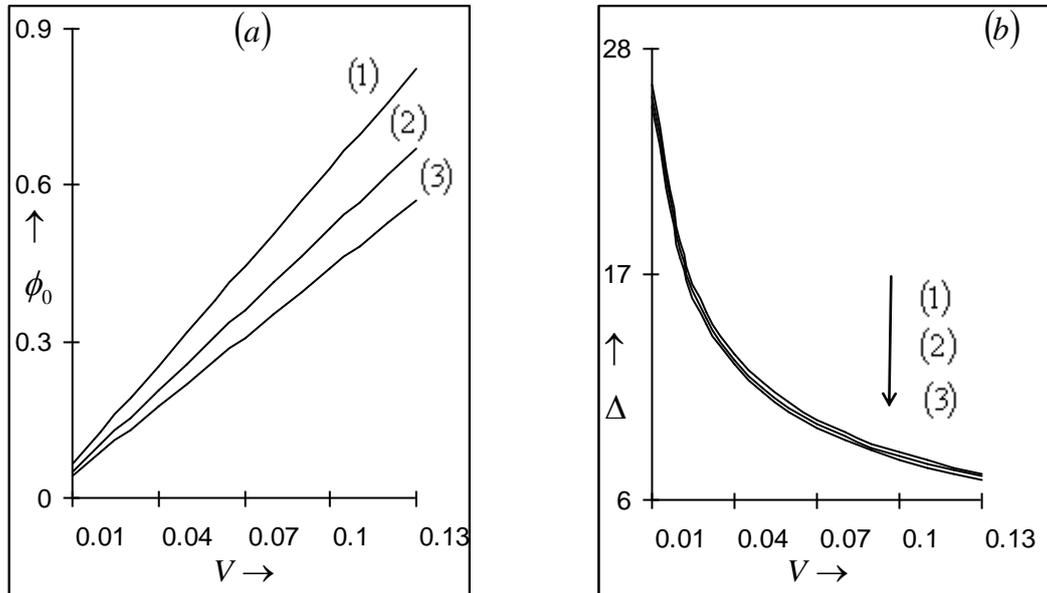


FIG.7. Amplitudes (a) and widths (b) of compressive soliton versus  $V$  for fixed  $\alpha = 0.40$  and  $Q = 0.00054$  for different values of  $\sigma = 0.25(1), 0.35(2), 0.45(3)$ .

**REFERENCES**

[1] Shukla, P. K., Mamun, A. A., "Introduction to Dusty Plasma Physics", Institute of Physics Publishing Ltd., 2002.  
 [2] Geortz, C. K., "Dusty plasmas in the solar system", Rev. Geophys., Vol. 27, pp. 271 – 292, 1989.  
 [3] Northrop, T. G., "Dusty plasmas" Phys. Scripta Vol. 45, pp. 475 - 490 1992.  
 [4] Verheest, F., "Waves in Dusty Plasma", (Kluwar Academic, Dordrecht ), 2000.  
 [5] Bliokh, P., Sinitin, V., Yaroshenko, V., Dusty and Self-Gravitational Plasmas in Space (Dordrecht: Kluwer Acad. Publ., 1995)  
 [6] Whipple, E. C., "Potentials of Surfaces in Space," Rep. Prog. Phys. Vol.44, PP.1197 – 1250, 1981.  
 [7] Robinson, P. A., Coakley, P., "Spacecraft Charging - Progress in the Study of Dielectrics and Plasmas" IEEE Trans. Electr. Insul. Vol.27, pp. 944 - 960, 1992.  
 [8] Thomas, H., Morfill, G. E., Dammel, V., "Plasma Crystal: Coulomb Crystallization in a Dusty Plasma", Phys. Rev. Lett., Vol.73, pp. 652 - 655, 1994.  
 [9] Selwyn, G. S., Jpn. J. Applied Phys. Part 1, Vol.32, pp. 3068, 1993.  
 [10] Walch, B., Horanyi, M., Robertson, S., Phys. Rev. Lett. Vol.75,pp. 538, 1995.  
 [11] Merlino, R. L., Barkan, A., Thompson, C., D' Angelo, N., "Laboratory studies of waves and instabilities in dusty plasmas", Phys. Plasmas Vol. 5, pp. 1607 – 1614,1998.  
 [12] Piel, A., Melzer, A., Plasma Phys. Controlled Fusion Vol. 44, R1 2002.  
 [13] Rao, N. N., Shukla, P.K., Yu, M.Y., "Dust-acoustic waves in dusty plasmas", Planet Space Sce. Vol. 38,pp. 543 – 546, 1990.  
 [14] Barkan, A., Merlino, R. L., D' Angelo, N., "Laboratory observation of the dust-acoustic wave mode", Phys. Plasmas Vol. 2, pp.3563– 3565,1995.  
 [15] D' Angelo, N., "Coulomb solids and low-frequency fluctuations in RF dusty plasmas", J. Phys.Vol. D28,pp. 1009, 1995.  
 [16] Nakamura, Y., Bailung, H., Shukla, P. K., "Observation of Ion-Acoustic Shocks in a Dusty Plasma", Phy. Rev. Lett. Vol. 83,pp. 1602 – 1605, 1999.  
 [17] Duan, W. S., Lu, K. P., Zhao, J.B., "Hot Dust Acoustic Solitary Waves in Dust Plasma with Variable Dust Charge", Chin. Phys. Lett. Vol.18,No. 8,pp. 1088 – 1089, 2001.  
 [18] Bharuthram, R., Shukla, P. K., "Large amplitude ion-acoustic solitons in a dusty plasma", Planet. Space Sci. Vol.40, pp. 973 – 977, 1992.  
 [19] Roychoudhury, R. Mukharjee, S., "Large-amplitude solitary waves in finite temperature dusty plasma", Phys.Plasmas Vol. 4, 2305, 1997.  
 [20] Shukla, P. K., Silin, V. P., "Dust ion-acoustic wave", Phys. Scr.Vol. 45, 508, 1992.  
 [21] Duan, W. S., "The Kadomtsev – Petviashvili (KP) equation of dust acoustic waves for hot dust plasmas", Chaos Soliton. Fract. Vol. 14,pp. 503 – 506, 2002.  
 [22] Duan, W. S., Hong, X.R., Shi, Y.R., Sun, J.A., "Envelop solitons in dusty plasmas for warm dust", Chaos Soliton.Fract.Vol.16,pp. 767 – 777, 2003.  
 [23] El-Labany, S. K., El-Taibany, W. F., "Dust acoustic solitary waves and double layers in a dusty plasma with an arbitrary streaming ion beam", Phys.Plasmas Vol. 10,pp. 989 (10 pages), 2003

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

**Vol. 3, Issue 1, January 2014**

- [24] El-Labany, S. K., El-Taibany, W. F., "Dust acoustic solitary waves and double layers in a dusty plasma with trapped electrons", Phys. Plasmas Vol. 12, pp. 4685 (11 pages), 2003.
- [25] Ghosh, S., "Dust acoustic solitary wave with variable dust charge: Role of negative ions", Phys. Plasmas Vol. 12, pp. 94504 (4 pages), 2005.
- [26] Zhang, L. P., Xue, Ju-kui, "Effects of the dust charge variation and non-thermal ions on multi-dimensional dust acoustic solitary structures in magnetized dusty plasmas", Chaos Soliton. Fract. Vol. 23, pp. 543 – 550, 2005.
- [27] El-Labany, S K, Moslem, W. M., Safy, F. M., "Effects of two-temperature ions, magnetic field, and higher-order nonlinearity on the existence and stability of dust-acoustic solitary waves in Saturn's F ring", Phys. Plasmas Vol.13, pp. 082903(12 pages), 2006.
- [28] Mamun, A. A., Shukla, P. K., "Cylindrical and spherical dust ionacoustic solitary waves", Phys. Plasmas Vol. 9, 1468 – 1470, 2002a.
- [29] Mamun A. A., Shukla, P. K., "Electrostatic solitary and shock structures in dusty plasmas", Phys. Scripta Vol. T98, pp. 107–114, 2002b.
- [30] Sayed, F., Mamun, A. A., "Effects of ion-fluid temperature on dust-ion-acoustic solitons", Pramana Vol.70, pp. 527 – 533, 2008.
- [31] Baluku, T. K., Hellberg, M. A., Kourakis, I., Saini, N. S., "Dust ion acoustic solitons in a plasma with kappa-distributed electrons", Phys. Plasmas Vol. 17, pp. 053702, (2010).
- [32] Das, A., Bandyopadhyay, A., Das, K. P., "Dust ion-acoustic solitary structures in non-thermal dusty plasma", J. Plasma Phys. Vol. 78, pp. 149 – 164, 2012.
- [33] Das, R., Karmakar, K., "Modified Korteweg-de Vries solitons in a dusty plasma with electron inertia and drifting effect of electrons", Can. J. Phys. Vol.91, pp. 839 – 843, 2013.