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Estimation of Weibull Parameters for Wind speed calculation at Kanyakumari in India

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Abstract: The study deals with analysis of four methods for determining the parameters of the Weibull distribution, using wind speed data collected in the Mupandal station at Kanya Kumari in India. The Weibull distribution is a twoparameter function commonly used to fit the wind speed frequency distribution. This family of curves has been shown to give a good fit to measured wind speed. Four methods for calculating the parameters of the Weibull wind speed distribution for wind energy analysis are presented : the Empirical method, the Maximum likelihood method, Modified maximum likelihood method, the proposed Energy pattern factor method. The application of each method is demonstrated using a sample wind speed data set and a comparison of the accuracy of each method is also performed using some statistical methods of analysis. The study helps to determine which one is effective in determining the parameters of Weibull distribution and to establish the wind energy resource

Keywords: Weibull distribution, Weibull's shape factor ,scale factor, Mean wind speed, Energy pattern factor, Accuracy, Statistical tests

I. INTRODUCTION

The installation of a number of wind turbine generators can effectively reduce environmental pollution fossil fuel consumption and the cost of overall electricity generation. Although wind is only an intermittent source of energy, it represents a reliable energy resources, wind power energy is the most popular and promising energy resources. At a specific wind farm the available electricity generated by a wind power generator system depends on mean wind speed and standard deviation. Since variation on annual mean wind speed is hard to predict, wind speed variations during a year can be well characterized in terms of the probability density function (pdf).

Over the last two decades a number of papers have appeared in the literature concerning efforts to develop an adequate statistical model for describing wind frequency distribution. The use of this frequency distribution approach can provide a simple method to predict the energy output of a wind energy conversion system. Much consideration have been given to the two parameter Weibull distribution because it has been found to fit a wide collection of wind data.

There are methods which have so far predominantly used for fitting the measured wind speed probability distribution in a given location over a period of time, typically monthly or yearly. In the literature it is common to fit these functions to compare which one fits the measured distribution best in particular location. During this comparison process, parameter on which the suitability of the fit is judged are required.

Several methods have been proposed to estimate Weibull parameters(Marks,2005; Rider,1961;Kao,1959; pang et al,2001; Pandy et al,2011; Sequro and Lambert,2000; Stevens and Smulders,1979; Seguro and Bhattacharya and Bhatacharjee 2012)[4]. In literature about wind energy, these methods are compared several times and in different ways. The suitability of the method may vary with the sample data size, sample data distribution, sample data format and goodness of fit test (Akdag and Ali,2009).[6]



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This study attempts to evaluate and compare four methods for estimating the Weibull parameters namely, shape (k) and scale (c) parameters and adjusting the Weibull distribution of wind speeds at Kanya Kumari district, situated at one of the extreme end of India. Mean wind speed data's of Kanya Kumari were collected from the meteorological society at various stations for several years 1986-2004 with mean average wind speed rating between 5.92 m/s to 7.32 m/s. Our study includes the mean wind speed data for 12-months observed at Muppandal station(1) at Kanya Kumari district with mast. height of 20 m at Latitude N at $8^{\circ}15^{\circ}30^{\circ}$, Longitude E at $77^{\circ}33^{\circ}20^{\circ}$.

II RELATED WORK

In Literature several distributions were applied to calculate the wind speed distribution. Justus et al[1976] applied the Weibull and Lognormal distribution to Wind speed data from more than a hundred stations of the USA and concluded that Weibull Distribution rendered the best fit. Corotis et al [1978] preffered the Rayleigh distribution a special case of Weibull. Hennessey [1978] found that the Energy output calculated by Rayleigh distribution is within 10% of the output based on the Weibull distribution . On the whole Weibull gave a good fit based on the shape and scale parameters In recent years the Weibull distribution has been one of the most widely used and recommended tool to determine the potential of Wind Energy. Moreover it is used as a benchmark to estimate the wind energy commercially.

III .WEIBULL DISTRIBUTION

The Weibull distribution (named after the Swedish physicist Weibull, who applied it when studying material in tension and fatigue in the 1930s) provides a close approximation to the probability laws of many natural phenomena. It has been used to represent wind speed distribution for application in wind load studies for some time. In recent years most attention has been focused on this method for wind energy application not only due to its greater flexibility and simplicity but also because it can give a good fit to experimental data. The Weibull distribution function , a two parameter function , for wind speed is expressed mathematically as

$$f(v) = \left(\frac{k}{c}\right) \left(\frac{v}{c}\right)^{k-1} exp\left(-\left(\frac{v}{c}\right)^k\right)$$
(1)

And the cumulative distribution function is

$$F(v) = 1 - exp\left(-\left(\frac{v}{c}\right)^k\right).$$
(2)

Where v is the wind speed, k is the shape parameter and c is the scale parameter.

IV. WIND SPEED DATA

Measured wind speed data are commonly available in time series format ,in which each data point represents either an instantaneous sample wind speed or an average wind speed over some time period. An example of such data(giving hourly averages over a 24 hr period is given) in Table -I. Sometimes wind speed data may be available in frequency distribution format ,Table-II. The methods described in following section can be used to estimate the Weibull parameters, given wind speed in either time-series or frequency distribution format.



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TABLE I WIND SPEED DATA IN TIME-SERIES FORMAT

Hour	Wind Speed (m/s)						
1	4.3124	7	4.2243	13	5.7124	19	8.3301
2	4.1627	8	4.3146	14	8.3242	20	9.3050
3	5.7024	9	4.2342	15	9.2672	21	6.5214
4	4.3240	10	6.7014	16	9.3250	22	4.2270
5	3.8642	11	6.8192	17	6.7030	23	3.8219
6	4.1634	12	6.8210	18	5.7184	24	4.3642

TABLE II WIND SPEED DISTRIBUTION IN FREQUENCY DISTRIBUTION FORMAT

Wind speed (m/s)	3-4	4-5	5-6	6-7	7-8	8-9	9-10
Frequency%	8.33	33.33	16.67	20.83	0	8.33	12.50

V. DETERMINATION OF WEIBULL PARAMETERS

For estimating the parameters of the Weibull wind speed distribution four methods are presented

5.1 Empirical method

The Empirical method is the special case of the moment method, where the parameters k and c are defined by

$$k = \left(\frac{\sigma}{\bar{v}}\right)^{-1.086}$$
(3)

$$c = \frac{\bar{v}}{\Gamma\left(1 + \frac{1}{k}\right)}$$
(4)

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$
(5)

5.2 The maximum likelihood method

The Weibull distribution can be fitted to time series wind data using the maximum likelihood method as suggested by Stevens and Smulders [1]. The shape parameter k and the scale parameter c are estimated using the following two equations:

$$k = \left(\frac{\sum_{i=1}^{n} v_i^{k} \ln(v_i)}{\sum_{i=1}^{n} v_i^{k}} - \frac{\sum_{i=1}^{n} \ln(v_i)}{n}\right)^{-1},$$

$$c = \left(\frac{1}{\sum_{i=1}^{n} v_i^{k}}\right)^{1/k},$$
(6)
(7)

Where v_i is the wind speed in time step i and n is the number of nonzero wind speed data points.

Eq.(6) must be solved using a iterative procedure (k=2 is a suitable initial guess), after which Eq.(7) can be solved explicitly. Care must be taken to apply Eq.(6) only to the nonzero wind speed data points.



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5.3 The modified maximum likelihood method

When wind speed data are available in frequency distribution format, a variation of maximum likelihood method can be applied. The Weibull parameters are estimated using the following two equations:

$$k = \left(\frac{\sum_{i=1}^{n} v_i^{k} \ln(v_i) P(v_i)}{\sum_{i=1}^{n} v_i^{k} P(v_i)} - \frac{\sum_{i=1}^{n} \ln(v_i) P(v_i)}{P(v \ge 0)}\right)^{-1},$$

$$c = \left(\frac{1}{P(v \ge 0)} \sum_{i=1}^{n} v_i^{k} P(v_i)\right)^{1/k},$$
(8)
(9)

where v_i is the wind speed, $P(v_i)$ represents the Weibull frequency, $p(v \ge 0)$ is the probability that the wind speed equals or exceeds zero.

Eq.(8) must be solved iteratively, after which eq.(9) can be solved explicitly.

5.4 Energy pattern factor method

This is a new method suggested by Akdag Ali(2009). It is related to the averaged data of wind speed. This method has simpler formulation, easier implementation and also requires less computation. The method is defined by the following equations :

$$E_{pf} = \frac{\overline{v^3}}{(v^3)},$$
(10)
$$k = 1 + \frac{3.69}{(v^3)}$$
(11)

$$\bar{\nu} = c\Gamma(1+1/k) \tag{11}$$

Eqn.(10) is known as the energy pattern factor method which can be solved numerically or approximately by power density technique using Eq.(11). Once k is determined, c can be estimated using Eqn.(12).

Month	Mean wind speed (m/s)	Month	Mean wind speed (m/s)
Jan	5.6138	July	9.4207
Feb	6.3581	Aug	10.1719
Mar	5.8483	Sep	9.2700
April	4.4753	Oct	8.8324
May	4.5120	Nov	6.6706
June	7.8833	Dec	4.6796

TABLE III MONTHLY WIND SPEED DATAS

TABLE IV THE WEIBULL PARAMETERS k and c

Weibull parameters	EM	MLM	MMLM	EPFM	
k	2.6760	2.5641	2.6490	2.6543	
с	7.8500	7.7904	7.8300	7.8521	



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VI. STATISTICAL ANALYSIS

Three tests were used to analyse the accuracy of the four methods : RMSE test , R^2 and Chi - Square tests defined by

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N} (y_i - x_i)^2\right]^{\frac{1}{2}}$$
$$\chi^2 = \frac{\sum_{i=1}^{N} (y_i - x_i)^2}{N - n}$$
$$R^2 = \frac{\sum_{i=1}^{N} (y_i - z_i)^2 - \sum_{i=1}^{N} (y_i - x_i)^2}{\sum_{i=1}^{N} (y_i - z_i)^2}$$

Where n is the number of observations, y_i is the frequency of observation, x_i is the frequency of Weibull, z_i is the mean wind speed, n is the number of constants used.

TABLE:V STATISTICAL TESTS

TESTS	EM	MLM	MMLM	EPFM
RMSE	0.0400	0.0390	0.0398	0.0397
ChiSquare	0.0015	0.0015	0.0016	0.0016
R ²	0.9174	0.9213	0.9182	0.9185

 TABLE VI

 COMPARISON BETWEEN the WIND SPEED DATA by the PREDICTED METHODS and MEASURED WIND SPEED

Methods	Mean wind speed	Error %
	(m/s)	
Measured	6.9780	-
EM	6.9811	0.0444
MLM	6.9164	-0.8827
MMLM	6.9572	-0.2980
EPFM	6.9769	-0.0157







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VII.RESULTS AND DISCUSSIONS

Four methods for estimating the parameters of the Weibull wind speed distribution for the wind analysis data of the Kanya Kumari district are presented. The application of each method is demonstrated using a sample wind speed data set and the accuracy of each method is compared with measured data obtained from metrological station for our Kanyakumari district of India[Table -VI]. Various tests were used to analyse the accuracy of the compared methods[Table-V].

VIII.CONCLUSION

According to the results, it might be concluded that suitability of these methods may vary with the sample data such as data size, sample data distribution, sample data format and of fit tests. The Energy pattern factor method is an efficient method for determining the k and c parameters to fit Weibull distribution for the wind speed data at Kanya Kumari district of India. This fact is also supported by means of the Fig[I]. It is also observed from the statistical analysis that the values of RMSE, Chi-Square, R^2 have magnitudes close to each other for all the methods for the Kanyakumari district.

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