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Bianchi Type-IX Radiating Cosmological Model in Self-Creation Cosmology

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ABSTRACT:A Bianchi type-IX cosmological model has been investigated in the presence of perfect fluid with disordered radiation in Barbers (Gen. Relat. Gravit.14, 117, 1982) second self-creation theory of gravitation. The field equations have been solved by applying special law of variation for generalized Hubble's parameter given by Bermann (NuovoCimento 74, 182, 1983). Some physical properties of the models are also discussed.

KEYWORDS:Self-Creation Cosmology, Radiating Model, Bianchi type-IX .

I. INTRODUCTION

Einstein's theory of general relativity (Einstein 1916) is a very successful gravitational theory in describing the gravitational phenomena which also served as a basis for the models of the universe. Einstein pointed out that general relativity does not account satisfactorily for inertial properties of matter i.e. Mach's principle is not substantiated by general relativity. There have been several attempts to generalize the general theory of gravitation by incorporating Mach's principle and other desired features which were lacking in the original theory. Barber [1] has proposed two self-creation theories of gravitation by modifying Brans-Dicke theory and general relativity. Brans [2] has pointed out that Barber's first theory is in agreement with experiment but is actually inconsistent in general as it violates equivalence principle. However second theory is a modification of general relativity to a variable G-theory and predicts local effects that are within the observational limits. In this theory, the scalar field does not gravitate directly but simply divides the matter tensor, acting as a reciprocal gravitational constant. Here the scalar field couples to the trace of the energy momentum tensor.

Pimentel [3] and Soleng [4] have discussed FRW models by using a power law relation between the expansion factor of the universe and the scalar field. Singh [5], Reddy [6] and Reddy et al. [7] have studied Bianchi type space-times solutions in Barber's second theory of gravitation while Reddy and Venkateswarlu [8] presented Bianchi type-VI_0 cosmological model in Barber's second self-creation theory of gravitation. Shanthi and Rao [9] studied Bianchi type II and III space-times in this theory, both in vacuum as well as in presence of stiff fluid. Ram and Singh [10] have discussed the spatially homogeneous and isotropic Robertson-Walker and Bianchi type-II models of the universe in Barber's self-creation theory in presence of perfect fluid by using gamma law equation of state. Pradhan and Pandey [11], Pradhan and Vishwakarma [12], Panigrahi and Sahu [13], Venkateshwarlu and Kumar [14], Singh and Kumar [15], Venkateswarlu*et al.* [16], Reddy and Naidu [17] and Katore*et al.* [18] are some of the authors who have studied various aspects of cosmological models in Barber's second self-creation theory. Katore*et al.* [19] have studied accelerating and decelerating hypersurface-homogeneous cosmological models in Barbers's second self-creation theory. Recently, Mahanta [20] have studied dark energy (DE) models with variable EoS parameter in self-creation theory of gravitation.

Bianchi type-IX cosmological models are very popular for relativistic studies. These models are also used to examine the role of certain anisotropic sources during the formation of large scale structures as we seen the universe today. Chakraborty [21], Raj Bali and Dave [22], Raj Bali and Yadav [23] have studied Bianchi type-IX string as well as viscous fluid models in general relativity. Pradhan [24] have studied some homogeneous Bianchi type-IX viscous fluid cosmological models with varying Λ . Tyagi*et al.* [25] have obtained Bianchi type-IX string cosmological models for perfect fluid distribution in general relativity. Ghate and Sontakke [26-27] have studied Bianchi type-IX cosmological models in different context.

In this paper, Bianchi type-IX cosmological model has been investigated in the presence of perfect fluid with disordered radiation. The solution to the Einstein field equations are obtained using the condition that expansion scalar is proportional to the shear scalar given by Berman. The physical and geometrical properties of model are also discussed.

II. METRIC AND FIELD EQUATIION

Bianchi type-IX metric is considered in the form

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 dy^2 + (b^2 \sin^2 y + a^2 \cos^2 y) dz^2 - 2a^2 \cos y dx dz \,, \tag{1}$$

where*a* and *b* are functions of cosmic time *t*.

The field equations in Barber's [1] second self-creation theory are

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi\phi^{-1} T_{ij} \,, \tag{2}$$

and

$$\phi^{k}_{;k} = \frac{8\pi}{3} \lambda T \,, \tag{3}$$

where$T$ is the trace of the energy-momentum tensor, λ is a coupling constant to be determined from the experiment $(|\lambda| \le 0.1)$ and semi-colon denotes covariant differentiation. In the limit as $\lambda \to 0$, this theory approaches the standard general relativity theory in every respect and $G = \phi^{-1}$.

The energy momentum tensor for T_{ij} for perfect fluid distribution is given by

$$T_{ij} = (\rho + p) u_i u_j + p g_{ij} \,, \tag{4}$$

together with

$$g_{ij} u^i u_j = -1 \,, \tag{5}$$

where u^i is the four velocity vector, p and ρ are proper pressure and energy density.

In a commoving coordinate system, from equation (4), we have

$$T^1_1 = T^2_2 = T^3_3 = p \,,\; T^4_4 = -\rho \tag{6}$$

and

$$T = 3p - \rho \,. \tag{7}$$

Here, we use the equation of state

$$\rho = 3p \,, \tag{8}$$

which represents disordered radiation of matter distribution.

The field equations (2) and (3), for the metric (1) with the help of equations (4)-(6) in Barber's second self-creation theory can be explicitly written as

$$2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{a^2}{4b^4} = \frac{8\pi}{\phi}\rho \,, \tag{9}$$

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{3a^2}{4b^4} = -\frac{8\pi}{\phi} p \,, \tag{10}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + \frac{a^2}{4b^4} = -\frac{8\pi}{\phi} p \,, \tag{11}$$

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$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right) = -\frac{8\pi}{3}\lambda T\,, \tag{12}$$

whereoverdot ($\dot{}$) denotes partial differentiation with respect to *t*.

III. SOLUTIONS OF THE FIELD EQUATIONS

Equations (9)-(12) are four highly non-linear equations in six unknowns a, b, ρ, p, ϕ and T. We can introduce more conditions either by an assumption corresponding to some physical situation or an arbitrary mathematical supposition, however these procedures have some drawbacks. Physical situation may lead to differential equations which will be difficult to integrate and mathematical supposition may lead to a non-physical situation. Two additional conditions relating these unknowns are required to obtain explicit solutions of the systems.

(i) Firstly, we assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$a = b^m\,, \tag{13}$$

where*m* is proportionality constant.

The motive behind assuming condition is explained with reference to Thorne [28], the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent (Kantowski and Sachs [29]; Kristian and Sachs [30]). To put more precisely, red-shift studies place the limit $\frac{\sigma}{H} \le 0.3$ on the ratio of shear σ to Hubble constant *H* in the neighborhood of our galaxy today. Collin *et al.* [31] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies the condition $\frac{\sigma}{\theta}$ is constant.

(ii) Secondly, the law of variation of Hubble's parameter yields a constant value of deceleration parameter. Such type of relations have already been considered by Berman [32] for solving FRW models.

We consider the constant deceleration parameter model defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = \text{constant}, \tag{14}$$

where the scale factor *R* is given by

$$R = \left(a b^2\right)^{\frac{1}{3}}. \tag{15}$$

Here the constant is taken as negative (*i.e.*, it is accelerating model of the universe).

The solution of equations (13) and (14), gives

$$R = \left(\alpha t + \beta\right)^{\frac{1}{1+q}}\,, \tag{16}$$

where $\alpha(\neq 0)$, and β are constants of integration.

This condition implies that the condition of expansion is $1 + q > 0$.

Solving the field equations (9)-(11) with the help of equations (13), (15) and (16), we obtain the expansion for metric coefficients as follows:

$$a = (\alpha t + \beta)^{\frac{3m}{(1+q)(m+2)}}\,, \tag{17}$$

$$b = (\alpha t + \beta)^{\frac{3}{(1+q)(m+2)}}\,. \tag{18}$$

Using equations (7), (8), (17) and (18), equation (12) leads to

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$$\phi = \left[\frac{\phi_0}{\alpha}\left(\frac{1+q}{q-2}\right)\right](\alpha t+\beta)^{\frac{(q-2)}{(1+q)}}. \tag{19}$$

Through a proper choice of coordinates and constants, Bianchi type-IX radiating cosmological model in Barbers second self-creation cosmology can be written as

$$\begin{aligned} ds^2 &= -dT^2 + T^{\frac{6m}{(1+q)(m+2)}}dx^2 + T^{\frac{6}{(1+q)(m+2)}}dy^2 + \left(T^{\frac{6}{(1+q)(m+2)}}\sin^2 y + T^{\frac{6m}{(1+q)(m+2)}}\cos^2 y\right)dz^2 \\ &\quad - 2T^{\frac{6m}{(1+q)(m+2)}}\cos y\,dx\,dz \end{aligned}. \tag{20}$$

IV. PHYSICAL PROPERTIES OF THE MODEL

The model (20) represents Bianchi type-IX disordered radiating cosmological model which is physically significant for the study of early stages of evolution of the universe.

The physical quantities spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2, scalar field ϕ, energy density ρ, pressure p, for the model (20) have the following expressions:

Spatial volume, $$V = T^{\frac{3}{1+q}}. \tag{21}$$

Hubble parameter, $$H = \frac{1}{(1+q)T}. \tag{22}$$

Expansion scalar, $$\theta = 3H = \frac{3}{(1+q)T}. \tag{23}$$

Mean Anisotropy Parameter, $$A_m = \frac{2(m-1)^2}{(m+2)^2} \tag{24}$$

Shear scalar, $$\sigma^2 = \frac{3(m-1)^2}{(1+q)^2(m+2)^2T^2}. \tag{25}$$

Scalar field, $$\phi = \left[\phi_0\left(\frac{1+q}{q-2}\right)\right]T^{\frac{q-2}{1+q}}. \tag{26}$$

Also, $$\frac{\sigma}{\theta} = \frac{(m-1)}{\sqrt{3}(m+2)} \neq 0. \tag{27}$$

The energy density, $$\rho = \frac{\phi_0(1+q)}{8\pi(q-2)}\left[\frac{9(2m+1)}{(1+q)^2(m+2)^2}T^{-\frac{(q+4)}{(1+q)}} + T^{\frac{mq-2m+2q-10}{(1+q)(m+2)}} - \frac{1}{4}T^{\frac{mq+4m+2q-16}{(1+q)(m+2)}}\right], \tag{28}$$

where is ϕ_0 constant of integration.

Pressure, $$p = \frac{\phi_0(1+q)}{24\pi(q-2)}\left[\frac{9(2m+1)}{(1+q)^2(m+2)^2}T^{-\frac{(q+4)}{(1+q)}} + T^{\frac{mq-2m+2q-10}{(1+q)(m+2)}} - \frac{1}{4}T^{\frac{mq+4m+2q-16}{(1+q)(m+2)}}\right]. \tag{29}$$

V. RESULTS AND DISCUSSION

Initially, when $T=0$, the spatial volume V is zero while Hubble parameter H, expansion scalar θ, shear scalar σ^2, energy density ρ, pressure p diverge. Also when $T\rightarrow\infty$, we observe that the spatial volume V and scalar field ϕ tend to infinity while H, θ, σ^2, ρ, and p become zero. Also $\lim_{T\rightarrow\infty}\frac{\sigma^2}{\theta^2}=constant\,(\neq 0)$ and $A_m\neq 0$, hence the model does not approach isotropy throughout the evolution.
In this model the particle horizon exists because,

$$\int_{T_0}^{T}\frac{dT}{V(t)}=\left(\frac{1+q}{q-2}\right)\left(T^{\frac{q-2}{1+q}}\right)_{T_0}^{T}$$

is a convergent integral.

VI. CONCLUSION

In this paper, we have investigated radiating Bianchi type-IX cosmological model in Barbers second self-creation theory of gravitation using the law of variation for the Hubbles parameter which yields a constant value of decelerating parameter stated by Berman. At initial epoch the model starts evolving with zero volume and expands for large time where the rate of expansion decreases with increase of time. Since, $\lim_{T\rightarrow\infty}\frac{\sigma^2}{\theta^2}=constant\,(\neq 0)$ indicating that the model does not approach isotropy for large time. It is also observed that, $A_m\neq 0$, which indicates the model is anisotropic and represents the early stages of the universe which matches the results of recent experiments which shows that there is certain amount of anisotropy in the universe. Hence the anisotropic models are important. It is well known that the scalar fields have considerable effects on cosmological scale. The radiating model obtained here, will help to study early era in self-creation cosmology. Also, this study of cosmological models and understanding our universe through these models has astrophysical importance and relevance.

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