

An Innovative Method for Finding Optimal Solution to Assignment Problems

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ABSTRACT: In this paper an innovative method named by TVAM is proposed to find an optimal solution for any assignment problem in single stage. Here the assignment problem is solved to get optimal solution directly. A numerical illustration is established and the optimality of the result yielded by this method is also checked. This method is more lucrative for decision makers and efficient while comparing with other existing methods because of its simplicity and less calculation time.

KEYWORDS: assignment problem;TVAM;optimal solution, maximization.

I.INTRODUCTION

The assignment problem is one of the earliest applications and is a special case of linear programming problem, which deals with available sales-force to different regions: vehicles to routes; products to factories; contracts to bidders; machines to jobs; development engineering to several construction sites and so on. Since all supplies, demands, and bounds on variables are integral, the assignment problem relies on a nice property of transportation problems that the optimal solution will be entirely integral. As you will see, the goal of the objective of the assignment problem (unlike transportation problems) may not have anything to do at all with moving anything.

Generally the management makes assignment on a one-to-one basis in such a manner that the group maximizes the revenue from the sales; the vehicles are deployed to various routes in such a way that the assignment cost is minimum and so on.

Applications of assignment problems are varied in the real world. Certainly it can be useful for the classic task of assigning employees to tasks or machines to production jobs, but its uses are more widespread. It could be used to assign fleets of aircrafts to particular trips, or assigning school buses to routes, or networking computers. In rare cases, it can even be used to [determine marriage partners](#).

A considerable number of methods have been so far presented for assignment problem in which the Hungarian method is more convenient method among them. Different methods have been presented for transportation problem and various articles have been published on the subject. Many of the authors [1] - [3] have done the transportation problems in different methods. Pandian and Natarajan [4] proposed new method for finding an optimal solution directly for transportation problem. An Optimal Solution for Transportation Problems solved by [5], [6] using zero suffix method and ASM method respectively. The more for less method to distribution related problems was established by [7].

By a complete assignment for a cost matrix $n \times n$, we mean an assignment plan containing exactly n assigned independent zeros, one in each row and one in each column. The main concept of assignment problem is to find the optimum allocation of a number of resources to an equal number of demand points. An assignment plan is optimal if optimizes the total cost or effectiveness of doing all the jobs.

Hence we proposed a new method named by TVAM, which is different from the preceding methods to apply in the assignment problems.

This proposed method has the following advantages

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- i. The assignment problem is solved in single stage.
- ii. Test of optimality like Hungarian method and other test of optimality no need.
- iii. Unique optimal solution is reached.

II. MATHEMATICAL FORMULATION OF ASSIGNMENT PROBLEM

$$\text{Let } X_{ij} = \begin{cases} 1 & \text{if } i^{\text{th}} \text{ job is assigned to } j^{\text{th}} \text{ facility} \\ 0 & \text{if } i^{\text{th}} \text{ job is not assigned to } j^{\text{th}} \text{ facility} \end{cases}$$

Determine X_{ij} $i, j = 1, 2, 3, \dots, m$ so as to

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^m C_{ij} x_{ij} \quad (1)$$

$$\sum_{i=1}^m x_{ij} = 1 \quad j = 1, 2, 3, \dots, m \quad (2)$$

$$\sum_{j=1}^m x_{ij} = 1 \quad i = 1, 2, 3, \dots, m \quad (3)$$

$$x_{ij} = 0 \text{ or } 1 \quad (4)$$

Where C_{ij} is the cost or effectiveness of assigning i^{th} job to j^{th} machine x_{ij} is to be some positive integer or zero and the only possible integer is one. So the condition of $x_{ij} = 0$ or 1 is automatically satisfied.

Associated to each assignment problem there is a matrix called cost or effectiveness matrix C_{ij} , where C_{ij} is the cost of assigning i^{th} job to j^{th} facility. In this paper we call it assignment matrix, and represent it as follows

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} & \dots & C_{1n} \\ C_{21} & C_{22} & C_{23} & \dots & C_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ C_{n1} & C_{n2} & C_{n3} & \dots & C_{nn} \end{pmatrix}$$

An assignment problem could thus be solved by simplex method. It also happens to be an $n \times n$ transportation problem with each $a_i = b_i = 1$. However, as an assignment problem is highly degenerate, it will be frustrating to attempt to solve it by simplex method. In fact a very convenient iterative procedure is available for solving an assignment problem.

III. THEOREM

If a constant is added or subtracted to every element of a row or column of the cost matrix of an assignment problem, the resulting assignment problems has the same optimal solution as the original problem and vice versa, which was established by [4].

Alternatively, if $x_{ij} = x_{ij}^*$ minimizes

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$$Z = \sum_{i=1}^m \sum_{j=1}^m C_{ij} x_{ij} \text{ over all } x_{ij} \text{ such that}$$

$$\sum_{i=1}^m x_{ij} = \sum_{j=1}^m x_{ij} = 1 \geq 0 \text{ then } x_{ij} = x_{ij}^* \text{ also}$$

$$Z^* = \sum_{i=1}^m \sum_{j=1}^m C_{ij}^* x_{ij}$$

where $C_{ij}^* = C_{ij} \pm u_i \pm v_j$ for all $i, j = 1, 2, 3, \dots, m$ and u_i, v_j are real number.

Proof:

$$\begin{aligned} Z^* &= \sum_{i=1}^m \sum_{j=1}^m C_{ij}^* x_{ij} \\ &= \sum_{i=1}^m \sum_{j=1}^m (C_{ij} \pm u_i \pm v_j) x_{ij} \\ &= \sum_{i=1}^m \sum_{j=1}^m C_{ij} x_{ij} \pm \sum_{i=1}^m u_i \sum_{j=1}^m x_{ij} \pm \sum_{i=1}^m x_{ij} \sum_{j=1}^m v_j \\ &= \sum_{i=1}^m \sum_{j=1}^m C_{ij} x_{ij} \pm \sum_{i=1}^m u_i \pm \sum_{i=1}^m v_i \end{aligned}$$

since $\sum_{i=1}^m x_{ij} = \sum_{j=1}^m x_{ij} = 1$

$$Z^* = Z \pm \sum_{i=1}^m u_i \pm \sum_{i=1}^m v_i$$

This shows that the minimization of the new objective function Z^* yields the same solution as the minimization of original objective function Z , because $\sum u_i$ and $\sum v_j$ are independent of x_{ij} .

IV. ALGORITHM

The TVAM method proceeds as follows

- Step 1 Construct the assignment table for the given assignment problem and then convert it into a balanced one, if it is not.
- Step 2 Suppose given problem is maximization (i.e. assign person to jobs in such a way that the total profit is maximized) first converted into an equivalent minimization problem. Take the highest cost element in that cost matrix, and subtract all the cost element of the cost matrix.
- Step 3 Find the difference between the smallest and nextsmallest element in each row (row penalties) and each column (column penalties) and write them in brackets against the corresponding row and corresponding column.
- Step 4 Identify the row or column with largest element, and choose the cell with smallest cost in that selected row or selected column and allocate the assignment. Cross out the row and column against the assignment cell.
- Step 5 Repeat steps 3 and 4 if each row and each column contain exactly one assignment.

V. EXAMPLE

Consider the following assignment problem. Assign the five jobs to the five machines so as to minimize the total cost.

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$$\begin{pmatrix} 20 & 30 & 25 & 15 & 35 \\ 25 & 10 & 40 & 12 & 28 \\ 15 & 18 & 22 & 32 & 24 \\ 29 & 8 & 34 & 10 & 40 \\ 35 & 23 & 17 & 26 & 45 \end{pmatrix}$$

By applying the proposed algorithm the optimal feasible solution is shown below.

	A	B	C	D	E	
[5]	15	10	1	1		5
15	[0]	30	2	2		2 2 2 2
0	3	7	3	[0]		0
21	0	26	4	3		2 2 2 2
18	6	[0]	5	2		6 6 6
			5	0	0	2 9
			10	0	10	2
				0	10	0

The optimal assignment schedule is Job 1 → A, Job 2 → B, Job 3 → E, Job 4 → D, Job 5 → C.
Finally the optimal assignment cost is 81.

VI.CONCLUSION

The proposed algorithm carries systematic procedure, and very easy to understand. It can be extended to travelling salesman problems to get optimal solution. From this paper, it can be concluded that TVAM Method provides an optimal solution directly, in fewer iterations, for the assignment problems. As this method consumes less time and is very easy to understand and apply, so it will be very helpful for decision makers.

REFERENCES

- [1] A. Edward Samuel, and M. Venkatachalapathy, "Modified Vogel's Approximation method for Fuzzy Transportation Problem", Applied Mathematical Science, Vol.5, No.28, pp.1367-1372, 2011.
- [2] NagarajBalakrishnan, "Modified Vogel's Approximation Method for Unbalanced Transportation Problem", Applied Mathematics Letters 3, pp.9-11, 1990.
- [3] A. Nagooragani, "Fuzzy Transportation problem, Proceedings of national Seminar on Recent Advancement in Mathematics, 2009.
- [4] P. Pandian, and G. Natarajan, "A new method for finding an optimal solution for Transportation problem," International Journal of Mathematical Sciences and Engineering Applications, Vol. 4, pp. 59-65, 2010.
- [5] V. J. Sudhakar, N. Arunsankar, and T. Karpagam, "A New approach for finding an optimal Solution for transportation problem", Applied European Journal of Scientific Research, Vol.68, pp. 254-257, 2012.
- [6] Abdul Quddoes, ShakeelJavaid, and M. M. Khalid, "A New method for finding an Optimal Solution for Transportation Problem", International Journal on Computer Science and Engineering, Vol.4, No.7, July 2012.
- [7] VeenaAdlakha, and Krzysztof Kowalski, "A heuristic method for more-for less in distribution related problems", Internal Journal of Mathematical Education in Science and Technology, Vol. 32, pp. 61-71, 2001.
- [8] H. A. Taha, Operations Research- Introduction, Prentice Hall of India, New Delhi, 2004.
- [9] J. K. Sharma, Operations Research- Theory and applications, Macmillan India LTD, New Delhi, 2005.
- [10] KantiSwarup, P. K. Gupta and Man Mohan, Operations Research, Sultan Chand & Sons, 12th Edition, 2004.
- [11] P. K. Gupta, D. S. Hira, Operations Research, S. Chand & Company Limited, 14th Edition, 1999.

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BIOGRAPHY



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