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A New Kind of Lacunary Interpolation through g-Splines

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ABSTRACT: In this paper, we construct A new technique of spline methods is used for $(0,3,4)$ - lacunary interpolation by g-splines and their local approximation with functions belonging to $C^{(4)}(I)$ and $C^{(5)}(I)$ using piecewise polynomials with certain specific properties. Our methods are of lower degree having better convergence property than the earlier investigations .

KEYWORDS – Spline function, lacunary interpolation, quintic splines piecewise polynomial.

I.INTRODUCTION

The spline interpolation is based on the following principle, the interpolation interval is divided into small subintervals. The polynomial coefficients are chosen to satisfy certain conditions. A “spline” was a common drafting tool a flexible rod, that was used to help draw smooth curves connecting widely spaced points. Spline interpolation method, as applied to the solution of differential equation employ some from approximating function such as polynomials to approximate the solution by evaluating the function for sufficient number of points in the domain of the solution.

Spline functions are a good tool for the numerical approximation of functions on one hand and they suggest new challenging and rewarding problem's on the other hand. Piecewise linear functions as well as step functions have along been important theoretical and practical tools for approximation of function. Lacunary interpolation by splines appears whenever observation gives scattered or irregular information about a function and its derivatives. The data in the problem of lacunary interpolation has also values of the functions and its derivatives but without Hermite conditions that only consecutive derivative is used at each node. Spline function are arise in many problems of mathematical Physics such as viscoelasticity, hydrodynamics, electromagnetic theory, mixed boundary problems in mathematical physics, biology and Engineering.

One uses polynomial for approximation because they can be evaluated. cubic spline interpolation is the most common piecewise polynomial method and is referred as “piecewise” since a unique polynomials is fitted between each pair of data points. The collection of polynomials that form the curve of polynomials that form the curve is collectively referred to as “ the spline”. The traditional and constrained cubic splines are few different groups of the same family. The group of traditional cubic splines can furthermore be divided into sub group natural, parabolic, runout, cubic run-out and damped cubic splines. The natural cubic spline is by far the most popular and widely used version of the cubic splines family.

Th Fawzy ([3] [4]) constructed special kinds of lacunary quintic g-splines and proved that for functions $f \in C^{(4)}$ the method converges faster that investigated by A.K. Verma[1] and for functions $f \in C^{(5)}$ the order of approximation is the same as the best order of approximation using quintic g- splines. In the last four decades Birkhoff has renewed this theory on lacunary interpolation or interpolation by splines. Several researchers have studied the use of splines to solve such interpolation problems [2] [5] [7]. All of these methods are global and require the solution of a large system of equations. In addition to the paper mentioned above dealing with best interpolation on approximation by splines there were also few papers that deal with constructive properties of space of splines interpolation. In my earlier work [6] [12] some kinds of lacunary interpolation by g-splines have been investigated. In this paper we will continue to discuss the problem.

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This paper is organized as follows- In Section 2, we construct a spline function of degree five which interpolates the lacunary data (0, 3, 4) some theorems about spline functions are also studied in section 2 and section 3. In section 3 we construct almost quartic spline interpolant – (0, 3, 4). Our methods are of lower degree having better convergence property than the earlier investigations [8] [9] [10] [11].

II. SPLINE INTERPOLANT (0, 3, 4) FOR $f \in C^{(5)}(I)$

Let

$$\Delta: 0 = x_0 < x_1 < \dots < x_{n-1} < x_n = 1$$

be a partition of the interval $I = [0,1]$ with $X_{k+1} - x_k = h_k, k = 0(1)n - 1$.

and

$s_{3,\Delta}$ be a piecewise polynomial of degree ≤ 5 , which is a solution of (0, 3, 4)- interpolation for functions $f \in C^{(5)}[x_0, x_n]$ in the form :

$$(2.1) s_{3,\Delta}(x) = s_{3,k}(x) = \sum_{j=0}^5 \frac{s_{kj}^{(3)}}{j!} (x - x_k)^j, x_k \leq x \leq x_{k+1} \text{ for } k = 0(1)n - 1,$$

for $j = 0,3,4, k = 0(1)n-1$

$$(2.2) s_{kj}^{(3)} = f_k^{(j)}, j = 0,3,4$$

$$(2.3) s_{k,5}^{(3)} = \frac{20}{h^5} [(f_{k+1} - f_k - \frac{h^3}{3!} f_k^{(3)} - \frac{h^4}{4!} f_k^{(4)}) - h (f_{k+1}^{(1)} - \frac{h^2}{2!} f_k^{(3)} - \frac{h^3}{3!} f_k^{(4)}) + \frac{h^2}{2!} (f_{k+1}^{(2)} - h f_k^{(2)})]$$

$$(2.4) s_{k,2}^{(3)} = \frac{10}{3h^2} [h(f_{k+1}^{(1)} - \frac{h^2}{2!} f_k^{(3)} - \frac{h^3}{3!} f_k^{(4)}) - (f_{k+1} - f_k - \frac{h^3}{3!} f_k^{(3)} - \frac{h^4}{4!} f_k^{(4)}) - \frac{h^5}{5!} (f_{k+1}^{(2)} - h f_k^{(2)})]$$

and

$$(2.5) s_{k,1}^{(3)} = \frac{1}{h} [(f_{k+1} - f_k - \frac{h^3}{3!} f_k^{(3)} - \frac{h^4}{4!} f_k^{(4)} - \frac{h^2}{2!} s_{k,2}^{(3)} - \frac{h^5}{5!} s_{k,5}^{(3)})]$$

The relation (2.3)-(2.5) have been obtained for condition :

$$s_{3,\Delta}(x) \in C^{[0,3,4]} [x_0, x_n]$$

If $f \in C^{(5)}(I)$ than owing to (2.3)-(2.5) and using Taylor's expansion, we have

$$(2.6) |s_{kj}^{(3)} - f_k^{(j)}| \leq C_{kj}^{(3)} h^{5-j} \omega(f^{(5)}; h), j = 1, 2, 5;$$

Where the constant are given by :

$$C_{k,1}^{(2)} = \frac{61}{720}, C_{k,2}^{(2)} = \frac{5}{36}, \text{ and } C_{k,5}^{(2)} = \frac{11}{6}$$

Using (2.1) - (2.6) and a little computation gives :

Theorem 2.1

Let $f \in C^{(5)}(I)$ and $s_{3,\Delta}^{(0,3,4)}(I)$ be the unique quintic spline interpolant (0, 3, 4) given in (2.1) - (2.5)

),
then

$$(2.7) || D^{(j)} (f - S_{3,\Delta}) || \leq c_{3,k}^{(j)} h^{5-j} \omega(f^{(5)}, h), j=0(1)5; k=0(1)n-1$$

Where the constants $c_{3,k}^{(1)}$, s are given by :

$$c_{3,k}^0 = \frac{61}{360}, c_{3,k}^{(1)} = \frac{83}{360}, c_{3,k}^2 = \frac{4}{9}, c_{3,k}^{(3)} = \frac{11}{12}, c_{3,k}^{(4)} = \frac{11}{6}, c_{3,k}^{(5)} = \frac{11}{6}.$$

III. ALMOST QUARTIC SPLINE INTERPOLANT (0, 3, 4) FOR $f \in C^{(4)}(I)$

Almost quartic spline interpolant is a piecewise polynomial of degree 4 in each subinterval except in the last one, where it is a polynomial of degree 5. In this case, we have

$$(3.1) S_{3,\Delta}^*(x) = S_{3,k}^*(x) = \sum_{j=0}^4 \frac{S_{kj}^{*(3)}}{j!} (x - x_k)^j, x_k \leq x \leq x_{k+1}, k = 0(1)n - 2$$

$$= \sum_{j=0}^5 \frac{S_{n-1,j}^{*(3)}}{j!} (x - x_{n-1})^j, x_{n-1} \leq x \leq x_n, k = n - 1$$

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The coefficients $S_{k,j}^{*(3)}$ are explicitly given in terms of the data.

In particular, for $K = O(1)n-1$, we prescribe

$$(3.2) \quad S_{k,j}^{*(3)} = f_k^{(j)}, \quad j = 0, 3, 4.$$

For $k = O(1)n-2$ and $j = 1, 2$, $S_{k,j}^{*(3)}$ are given by

$$(3.3) \quad S_{k,1}^{*(3)} = \frac{2}{h} \left[(f_{k+1} - f_k - \frac{h^3}{3!} f_k^{(3)} - \frac{h^4}{4!} f_k^{(4)}) - \frac{h}{2} (f_{k+1} - f_k - \frac{h^2}{2!} f_k^{(3)} - \frac{h^3}{3!} f_k^{(4)}) \right]$$

and

$$(3.4) \quad S_{k,2}^{*(3)} = -\frac{2}{h^2} \left[-h \left(f'_{k+1} - \frac{h^2}{2!} f_k^{(3)} - \frac{h^3}{3!} f_k^{(4)} \right) + \left(f_{k+1} - f_k - \frac{h^3}{3!} f_k^{(3)} - \frac{h^4}{4!} f_k^{(4)} \right) \right]$$

For $k=n-1$ and $j=1, 2$ and 5 , we have

$$(3.5) \quad S_{n-1,5}^{*(3)} = \frac{20}{h^5} \left[\{f_n - f_{n-1} - \frac{h^3}{3!} f_{n-1}^{(3)} - \frac{h^4}{4!} f_{n-1}^{(4)}\} - h \{f_n^{(1)} - \frac{h^2}{2!} f_{n-1}^{(3)} - \frac{h^3}{3!} f_{n-1}^{(4)}\} + \frac{h^2}{2} \{f_n^{(2)} - h f_{n-1}^{(3)}\} \right]$$

$$(3.6) \quad S_{n-1,2}^{*(3)} = \frac{10}{3h^2} \left[h \{f_n^{(1)} - \frac{h^2}{2!} f_{n-1}^{(3)} - \frac{h^3}{3!} f_{n-1}^{(4)}\} - \{f_n - f_{n-1} - \frac{h^3}{3!} f_{n-1}^{(3)} - \frac{h^4}{4!} f_{n-1}^{(4)}\} - \frac{h^5}{5!} \{f_n^{(2)} - h f_{n-1}^{(3)}\} \right]$$

and

$$(3.7) \quad S_{n-4,1}^{*(3)} = \frac{1}{h} \left[f_n - f_{n-1} - \frac{h^3}{3!} f_{n-1}^{(3)} - \frac{h^4}{4!} f_{n-1}^{(4)} - \frac{h^2}{2!} S_{n-1,2}^{*(3)} - \frac{h^5}{5!} S_{n-1,5}^{*(3)} \right]$$

(4.10) and (4.11) are obtained from the condition.

$$(3.8) \quad S_{3,\Delta}^* \in C^{(1)} [x_0, x_n],$$

While the relations (4.12)-(4.14) are obtained from the interpolating conditions.

As in (4.6) for $f \in C^{(4)} [I]$, one can establish

$$(3.9) \quad |S_{k,j}^{*(3)} - f_k^{(j)}| \leq C_{k,j}^{*(3)} h^{4-j} \omega(f^{(4)}, h),$$

Where the constants $C_{k,j}^{*(3)}$ are given by

$$C_{k,j}^{*(3)} = \begin{cases} \frac{1}{4}, & j = 1 \\ \frac{5}{12}, & j = 2 \end{cases}; \quad k=O(1)n-2$$

$$|S_{n-1,1}^{*(3)} - f^{(1)}(x_{n-1})| \leq \frac{61}{144}$$

$$h^2 \omega(f^{(4)}, h) + \frac{3}{8} h^3 \|f^{(4)}\|$$

$$|S_{n-1,2}^{*(3)} - f^{(2)}(x_{n-1})| \leq \frac{25}{36} h^2 \omega(f^{(4)}, h) + \frac{1}{3} h^2 \|f^{(4)}\|$$

$$|S_{n-1,5}^{*(3)}| \leq \frac{25}{6} \frac{1}{h} \omega(f^{(4)}, h) + \frac{5}{h} \|f^{(4)}\|$$

Finally we have

Theorem 3.1

Let $f \in C^{(4)} [I]$ and $S_{3,\Delta}^*$ be the unique almost quartic spline interpolant $(0, 3, 4)^*$, given by (4.8), then

$$(3.10) \quad \|\mathcal{D}^{(j)}(f - S_{3,\Delta}^*)\|_{L_\infty [x_k, x_{k+1}]} \leq C_{3,k}^{*(j)} h^{4-j} \omega(f^{(4)}, h),$$

Where the constants $C_{3,k}^{*(j)}$ are given by :

	$C_{3,k}^{*(0)}$	$C_{3,k}^{*(1)}$	$C_{3,k}^{*(2)}$	$C_{3,k}^{*(3)}$	$C_{3,k}^{*(4)}$
$k=O(1)n-2$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{11}{12}$	1	1
$K=n-1$	$\frac{117}{144} + \frac{13}{24} \ f^{(4)}\ $	$\frac{105}{144} + \frac{17}{24} \ f^{(4)}\ $	$\frac{43}{36} + \frac{1}{3} \ f^{(4)}\ $	1	1

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IV. CONCLUSION

In this paper, we have studied the existence and uniqueness of the quintic lacunary g-splines and almost quartic spline interpolant (0,3,4). Also we conclude that this new technique which we have used in the proving of two important theorems which are solutions of (0,3,4)- interpolation and obtained their local approximations with functions belonging to $C^{(4)}(I)$ and $C^{(5)}(I)$. Now we proved that our methods are of lower degree having better convergence property than the earlier investigations.

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