

Approximate Solution for Fuzzy Differential Algebraic Equations of Fractional Order Using Fractional Differential Transform Method

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ABSTRACT: In this paper we shall present an approximate solution for fuzzy differential algebraic equations of fractional order (FFDAEs) based on the fractional differential transform Method (FDTM) which is proposed to solve (FFDAEs). The fuzziness will appear in the boundary conditions, to be fuzzy numbers. The solution of the proposed model of equations are calculated in the form of a convergent series with easily computable components. Some examples are solved as an illustrations, the numerical results shows that the followed approach is easy to implement and accurate when applied to (FFDAEs).

KEYWORDS: fractional calculus, fuzzy set theory, fractional differential algebraic equation, fractional differential transform method.

I. INTRODUCTION

The subject of fractional calculus (that is calculus of integrals and derivatives of any arbitrary real or complex order) has gained considerable popularity and importance during the past three decades or so, due mainly to its demonstrated applications in numerous seemingly diverse and widespread fields of science and engineering. It does indeed provide several potentially useful tools for solving differential and integral equations, and various other problems involving special functions of mathematical physics as well as their extensions and generalizations in one and more variables [1]. In this paper the approximate solution of (FFDAEs) will be investigated. The concept of fuzzy sets which was originally introduced by Zadeh [2] leads to the definition of the fuzzy number and its implementation in fuzzy control and approximate reasoning problems. The basic arithmetic structure for fuzzy numbers was later developed by Mizumoto and Tanaka [3], Nahmias and Ralescu [4,5] all of which observed the fuzzy number as a collection of α levels, $0 < \alpha \leq 1$.

The basic concepts on fuzzy sets, fuzzy differentials and fuzzy differential equations can be found in [6,7,8,9,10]. The study of fuzzy differential equations (FDEs) forms a suitable setting for the mathematical modeling of real world problems in which uncertainty or vagueness pervades. The proposed method that will be used to approximate the solution (FFDAEs) is The concept of differential transform (DTM) was first proposed by Zhou [11] and it was applied to solve linear and nonlinear initial value problems in electric circuit analysis.

This method provides an iterative procedure to obtain the spectrum of the analytical solutions. The DTM has been successfully applied to a wide class of differential equations arising in many areas of science and engineering. Since many physical phenomena are more faithfully modeled by fractional differential equations (FDEs), Arikoglu and Ozkol [12], developed the fractional differential transform method (FDTM) for their efficient solution. Also Odibat and Shawagfeh [13] suggested the same technique as a generalized Taylor's formula for solving FDEs. The differential transform method evaluates the approximate solution by the finite Taylor series, but, in the differential transform

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method, the derivative is not computed directly; instead, the relative derivatives are calculated by an iteration procedure. New equations are obtained from the original equations by applying differential transformations.

This paper is organized as follows: In section two, we recall the basic concept of fuzzy set theory, in section three, the fractional order definitions of derivatives and integration are considered. Fractional differential transform method and some of its related theorems are given in section four our approach is presented in section five, and two numerical examples are solved in section six to illustrate the ability and efficiency of the proposed method.

II. SOME BASIC CONCEPT OF FUZZY SET THEORY

In this section, we shall present some basic definitions of fuzzy set theory including the definition of fuzzy numbers and fuzzy functions.

Definition (1), [14]:

Let X be any set of elements, a fuzzy set \tilde{A} is characterized by a membership function $\mu_{\tilde{A}}(X): X \rightarrow [0,1]$ and may be written as the set of points $\tilde{A} = \{(x, \mu_{\tilde{A}}) \mid x \in X, 0 \leq \mu_{\tilde{A}}(x) \leq 1\}$.

Definition (2), [14]:

The crisp set of elements that belong to the set A at least to the degree α is called the weak α -level set (or weak α -cut), and is defined by:

$A_{\alpha} = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$ While the strong α -level set (or strong α -cut) is defined by:

$A'_{\alpha} = \{x \in X: \mu_{\tilde{A}}(x) > \alpha\}$

Definition (3), [9]

A fuzzy subset \tilde{A} of a universal space X is convex if and only if the sets A_{α} are convex, $\forall \alpha \in [0,1]$. or equivalently, we can define convex fuzzy set directly by using its membership function to satisfy:

$\mu_{\tilde{A}}[\lambda x_1 + \mu_{\tilde{A}}(1 - \lambda)x_2] \geq \text{Min}\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, for all $x_1, x_2 \in X$ and $\lambda \in [0,1]$.

Remark (1), [15]:

A fuzzy number \tilde{M} may be uniquely represented in terms of its α -level sets, as the following closed intervals of the real line:

$$M_{\alpha} = [m - \sqrt{1 - \alpha}, m + \sqrt{1 - \alpha}]$$

Or

$$M_{\alpha} = [\alpha m, \frac{1}{\alpha} m]$$

Where m is the mean value of \tilde{M} and $\alpha \in (0,1]$. This fuzzy number may be written as $\tilde{M} = [\underline{M}, \overline{M}]$, where \underline{M} refers to the greatest lower bound of \tilde{M} and \overline{M} to the least upper bound of \tilde{M} .

Remark (2), [15]:

Similar to the second approach given in remark (1), one can fuzzyfy any crisp or non fuzzy function f , by letting:

$\underline{f}(X) = \alpha f(x), \overline{f} = \frac{1}{\alpha} f(x), x \in X, \alpha \in (0,1]$, and hence the fuzzy function in terms of its β -levels is given by

$$f_{\alpha} = [\underline{f}, \overline{f}].$$

III. FRACTIONAL ORDER DERIVATIVE AND INTEGRATION

In this section we shall present some definitions of fractional order derivatives and integral which will be used further in this paper [12].

Definition (4), [16]:

The Riemann-Liouville fractional integral operator of order $q > 0$ is defined as:

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$$I^q f(x) = \frac{1}{\Gamma(q)} \int_0^x (x-t)^{q-1} f(t) dt, q > 0, x > 0 \quad \dots(1)$$

$$I^0 f(x) = f(x)$$

Definition (5)[16]:

The Riemann-Liouville fractional derivative operator of order $q > 0$ is defined as:

$$D_x^q f(x) = \frac{1}{\Gamma(m-q)} \frac{d^m}{dx^m} \int_0^x (x-t)^{m-q-1} f(t) dt \quad \dots(2)$$

Where m is an integer and $m-1 < q \leq m$

Definition (6)[16]:

Caputo fractional derivative operator of order $q > 0$ is defined as:

$${}^c D_x^q f(x) = \frac{1}{\Gamma(m-q)} \int_0^x (x-t)^{m-q-1} \frac{d^m}{dx^m} f(t) dt \quad \dots(3)$$

Caputo fractional derivative has a useful property:

$$I^q {}^c D_x^q f(x) = f(x) - \sum_{k=0}^{m-1} f^{(k)}(0^+) \frac{x^k}{k!} \quad \dots(4)$$

Where m is an integer and $m-1 < q \leq m$

And similar to integer order differentiation Caputo's fractional differentiation is a linear operation i.e:

$${}^c D_x^q f(x) [\lambda f(t) + \mu g(t)] = \lambda {}^c D_x^q f(x) + \mu {}^c D_x^q g(x).$$

Where λ and μ are constants, for Caputo derivative also we have

$${}^c D_x^q c = 0, \text{ c is constant}$$

$${}^c D_x^q f(x) \begin{cases} 0, & \text{for } n \in N_0 \text{ and } n < [q] \\ \frac{c^{(n+1)}}{c^{(n+1-q)}}, & \text{for } n \in N_0 \text{ and } n \geq [q] \end{cases}$$

IV. FRACTIONAL DIFFERENTIATION TRANSFORM METHOD[17]

DTM has been developed as follows:

The fractional differentiation in Riemann-Liouville sense was defined by

$$D_{x_0}^q f(x) = \frac{1}{\Gamma(m-q)} \frac{d^m}{dx^m} \left[\int_{x_0}^x \frac{f(t)}{(x-t)^{1+q-m}} dt \right],$$

For $n-1 < m \leq n, n \in \mathbb{Z}^+, x > x_0$. Let us expand the analytical and continuous function $f(x)$ in terms of fractional power series as follows:

$$f(x) = \sum_{k=0}^{\infty} F(k) (x-x_0)^{k/\alpha}$$

Where α is the order of fraction and $F(k)$ is the fractional differential transform of $f(x)$. In order to avoid fractional initial and boundary conditions, we shall define the fractional derivatives in the Caputo sense.

The relationship between the Riemann-Liouville operator and Caputo operator is given by :

$${}^c D_x^q f(x) = {}^c D_{x_0}^q [f(x) - \sum_{k=0}^{m-1} \frac{1}{k!} (x-x_0)^k f^{(k)}(x_0)] \quad \dots(5)$$

Setting

$$f(x) = f(x) - \left[\sum_{k=0}^{m-1} \frac{1}{k!} (x-x_0)^k f^{(k)}(x_0) \right]$$

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In equation (2) and using equation (5), we obtain fractional derivative in the Caputo sense as follows:

$${}^c D_{x_0}^q f(x) = \frac{1}{\Gamma(m-q)} \frac{d^m}{dx^m} \left[\int_{x_0}^x \frac{f(t) - \sum_{k=0}^{m-1} \frac{1}{k!} (t-x_0)^k f^{(k)}(x_0)}{(x-t)^{1+q-m}} dt \right] \quad \dots(6)$$

Since the initial conditions are implemented to the integer order derivatives, the transformations of the initial conditions are defined as follows:

$$F(k) = \begin{cases} k/\alpha \in z^+, \frac{1}{(k/\alpha)!} \left[\frac{d^{k/\alpha} f(x)}{dx^{k/\alpha}} \right]_{x=x_0}, \text{ or } k = 0, 1, 2, \dots, q\alpha - 1 \\ k/\alpha \notin z^+, 0 \end{cases}$$

where q is the order of fractional derivative.

Following we present some important theorems which are relating to the FDTM and for their proofs see [12]

Theorem (1):

if $f(x) = g(x) \pm h(x)$, then $F(k) = G(k) \pm H(k)$, where F, G and H are the differential transform of f, g and h respectively.

Theorem (2):

if $f(x) = g(x)h(x)$, then $F(k) = \sum_{r=0}^k G(r)H(k-r)$, where F, G and H are the differential transform of f, g and h respectively.

Theorem(3):

If $f(x) = g_1(x) g_2(x) \dots g_{n-1}(x)g_n(x)$, then

$F(k) = \sum_{k_{n-1}=0}^k \sum_{k_{n-2}=0}^{k_{n-1}} \dots \sum_{k_2=0}^{k_3} \sum_{k_1=0}^{k_2} G_1(K_1)G_2(K_2 - K_1) \dots G_{n-1}(K_{n-1} - K_{n-2})G_n(k - K_{n-1})$ where F, G_1, G_2, \dots, G_n are the differential transform of f, g_1, g_2, \dots, g_n respectively.

Theorem(4):

If $f(x) = (x - x_0)^q$ then $F(k) = \delta(k - \alpha q)$, where:

$$\delta(k) = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases}$$

Theorem (5):

If $f(x) = D_{x_0}^q [g(x)]$, then:

$$F(K) = \frac{\Gamma(q + 1 + k/\alpha)}{\Gamma(1 + k/\alpha)} G(K + \alpha q).$$

V. THE PROPOSED APPROACH

In this section we shall employ the FDTM to solve fuzzy differential algebraic equations of fractional order and for this purpose let us consider the following system of fuzzy differential algebraic equation of fractional order with variable coefficients:

$$A(t) [{}^c D_{x_0}^q \tilde{x}(t)] + B(t)\tilde{x}(t) = f(t), 0 < q < 1 \quad \dots(7)$$

With initial value

$$\tilde{x}(t_0) = \tilde{x}_{t_0} \quad \dots(8)$$

Where ${}^c D_{x_0}^q$ represent the caputo fractional derivative of order q and with $A(t)$ is $n \times n$ singular matrix, $B(t)$ is $n \times n$ nonsingular matrix, f is $n \times 1$ vector function, \tilde{x}_{t_0} is $n \times 1$ known fuzzy number vector. since the initial value vector is fuzzy then the solution $\tilde{x}(t)$ will be a fuzzy vector solution and therefore $\tilde{x}(t)$ will be written as $\tilde{x}(t) = [\underline{x}(t), \bar{x}(t)]$ and to find $\bar{x}(t)$ we must solve

the following problem

$$A(t) [{}^c D_{x_0}^q \bar{x}(t)] + B(t)\bar{x}(t) = f(t), 0 < q < 1 \quad \dots (9)$$

$$\text{With the initial value } \bar{x}(t_0) = \bar{x}_{t_0} \quad \dots(10)$$

Similarly in order to find $\underline{x}(t)$ we must solve

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$$A(t)[{}^cD_{x_0}^q \underline{x}(t)] + B(t)\underline{x}(t) = f(t), 0 < q < 1 \quad \dots (11)$$

With initial value $\underline{x}(t_0) = \underline{x}_{t_0} \quad \dots (12)$

VI. ILLUSTRATIVE EXAMPLES

In this section, two examples are given in order to illustrate the proposed method.

Example1:

Consider the following fuzzy differential algebraic equations of fractional order

$$\begin{pmatrix} 1 & -x \\ 0 & 0 \end{pmatrix} \begin{pmatrix} {}^cD_x^q \underline{v}_1(x) \\ {}^cD_x^q \underline{v}_2(x) \end{pmatrix} + \begin{pmatrix} 1-(1+x) & \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \underline{v}_1(x) \\ \underline{v}_2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ \sin x \end{pmatrix} \quad \dots (13)$$

With the following initial conditions :

$$\begin{pmatrix} \underline{v}_1(0) \\ \underline{v}_2(0) \end{pmatrix} = \begin{pmatrix} \underline{1} \\ \underline{0} \end{pmatrix} \quad \dots (14)$$

The solution $\underline{v}(x)$ can be written as $\underline{v}(x) = [\underline{V}, \overline{V}]$, and to find \underline{V}_1 we must solve:

$$\begin{pmatrix} 1 & -x \\ 0 & 0 \end{pmatrix} \begin{pmatrix} {}^cD_x^q \underline{V}_1(x) \\ {}^cD_x^q \underline{V}_2(x) \end{pmatrix} + \begin{pmatrix} 1 & -(1+x) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \underline{V}_1(x) \\ \underline{V}_2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ \sin x \end{pmatrix} \quad \dots (15)$$

With the following initial conditions

$$\begin{pmatrix} \underline{V}_1(0) \\ \underline{V}_2(0) \end{pmatrix} = \begin{pmatrix} 1 & -\sqrt{1-B} \\ -\sqrt{1-B} \end{pmatrix} \quad \dots (16)$$

Equivalently eq. (15) can be written as

$${}^cD_x^q \underline{V}_1(x) - x [{}^cD_x^q \underline{V}_2(x)] + \underline{V}_1(x) - (1+x)\underline{V}_2(x) = 0 \quad \dots (17)$$

$$\underline{V}_2(x) = \sin(x) \quad \dots (18)$$

By using the basic properties of fractional differential transform method, and applying the transformation on (17), (18) we have

$$\underline{V}_1(k + \alpha q) = \frac{\Gamma(1+k/\alpha)}{\Gamma(q+1+k/\alpha)} \left\{ \sum_{r=0}^k \frac{\Gamma(q+1+(k-r)/\alpha)}{\Gamma(1+(k-r)/\alpha)} \delta(r-\alpha) \underline{V}_2(k-r+\alpha q) - \underline{V}_1(k) + \underline{V}_2(k) + \sum_{r=0}^k \delta(r-\alpha) \underline{V}_2(k-r) \right\} \quad \dots (19)$$

$$\underline{V}_2(k) = \begin{cases} 0 & , \text{ if } k/\alpha \notin \mathbb{Z}^+ \\ \frac{\sin(\frac{k\pi}{2\alpha})}{(k/\alpha)!} & , \text{ if } k/\alpha \in \mathbb{Z}^+ \end{cases}$$

Similarly in order to find \overline{V}_1 we must solve:

$$\begin{pmatrix} 1 & -x \\ 0 & 0 \end{pmatrix} \begin{pmatrix} {}^cD_x^q \overline{V}_1(x) \\ {}^cD_x^q \overline{V}_2(x) \end{pmatrix} + \begin{pmatrix} 1 & -(1+x) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \overline{V}_1(x) \\ \overline{V}_2(x) \end{pmatrix} \quad \dots (20)$$

With the following initial conditions

$$\begin{pmatrix} \overline{V}_1(0) \\ \overline{V}_2(0) \end{pmatrix} = \begin{pmatrix} 1 + \sqrt{1-B} \\ \sqrt{1-B} \end{pmatrix}$$

Equivalently eq. (20) can be written as

$${}^cD_x^q \overline{V}_1(x) - x [{}^cD_x^q \overline{V}_2(x)] + \overline{V}_1(x) - (1+x)\overline{V}_2(x) = 0 \quad \dots (21)$$

$$\overline{V}_2(x) = \sin x \quad \dots (22)$$

$$\overline{V}_1(k + \alpha q) = \frac{\Gamma(1+k/\alpha)}{\Gamma(q+1+k/\alpha)} \left\{ \sum_{r=0}^k \frac{\Gamma(q+1+(k-r)/\alpha)}{\Gamma(1+(k-r)/\alpha)} \delta(r-\alpha) \overline{V}_2(k-r+\alpha q) - \overline{V}_1(k) + \overline{V}_2(k) + \sum_{r=0}^k \delta(r-\alpha) \overline{V}_2(k-r) \right\} \quad \dots (23)$$

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$$\bar{V}_2(k) = \begin{cases} 0 & , \text{ if } k/\alpha \notin z^+ \\ \frac{\sin(\frac{k\pi}{2\alpha})}{(k/\alpha)!} & , \text{ if } k/\alpha \in z^+ \end{cases} \dots (24)$$

Following tables (1) and (2) represent the approximate values of $\underline{V}_1(x)$ and $\bar{V}_1(x)$ using FDTM up to 15 terms for $q = 1$ and $q = \frac{1}{2}$ and $B = 0.25, 0.5$ and 1 respectively .

Table(1)

The approximate values of $\underline{V}_1(x)$ and $\bar{V}_1(x)$ for $q=1$ and $B = 0.25, 0.5$ and 1 .

X	B = 0.25		B = 0.5		B = 1		
	$\underline{V}_1(x)$	$\bar{V}_1(x)$	$\underline{V}_1(x)$	$\bar{V}_1(x)$	$\underline{V}_1(x)$	$\bar{V}_1(x)$	Exact solution of $v_1(x)$
0	0.1339745	1.8660254	0.2928932	1.7071067	1	1	1
0.1	0.04464438	1.78503549	0.20428864	1.62590312	0.914820756	0.914820756	0.91482076
0.2	-0.02334599	1.74071197	0.1388567	1.58315008	0.8584648713	0.8584648713	0.858464619
0.3	-0.06999662	1.73086571	0.09595432	1.57973773	0.8294863325	0.8294863325	0.829474283
0.4	-0.00953075	1.7531757	0.07492739	1.61613127	0.826227253	0.826227253	0.826087383
0.5	-0.09927864	1.80550443	0.07462822	1.69271432	0.8471297026	0.8471297026	0.846243429
0.6	-0.08191003	1.88669956	0.09353874	1.81061571	0.891531253	0.891531253	0.88759712
0.7	-0.04320168	1.9982344	0.12979985	1.97337286	0.9612938756	0.9612938756	0.947537684
0.8	1.01684642	2.14714079	0.18124689	2.18988528	1.0637235914	1.0637235914	1.023213837
0.9	1.09823426	2.35078858	0.24545152	2.47921275	1.216333588	1.216333588	1.111563878

Table(2)

The approximate values of $\underline{V}_1(x)$ and $\bar{V}_1(x)$ for $q = \frac{1}{2}$ and $B = 0.25, 0.5$ and 1 .

X	B = 0.25		B = 0.5		B = 1	
	$\underline{V}_1(x)$	$\bar{V}_1(1)$	$\underline{V}_1(x)$	$\bar{V}_1(1)$	$\underline{V}_1(x)$	$\bar{V}_1(1)$
0	0.1339746	1.8660254	0.29289321	1.70710678	1	1
0.1	-0.2214142	1.63403766	0.21157475	1.47671671	0.64317517	0.64317517
0.2	-0.368621	1.63773336	0.23117698	1.48697194	0.495373485	0.495373485
0.3	-0.4815768	1.69683886	0.2873167	1.55656262	0.381961264	0.381961264
0.4	-0.5768029	1.79209453	0.37255331	1.66530889	0.286350339	0.286350339
0.5	-0.6606988	1.91472875	0.48245303	1.80371046	0.202115423	0.202115423
0.6	-0.7365465	2.05904024	0.61272988	1.96547408	0.125961238	0.125961238
0.7	-0.8062957	2.2204994	0.75824999	2.14557722	0.055930237	0.055930237
0.8	-0.8712166	2.39506954	0.91241203	2.33956268	-0.00925302	-0.00925302
0.9	-0.9321917	2.57892645	1.06662369	2.54323672	-0.07047449	-0.07047449

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Example 2:

Consider the following fuzzy fractional differential algebraic equations

$$\begin{pmatrix} 1 & -x & x^2 \\ 0 & 1 & -x \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} {}^c D_x^q \tilde{v}_1(x) \\ {}^c D_x^q \tilde{v}_2(x) \\ {}^c D_x^q \tilde{v}_3(x) \end{pmatrix} + \begin{pmatrix} 1 & -(x+1) & -(x^2+2x) \\ 0 & 1 & -(x+1) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{v}_1(x) \\ \tilde{v}_2(x) \\ \tilde{v}_3(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sin x \end{pmatrix} \quad \dots(25)$$

With the following initial conditions:

$$\begin{pmatrix} \tilde{v}_1(x) \\ \tilde{v}_2(x) \\ \tilde{v}_3(x) \end{pmatrix} = \begin{pmatrix} \tilde{\gamma} \\ \tilde{\gamma} \\ \tilde{0} \end{pmatrix} \quad \dots(26)$$

The solution $\tilde{v}(x)$ can be written as $\tilde{v}(x) = [V, \bar{V}]$, and to find V we must solve:

$$\begin{pmatrix} 1 & -x & x^2 \\ 0 & 1 & -x \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} {}^c D_x^q V_1(x) \\ {}^c D_x^q V_2(x) \\ {}^c D_x^q V_3(x) \end{pmatrix} + \begin{pmatrix} 1 & -(x+1) & -(x^2+2x) \\ 0 & 1 & -(x+1) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_1(x) \\ V_2(x) \\ V_3(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sin x \end{pmatrix} \quad \dots(27)$$

With following initial conditions

$$\begin{pmatrix} V_1(0) \\ V_2(0) \\ V_3(0) \end{pmatrix} = \begin{pmatrix} 1 - \sqrt{1-B} \\ 1 - \sqrt{1-B} \\ -\sqrt{1-B} \end{pmatrix} \quad \dots(28)$$

Equivalently eq. (27) can be written as

$${}^c D_x^q V_1(x) - x [{}^c D_x^q V_2(x)] + x^2 [{}^c D_x^q V_3(x)] + V_1(x) - xV_2(x) - V_2(x) + x^2V_3(x) + 2xV_3(x) = 0 \quad \dots(29)$$

$${}^c D_x^q V_2(x) - x [{}^c D_x^q V_3(x)] - V_2(x) + xV_3(x) - V_3(x) = 0 \quad \dots(30)$$

$$V_3(x) = \sin(x) \quad \dots(31)$$

By using the basic properties of fractional differential transform method, and applying the transformation on (29), (30) and (31) we have

$$\underline{V}_1(k + \alpha q) = \frac{[1+k/\alpha]}{[q+1+k/\alpha]} \left\{ 2 \sum_{r=0}^k \delta(r-\alpha) \underline{V}_2(k-r) - 2 \sum_{r=0}^k \delta(r-2\alpha) \underline{V}_3(k-r) - \sum_{r=0}^k \delta(r-\alpha) \underline{V}_3(k-r) - \underline{V}_1(k) + \underline{V}_2(k) \right\} \quad \dots(32)$$

$$\underline{V}_2(k + \alpha q) = \frac{[1+k/\alpha]}{[q+1+k/\alpha]} \left\{ \sum_{r=0}^k \frac{[q+1+(k-r)/\alpha]}{[1+(k-r)/\alpha]} \delta(r-\alpha) \underline{V}_3(k-r+\alpha q) + \underline{V}_2(k) - \sum_{r=0}^k \delta(r-\alpha) \underline{V}_3(k-r) + \underline{V}_3(k) \right\} \quad \dots(33)$$

$$\underline{V}_3(k) = \begin{cases} 0 & , \text{ if } k/\alpha \notin z^+ \\ \frac{\sin(\frac{k\pi}{2\alpha})}{(k/\alpha)!} & , \text{ if } k/\alpha \in z^+ \end{cases} \quad \dots(34)$$

Similarly in order to find \bar{V} we must solve:

$$\begin{pmatrix} 1 & -x & x^2 \\ 0 & 1 & -x \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} {}^c D_x^q \bar{V}_1(x) \\ {}^c D_x^q \bar{V}_2(x) \\ {}^c D_x^q \bar{V}_3(x) \end{pmatrix} + \begin{pmatrix} 1 & -(x+1) & -(x^2+2x) \\ 0 & 1 & -(x+1) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{V}_1(x) \\ \bar{V}_2(x) \\ \bar{V}_3(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sin x \end{pmatrix} \quad \dots(35)$$

With following initial conditions

$$\begin{pmatrix} \bar{V}_1(0) \\ \bar{V}_2(0) \\ \bar{V}_3(0) \end{pmatrix} = \begin{pmatrix} 1 + \sqrt{1-B} \\ 1 + \sqrt{1-B} \\ \sqrt{1-B} \end{pmatrix} \quad \dots(36)$$

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Equivalently eq. (35) can be written as

$${}^c D_x^q \bar{V}_1(x) - x[{}^c D_x^q \bar{V}_2(x)] + x^2[{}^c D_x^q \bar{V}_3(x)] + \bar{V}_1(x) - x\bar{V}_1(x) - \bar{V}_2(x) + x^2\bar{V}_3(x) + 2x\bar{V}_3(x) = 0 \quad \dots(37)$$

$${}^c D_x^q \bar{V}_2(x) - x[{}^c D_x^q \bar{V}_3(x)] - \bar{V}_2(x) + x\bar{V}_3(x) - \bar{V}_3(x) = 0 \quad \dots(38)$$

$$\bar{V}_3(x) = \sin(x) \quad \dots(39)$$

By using the basic properties of fractional differential transform method, and applying the transformation on (37), (38) and (39) we have

$$\bar{V}_3(k + \alpha q) = \frac{\Gamma(1+k/\alpha)}{\Gamma(q+1+k/\alpha)} \{ 2 \sum_{r=0}^k \delta(r-\alpha) \bar{V}_2(k-r) - 2 \sum_{r=0}^k \delta(r-2\alpha) \bar{V}_3(k-r) - \sum_{r=0}^k \delta(r-\alpha) \bar{V}_3(k-r) - \bar{V}_1(k) + \bar{V}_2(k) \} \quad \dots(40)$$

$$\bar{V}_2(k + \alpha q) = \frac{\Gamma(1+k/\alpha)}{\Gamma(q+1+k/\alpha)} \{ \sum_{r=0}^k \frac{\Gamma(q+1+(k-r)/\alpha)}{\Gamma(1+(k-r)/\alpha)} \delta(r-\alpha) \bar{V}_3(k-r + \alpha q) + \bar{V}_2(k) - \sum_{r=0}^k \delta(r-\alpha) \bar{V}_3(k-r) \bar{V}_3(k) \} \quad \dots(41)$$

$$\bar{V}_3(k) = \begin{cases} 0 & , \text{ if } k/\alpha \notin z^+ \\ \frac{\sin\left(\frac{k\pi}{2\alpha}\right)}{(k/\alpha)!} & , \text{ if } k/\alpha \in z^+ \end{cases} \quad \dots(42)$$

Following tables (3), (4), (5) and (6) represent the approximate values of $\bar{V}_1(x)$, $\bar{V}_1(x)$, $\bar{V}_2(x)$ and $\bar{V}_2(x)$ using FDTM up to 10 terms for $q = 1$ and $q = \frac{1}{2}$ and $B = 0.25, 0.5$ and 1 respectively.

Table(3)

The approximate values of $\bar{V}_1(x)$ and $\bar{V}_1(x)$ for $q=1$ and $B = 0.25, 0.5$ and 1 .

X	B = 0.25		B = 0.5		B = 1		Exact solution of v1(x)
	$\bar{V}_1(x)$	$\bar{V}_1(x)$	$\bar{V}_1(x)$	$\bar{V}_1(x)$	$\bar{V}_1(x)$	$\bar{V}_1(x)$	
0	0.1339746	1.8660254	0.29289322	1.70710678	1	1	1
0.1	0.13604123	1.88536513	0.29739045	1.72452201	1.149927072	1.149927072	1.01535451
0.2	0.14261532	1.94328528	0.3113488	1.77866638	1.59952826	1.59952826	1.063011305
0.3	0.15475002	2.039205	0.33558998	1.87372048	2.3475931	2.3475931	1.145775863
0.4	0.17445346	2.1717411	0.37111033	2.01593853	3.39232705	3.39232705	1.267049925
0.5	0.20507964	2.3384119	0.41909835	2.21404796	4.73108905	4.73108905	1.430891295
0.6	0.25174366	2.53532135	0.48095493	2.47968748	6.36038021	6.36038021	1.642082916
0.7	0.32176679	2.7681942	0.55831658	2.82788973	8.27577726	8.27577726	1.906212199
0.8	0.42515649	2.99513419	0.65308196	3.27761478	10.47185345	10.47185345	2.229761706
0.9	0.57512737	3.2399708	0.76744219	3.85234158	12.94208569	12.94208569	2.62021246
1	0.78866927	3.4787196	0.90391514	4.58072524	15.67874675	15.67874675	3.08616126

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Table(4)

The approximate values of $\underline{V}_2(x)$ and $\overline{V}_2(x)$ for $q=1$ and $B = 0.25, 0.5$ and 1 .

X	B = 0.25		B = 0.5		B = 1		
	$\underline{V}_2(x)$	$\overline{V}_2(x)$	$\underline{V}_2(x)$	$\overline{V}_2(x)$	$\underline{V}_2(x)$	$\overline{V}_2(x)$	Exact solution of $v_2(x)$
0	0.1339746	1.8660254	0.29289322	1.70710678	1	1	1
0.1	0.0721293	1.97613207	0.26296973	1.96792234	1.10481242	1.221169252	1.11515426
0.2	0.03577676	2.10395317	0.25605309	2.27100947	1.21833111	1.49376145	1.261136624
0.3	0.02913046	2.24517354	0.27188852	2.62173024	1.33879725	1.82724414	1.43851487
0.4	0.05652258	2.39469989	0.30986995	3.02568622	1.46394264	2.23401431	1.647592035
0.5	0.12247194	2.54663445	0.36905862	3.48879926	1.59099149	2.72692548	1.88843404
0.6	0.23176317	2.69424989	0.44820757	4.0174076	1.71666665	3.31997349	2.160904284
0.7	0.38953741	2.82996482	0.54579196	4.61835818	1.83719954	4.02847945	2.464705088
0.8	0.60139491	2.94531902	0.66004492	5.29915505	1.94834342	4.86929414	2.799423801
0.9	0.87350975	3.03094743	0.78899851	6.0680651	2.0453894	5.861025225	3.16459733
1	1.21275701	3.076551881	0.93052937	6.93428098	2.12318425	7.02428902	3.559752813

Table(5)

The approximate values of $\underline{V}_1(x)$ and $\overline{V}_1(x)$ for $q = \frac{1}{2}$ and $B = 0.25, 0.5$ and 1 .

X	B = 0.25		B = 0.5		B = 1	
	$\underline{V}_1(x)$	$\overline{V}_1(x)$	$\underline{V}_1(x)$	$\overline{V}_1(x)$	$\underline{V}_1(x)$	$\overline{V}_1(x)$
0	0.1339746	1.8660254	0.29289322	1.70710678	1	1
0.1	0.19822778	2.25256619	0.27981233	1.72452201	1.16653749	1.16653749
0.2	0.25992262	2.79969677	0.29587197	1.77866638	1.42115155	1.42115155
0.3	0.32927844	3.50865123	0.34329279	1.87372048	1.76509557	1.76509557
0.4	0.41468562	4.39488487	0.43013942	2.01593853	2.2086986	2.2086986
0.5	0.52418635	5.47879671	0.56659479	2.21404796	2.76505343	2.76505343
0.6	0.66576272	6.78430131	0.76439272	2.47968748	3.44899744	3.44899744
0.7	0.84743563	8.33840049	1.03658768	2.82788973	4.27674418	4.27674418
0.8	1.07729934	10.17105066	1.39740276	3.27761478	5.2657018	5.2657018
0.9	1.36352704	12.31514625	1.86209726	3.85234158	6.43436283	6.43436283
1	1.71436135	14.80655719	2.44683663	4.58072524	7.80222595	7.80222595

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Table(6)

The approximate values of $\underline{V}_2(x)$ and $\overline{V}_2(x)$ for $q = \frac{1}{2}$ and $B = 0.25, 0.5$ and 1 .

X	B = 0.25		B = 0.5		B = 1	
	$\underline{V}_2(x)$	$\overline{V}_2(x)$	$\underline{V}_2(x)$	$\overline{V}_2(x)$	$\underline{V}_2(x)$	$\overline{V}_2(x)$
0	0.1339746	1.8660254	0.29289318	1.70710678	1	1
0.1	0.04503774	2.20400664	0.243126679	2.00591949	1.12452219	1.12452219
0.2	-0.05151516	2.61286011	0.1929458	2.36853043	1.28067255	1.28067255
0.3	-0.14549665	3.11467432	0.1536301	2.81713517	1.4845907	1.4845907
0.4	-0.22302687	3.73914207	0.14051302	3.38510174	1.75807368	1.75807368
0.5	-0.26636905	4.5251482	0.17328688	4.1242248	2.1294704	2.1294704
0.6	-0.25403538	5.52256467	0.27609563	5.11649637	2.63452849	2.63452849
0.7	-0.16140223	6.79414075	0.47720783	6.49231915	3.31692165	3.31692165
0.8	0.36930946	8.4173441	0.80801659	8.45790993	4.2279982	4.2279982
0.9	0.36930976	10.48598639	1.30107218	11.33572349	5.42502178	5.42502178
1	0.85346577	13.11145651	1.98681935	15.62311118	6.9668036	6.9668036

VII. CONCLUSION

In this paper the approximate solution of the fuzzy differential algebraic equations of fractional order was found by using the fractional differential transform method. It seems from the results of the illustrative examples that the proposed method gave an accurate result and the method followed to find the approximate solution is efficient and can be considered for such types of problems.

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