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Conformal Mapping and Its Applications

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ABSTRACT: In mathematics, a conformal map is a function that locally preserves angles, but not necessarily lengths. The conformal property may be described in terms of the Jacobian derivative matrix of a coordinate transformation. The transformation is conformal whenever the Jacobian at each point is a positive scalar times a rotation matrix (orthogonal with determinant one). Some authors define conformality to include orientation-reversing mappings whose Jacobians can be written as any scalar times any orthogonal matrix.^[1] For mappings in two dimensions, the (orientation-preserving) conformal mappings are precisely the locally invertible complex analytic functions. In three and higher dimensions, Liouville's theorem sharply limits the conformal mappings to a few types. The notion of conformality generalizes in a natural way to maps between Riemannian or semi-Riemannian manifolds. A map of the Riemann sphere onto itself is conformal if and only if it is a Möbius transformation. The complex conjugate of a Möbius transformation preserves angles, but reverses the orientation. For example, circle inversions. In plane geometry there are three types of angles that may be preserved in a conformal map.^[3] Each is hosted by its own real algebra, ordinary complex numbers, split-complex numbers, and dual numbers. The conformal maps are described by linear fractional transformations in each case.^[4]

KEYWORDS: conformal, mathematics, mapping, applications, dimensions, transformations, geometry, rotation

1. INTRODUCTION

A diffeomorphism between two Riemannian manifolds is called a conformal map if the pulled back metric is conformally equivalent to the original one. For example, stereographic projection of a sphere onto the plane augmented with a point at infinity is a conformal map. One can also define a conformal structure on a smooth manifold, as a class of conformally equivalent Riemannian metrics. A classical theorem of Joseph Liouville shows that there are far fewer conformal maps in higher dimensions than in two dimensions.¹

Consider the transformations $u = u(x, y)$ and $v = v(x, y)$, which maps a point $P(x_0, y_0)$ in the z -plane to a point $P'(u_0, v_0)$ in the w -plane. Let curves C_1 and C_2 intersect at point $z_0 = (x_0, y_0)$ in z -plane is mapped into curves C_1' and C_2' in w -plane intersecting at $w_0 = (u_0, v_0)$.

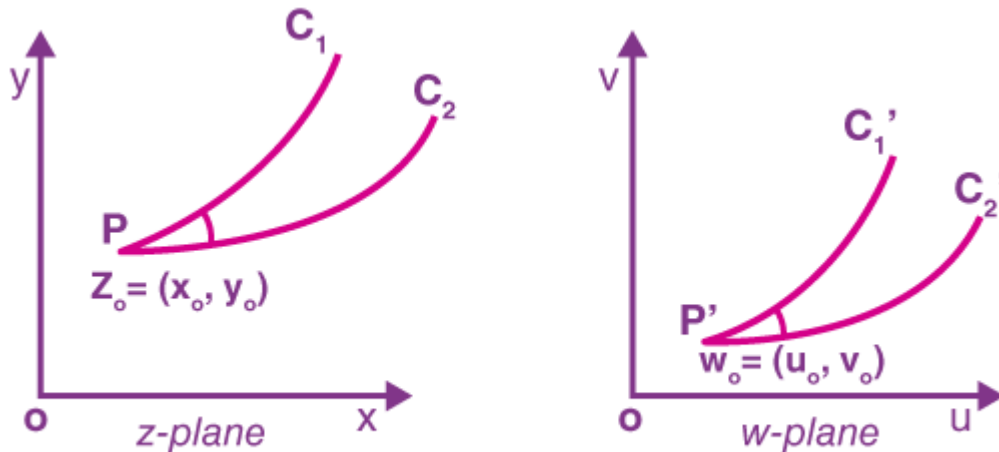
If the transformation is such that the angle between C_1 and C_2 at z_0 is equal both in magnitude and sense to the angle between the curve C_1' and C_2' at w_0 , it is known as conformal mapping at $z_0 = (x_0, y_0)$.²

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Note: i) The scaling factor and the rotation angle for a given transformation $f(z)$ depend only on z and not on curves passing through z .

ii) Conformality depends on the given point. It is different at various points in the plane.³

Any conformal map from an open subset of Euclidean space into the same Euclidean space of dimension three or greater can be composed from three types of transformations: a homothety, an isometry, and a special conformal transformation. Applications of conformal mapping exist in aerospace engineering,^[5] in biomedical sciences^[6] (including brain mapping^[7] and genetic mapping^{[8][9][10]}), in applied math (for geodesics^[11] and in geometry^[12]), in earth sciences (including geophysics,^[13] geography,^[14] and cartography),^[15] in engineering,^{[16][17]} and in electronics.^[18] In cartography, several named map projections, including the Mercator projection and the stereographic projection are conformal. They are specially useful for use in marine navigation because of its unique property of representing any course of constant bearing as a straight segment. Such a course, known as a rhumb (or, mathematically, a loxodrome) is preferred in marine navigation because ships can sail in a constant compass direction. Conformal mappings are invaluable for solving problems in engineering and physics that can be expressed in terms of functions of a complex variable yet exhibit inconvenient geometries. By choosing an appropriate mapping, the analyst can transform the inconvenient geometry into a much more convenient one⁴. For this reason, any function which is defined by a potential can be transformed by a conformal map and still remain governed by a potential. Examples in physics of equations defined by a potential include the electromagnetic field, the gravitational field, and, in fluid dynamics, potential flow, which is an approximation to fluid flow assuming constant density, zero viscosity, and irrotational flow.⁵ One example of a fluid dynamic application of a conformal map is the Joukowski transform that can be used to examine the field of flow around a Joukowski airfoil. Conformal maps are also valuable in solving nonlinear partial differential equations in some specific geometries. Such analytic solutions provide a useful check on the accuracy of numerical simulations of the governing equation. For example, in the case of very viscous free-surface flow around a semi-infinite wall, the domain can be mapped to a half-plane in which the solution is one-dimensional and straightforward to calculate.⁶ Maxwell's equations are preserved by Lorentz transformations which form a group including circular and hyperbolic rotations. The latter are sometimes called Lorentz boosts to distinguish them from circular rotations.⁷ All these transformations are conformal since hyperbolic rotations preserve hyperbolic angle, (called rapidity) and the other rotations preserve circular angle. The introduction of translations in the Poincare group again

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preserves angles. A larger group of conformal maps for relating solutions of Maxwell's equations was identified by Ebenezer Cunningham (1908) and Harry Bateman (1910).⁸ Their training at Cambridge University had given them facility with the method of image charges and associated methods of images for spheres and inversion. As recounted by Andrew Warwick (2003) *Masters of Theory*: Warwick highlights this "new theorem of relativity" as a Cambridge response to Einstein, and as founded on exercises using the method of inversion, such as found in James Hopwood Jeans textbook *Mathematical Theory of Electricity and Magnetism*.⁹ In general relativity, conformal maps are the simplest and thus most common type of causal transformations. Physically, these describe different universes in which all the same events and interactions are still (causally) possible, but a new additional force is necessary to effect this (that is, replication of all the same trajectories would necessitate departures from geodesic motion because the metric tensor is different).¹⁰ It is often used to try to make models amenable to extension beyond curvature singularities, for example to permit description of the universe even before the Big Bang. In cartography, a conformal map projection is one in which every angle between two curves that cross each other on Earth (a sphere or an ellipsoid) is preserved in the image of the projection, i.e. the projection is a conformal map in the mathematical sense. For example, if two roads cross each other at a 39° angle, then their images on a map with a conformal projection cross at a 39° angle. A conformal projection can be defined as one that is locally conformal at every point on the Earth. Thus, every small figure on the earth is nearly similar to its image on the map. The projection preserves the ratio of two lengths in the small domain. All Tissot's indicatrices of the projections are circles.¹¹ Conformal projections preserve only small figures. Large figures are distorted by even conformal projections. In a conformal projection, any small figure is similar to the image, but the ratio of similarity (scale) varies by location, which explains the distortion of the conformal projection. In a conformal projection, parallels and meridians cross rectangulary on the map. The converse is not necessarily true. The counterexamples are equirectangular and equal-area cylindrical projections (of normal aspects). These projections expand meridian-wise and parallel-wise by different ratios respectively. Thus, parallels and meridians cross rectangulary on the map,¹² but these projections do not preserve other angles; i.e. these projections are not conformal. As proven by Leonhard Euler in 1775, a conformal map projection cannot be equal-area, nor can an equal-area map projection be conformal.¹¹ This is also a consequence of Carl Gauss's 1827 Theorema Egregium.¹³

Many large-scale maps use conformal projections because figures in large-scale maps can be regarded as small enough. The figures on the maps are nearly similar to their physical counterparts.¹⁴

A non-conformal projection can be used in a limited domain such that the projection is locally conformal. Glueing many maps together restores roundness. To make a new sheet from many maps or to change the center, the body must be re-projected. Seamless online maps can be very large Mercator projections, so that any place can become the map's center, then the map remains conformal.¹⁵ However, it is difficult to compare lengths or areas of two far-off figures using such a projection. The Universal Transverse Mercator coordinate system and the Lambert system in France are projections that support the trade-off between seamlessness and scale variability. Maps reflecting directions, such as a nautical chart or an aeronautical chart, are projected by conformal projections. Maps treating values whose gradients are important, such as a weather map with atmospheric pressure, are also projected by conformal projections.¹⁶

Small scale maps have large scale variations in a conformal projection, so recent world maps use other projections. Historically, many world maps are drawn by conformal projections, such as Mercator maps or hemisphere maps by stereographic projection.¹⁷ Conformal maps containing large regions vary scales by locations, so it is difficult to compare lengths or areas. However, some techniques require that a length of 1 degree on a meridian = 111 km = 60 nautical miles. In non-conformal maps, such techniques are not available because the same lengths at a point vary the lengths on the map. In Mercator or stereographic projections, scales vary by latitude, so bar scales by latitudes are often appended. In complex projections such as of oblique aspect. Contour charts of scale factors are sometimes appended.¹⁸

II.DISCUSSION

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In mathematics, conformal geometry is the study of the set of angle-preserving (conformal) transformations on a space. In a real two dimensional space, conformal geometry is precisely the geometry of Riemann surfaces. In space higher than two dimensions, conformal geometry may refer either to the study of conformal transformations of what are called "flat spaces" (such as Euclidean spaces or spheres), or to the study of conformal manifolds which are Riemannian or pseudo-Riemannian manifolds with a class of metrics that are defined up to scale. Study of the flat structures is sometimes termed Möbius geometry, and is a type of Klein geometry. a conformal metric may be regarded as a metric that is only defined "up to scale". Often conformal metrics are treated by selecting a metric in the conformal class, and applying only "conformally invariant" constructions to the chosen metric.¹⁹

Conditions for Conformal Mapping

The sufficient condition for a transformation $w = f(z)$ to be a conformal mapping is:

Let $f(z)$ be an analytic function of z in a domain D of the z -plane and let $f'(z) \neq 0$ inside D . Then the mapping $w = f(z)$ is conformal at all points of D .

The necessary condition for a transformation being a conformal mapping is:

If $w = f(z)$ represents a conformal transformation of a domain D of z -plane into a domain D' of w -plane, then $f(z)$ is an analytic function of z in D' .

A conformal metric is conformally flat if there is a metric representing it that is flat, in the usual sense that the Riemann curvature tensor vanishes. It may only be possible to find a metric in the conformal class that is flat in an open neighborhood of each point. When it is necessary to distinguish these cases, the latter is called locally conformally flat, although often in the literature no distinction is maintained. The n -sphere is a locally conformally flat manifold that is not globally conformally flat in this sense, whereas a Euclidean space, a torus, or any conformal manifold that is covered by an open subset of Euclidean space is (globally) conformally flat in this sense. A locally conformally flat manifold is locally conformal to a Möbius geometry, meaning that there exists an angle preserving local diffeomorphism from the manifold into a Möbius geometry. In two dimensions, every conformal metric is locally conformally flat. In dimension $n > 3$ a conformal metric is locally conformally flat if and only if its Weyl tensor vanishes; in dimension $n = 3$, if and only if the Cotton tensor vanishes.²⁰

Conformal geometry has a number of features which distinguish it from (pseudo-)Riemannian geometry. The first is that although in (pseudo-)Riemannian geometry one has a well-defined metric at each point, in conformal geometry one only has a class of metrics. Thus the length of a tangent vector cannot be defined, but the angle between two vectors still can. Another feature is that there is no Levi-Civita connection because if g and $\lambda^2 g$ are two representatives of the conformal structure, then the Christoffel symbols of g and $\lambda^2 g$ would not agree. Those associated with $\lambda^2 g$ would involve derivatives of the function λ whereas those associated with g would not. Despite these differences, conformal geometry is still tractable. The Levi-Civita connection and curvature tensor, although only being defined once a particular representative of the conformal structure has been singled out, do satisfy certain transformation laws involving the λ and its derivatives when a different representative is chosen. In particular, (in dimension higher than 3) the Weyl tensor turns out not to depend on λ , and so it is a conformal invariant. Moreover, even though there is no Levi-Civita connection on a conformal manifold, one can instead work with a conformal connection, which can be handled either as a type of Cartan connection modelled on the associated Möbius geometry, or as a Weyl connection. This allows one to define conformal curvature and other invariants

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of the conformal structure. Möbius geometry is the study of "Euclidean space with a point added at infinity", or a "Minkowski (or pseudo-Euclidean) space with a null cone added at infinity". That is, the setting is a compactification of a familiar space; the geometry is concerned with the implications of preserving angles.²¹

At an abstract level, the Euclidean and pseudo-Euclidean spaces can be handled in much the same way, except in the case of dimension two. The compactified two-dimensional Minkowski plane exhibits extensive conformal symmetry. Formally, its group of conformal transformations is infinite-dimensional. By contrast, the group of conformal transformations of the compactified Euclidean plane is only 6-dimensional. In two dimensions, the group of conformal automorphisms of a space can be quite large (as in the case of Lorentzian signature) or variable (as with the case of Euclidean signature). The comparative lack of rigidity of the two-dimensional case with that of higher dimensions owes to the analytical fact that the asymptotic developments of the infinitesimal automorphisms of the structure are relatively unconstrained. In Lorentzian signature, the freedom is in a pair of real valued functions. In Euclidean, the freedom is in a single holomorphic function.

In the case of higher dimensions, the asymptotic developments of infinitesimal symmetries are at most quadratic polynomials.^[2] In particular, they form a finite-dimensional Lie algebra. The pointwise infinitesimal conformal symmetries of a manifold can be integrated precisely when the manifold is a certain model conformally flat space (up to taking universal covers and discrete group quotients).^[3]

The general theory of conformal geometry is similar, although with some differences, in the cases of Euclidean and pseudo-Euclidean signature.^[4] In either case, there are a number of ways of introducing the model space of conformally flat geometry. Unless otherwise clear from the context, this article treats the case of Euclidean conformal geometry with the understanding that it also applies, *mutatis mutandis*, to the pseudo-Euclidean situation.²²

III.RESULTS

The inversive model of conformal geometry consists of the group of local transformations on the Euclidean space E^n generated by inversion in spheres. By Liouville's theorem, any angle-preserving local (conformal) transformation is of this form.^[5] From this perspective, the transformation properties of flat conformal space are those of inversive geometry. Intuitively, the conformally flat geometry of a sphere is less rigid than the Riemannian geometry of a sphere. Conformal symmetries of a sphere are generated by the inversion in all of its hyperspheres. On the other hand, Riemannian isometries of a sphere are generated by inversions in geodesic hyperspheres (see the Cartan–Dieudonné theorem.) The Euclidean sphere can be mapped to the conformal sphere in a canonical manner, but not vice versa. An embedding of the Euclidean sphere into N^+ , as in the previous section, determines a conformal scale on S . Conversely, any conformal scale on S is given by such an embedding. Thus the line bundle $N^+ \rightarrow S$ is identified with the bundle of conformal scales on S : to give a section of this bundle is tantamount to specifying a metric in the conformal class $[g]$. Consider first the case of the flat conformal geometry in Euclidean signature. The n -dimensional model is the celestial sphere of the $(n+2)$ -dimensional Lorentzian space $R^{n+1,1}$. Here the model is a Klein geometry: a homogeneous space G/H where $G = SO(n+1, 1)$ acting on the $(n+2)$ -dimensional Lorentzian space $R^{n+1,1}$ and H is the isotropy group of a fixed null ray in the light cone. Thus the conformally flat models are the spaces of inversive geometry. For pseudo-Euclidean of metric signature (p, q) , the model flat geometry is defined analogously as the homogeneous space $O(p+1, q+1)/H$, where H is again taken as the stabilizer of a null line. Note that both the Euclidean and pseudo-Euclidean model spaces are compact.¹⁹

Conformal geometric algebra (CGA) is the geometric algebra constructed over the resultant space of a map from points in an n -dimensional base space $R^{p,q}$ to null vectors in $R^{p+1,q+1}$. This allows operations on the base space, including reflections, rotations and translations to be represented using versors of the geometric algebra; and it is found that points, lines, planes, circles and spheres gain particularly natural and computationally amenable representations.

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The effect of the mapping is that generalized (i.e. including zero curvature) k -spheres in the base space map onto $(k + 2)$ -blades, and so that the effect of a translation (or any conformal mapping) of the base space corresponds to a rotation in the higher-dimensional space. In the algebra of this space, based on the geometric product of vectors, such transformations correspond to the algebra's characteristic sandwich operations, similar to the use of quaternions for spatial rotation in 3D, which combine very efficiently. A consequence of rotors representing transformations is that the representations of spheres, planes, circles and other geometrical objects, and equations connecting them, all transform covariantly. A geometric object (a k -sphere) can be synthesized as the wedge product of $k + 2$ linearly independent vectors representing points on the object; conversely, the object can be decomposed as the repeated wedge product of vectors representing $k + 2$ distinct points in its surface. Some intersection operations also acquire a tidy algebraic form: for example, for the Euclidean base space \mathbb{R}^3 , applying the wedge product to the dual of the tetravectors representing two spheres produces the dual of the trivector representation of their circle of intersection.²⁰

As this algebraic structure lends itself directly to effective computation, it facilitates exploration of the classical methods of projective geometry and inversive geometry in a concrete, easy-to-manipulate setting. It has also been used as an efficient structure to represent and facilitate calculations in screw theory. CGA has particularly been applied in connection with the projective mapping of the everyday Euclidean space \mathbb{R}^3 into a five-dimensional vector space $\mathbb{R}^{4,1}$, which has been investigated for applications in robotics and computer vision. It can be applied generally to any pseudo-Euclidean space - for example, Minkowski space $\mathbb{R}^{3,1}$ to the space $\mathbb{R}^{4,2}$.²¹

IV.CONCLUSIONS

In the mathematical theory of conformal mappings, the area theorem gives an inequality satisfied by the power series coefficients of certain conformal mappings. The theorem is called by that name, not because of its implications, but rather because the proof uses the notion of area. The inequalities satisfied by power series coefficients of conformal mappings were of considerable interest to mathematicians prior to the solution of the Bieberbach conjecture. The area theorem is a central tool in this context. Moreover, the area theorem is often used in order to prove the Koebe 1/4 theorem, which is very useful in the study of the geometry of conformal mappings. Conformal mapping is a function defined on the complex plane which transforms a given curve or points on a plane, preserving each angle of that curve. If $f(z)$ is a complex function defined for all z in \mathbb{C} , and $w = f(z)$, then f is known as a transformation which transforms the point $z = x + iy$ in z -plane to $w = u + iv$ in w -plane. If this transformation preserves the angles between curves in both magnitude and sense (clockwise or counterclockwise), then the mapping is called conformal mappings.

If there is a transformation such that the magnitude of the angles between the curves is identical, but the sense (clockwise or counterclockwise) is opposite. In that case, the transformation is called isogonal.²²

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