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K-Ordered Hamiltonian Graphs

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ABSTRACT: In this chapter, we review the following results proved in [5,8,10]: (i) For $k \geq 3$, every $(k+1)$ -hamiltonian - connected graph is k -ordered. Determine $f(k,n)$ if n is sufficiently large in terms of k . Let $g(k,n) = n/2 + k/2 - 1$ (ii) $f(k,n) = g(k,n)$ if $n \geq 1$ (iii) $f(k,n) \geq g(k,n)$ for any $n \geq 2k$ and $f(k,n) > g(k,n)$ if $2k \leq n \leq 3k-6$ (iv) if G is a graph of order n with $3 \leq k \leq n/2$ and $\deg(u) + \deg(v) \geq n + (3k-9)/2$ for every pair u, v of non-adjacent vertices of G , then G is k -ordered Hamiltonian.

I. INTRODUCTION

In this section, the concept of K -hamiltonian connected graphs is introduced. The result that, for $k \geq 3$, every $(k+1)$ -hamiltonian connected graph is k -ordered is established[2]. Also necessary and sufficient conditions for a graph to be k -ordered are discussed[5]. The notations used are recalled here. We write $v_1, (x_1), v_2, (x_2), \dots, v_{k-1}, (x_{k-1}), v_k$ to indicate a v_1 - v_k path that contains the k vertices v_1, v_2, \dots, v_k and possibly upto $k-1$ additional vertices, namely x_1, x_2, \dots, x_{k-1} . Thus for $1 \leq j \leq k-1$, (x_j) indicates that some vertex, which we denote by x_i , may (or) may not be present on the path. So for each i ($1 \leq i \leq k-1$), the vertices v_1 and v_{i+1} are either adjacent or v_i, v_{i+1} is a path. For example, $u, v(x), y$ indicates the path u, v, y or the path u, v, x, y [6].

Proposition 1.1: Let G be a Hamiltonian graph of order $n \geq 3$. If G is k -ordered, $3 \leq k \leq n$, then G is $(k-1)$ -connected.
Proof: Suppose, to the contract that G is not $(k-1)$ connected. Then there exists a cut set $S = v_1, v_2, \dots, v_k$ of G , where $k \leq k-2$. Let v_{k+1} and v_{k+2} be vertices belonging to distinct components of $G-S$. Consequently, every v_{k+1} - v_{k+2} path in G contains vertices of S , so G contains no Hamiltonian cycle containing the vertices of the sequence $v_1, v_2, \dots, v_k, v_{k+1}, v_{k+2}$ in this order. Hence G is not $(k+2)$ -ordered and so is not k -ordered[1].
corollary 1.2: If G is a k -ordered Hamiltonian graph, then $\delta(G) \geq k-1$.

II. SUFFICIENT CONDITIONS FOR K-ORDERED GRAPHS

Many sufficient conditions have been given for Hamiltonian graphs. Two of the best known are due to Dirac and Ore.

Theorem 1.3 (Dirac): Let G be a graph of order $n \geq 3$. If $\deg v \geq n/2$ for every vertex v of G , then G is Hamiltonian.

Theorem 1.4 (Ore): Let G be a graph of order $n \geq 3$. If for every pair u, v of non-adjacent vertices of G , $\deg u + \deg v \geq n$, then G is hamiltonian.

Lemma 1.5: Let G be a graph of order $n \geq 3$ and let $S: v_1, v_2, \dots, v_k$ be a sequence of k distinct vertices of G , where $3 \leq k \leq n$. If

$$\deg u + \deg v \geq n + 2k - 7$$

for every pair u, v of non adjacent vertices of G , then G contains a $v_1 - v_k$ path of the type $v_1, (x_1), v_2, (x_2), \dots, (x_{k-1}), v_k$ or a $v_k - v_{k-1}$ path of the type $v_k, v_1, (x_1), v_2, (x_2), \dots, (x_{k-2}), v_{k-1}$.

Proof: Consider the vertices v_1 and v_2 . If v_1 and v_2 are adjacent, then G contains the path v_1, v_2 ; otherwise, $\deg v_1 + \deg v_2 \geq n + 2k - 7 \geq n - 1$, which implies the existence of a vertex x_1 mutually adjacent to v_1 and v_2 and so v_1, x_1, v_2 is a path in G . In any case, G contains a path $v_1, (x_1), v_2$. Now, consider the vertex v_3 . If $v_2 v_3 \in E(G)$, then G contains the path $v_1, (x_1), v_2, v_3$. Suppose then that $v_2 v_3 \notin E(G)$. Thus $\deg v_2 + \deg v_3 \geq n + 2k - 7$ by hypothesis. Since G has order n , the vertices v_2 and v_3 are mutually adjacent to at least $2k - 5$ vertices. If $k \geq 4$, then G contains a vertex x_2 distinct from v_1 and x_1 (if it exists) such that x_2 is mutually adjacent to v_2 and v_3 . Hence G contains the path $v_1, (x_1), v_2, (x_2), v_3$. Suppose then that $k=3$ and that G contains to vertex distinct from v_1 and x_1 that is mutually adjacent to v_2 and

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v_3 . If v_3 is adjacent to v_1 , then $v_3, v_1, (x_1), v_2$ is a path of G . If v_3 is not adjacent to v_1 , then v_2 and v_3 are mutually adjacent to 1 and to no other vertex, while v_2 is adjacent to v_1 . However, then v_1, v_2, x_1, v_3 is a path in G .

Proceeding inductively, assume that we have constructed a path $v_1, (x_1), v_2, (x_2), \dots, (x_{j-2}), v_{j-1}$ in G . Suppose first that $j \leq k-1$. It is to show that G contains a path $v_1, (x_1), v_2, (x_2), \dots, v_j$. If v_{j-1} is adjacent to v_j . If v_{j-1} is adjacent to v_j , then G contains such a path. Suppose then that $v_{j-1} v_j \notin E(G)$. If G contains a vertex x_{j-1} distinct from the vertices on the path $v_1, (x_1), v_2, (x_2), \dots, (x_{j-2})$ such that x_{j-1} is mutually adjacent to v_{j-1} and v_j then G contains a path of the designed type, otherwise v_{j-1} and v_j are mutually adjacent to at most $2j-4$ vertices and G contains at least $n+2k-2j-1$ vertices. Since $k \geq j+1$, this produces a contradiction and hence the desired claim is verified.

Finally, suppose that $j=k$, that is [7], assume that there is a path $v_1, (x_1), v_2, (x_2), \dots, (x_{k-2}), v_{k-1}$ is constructed in G . It is to show that G contains either a path $v_1, (x_1), v_2, (x_2), \dots, v_{k-1}, (x_{k-1}), v_k$ or a path $v_k, v_1, (x_1), v_2, (x_2), \dots, (x_{k-2}), v_{k-1}$. If either $v_k v_{k-1} \in E(G)$ or $v_k v_1 \in E(G)$, then the desired result is obtained, thus assume that v_k is adjacent to neither v_{k-1} nor v_1 . Also, if v_{k-1} and v_k are mutually adjacent to some vertex other than the (at most) 2_{k-5} vertices on the path $(x_1), v_2, (x_2), \dots, v_{k-2}, (x_{k-2})$, the proof is complete; so assume that this is not the case. Hence v_{k-1} and v_k are mutually adjacent to at most 2_{k-5} vertices. Since G has order n , it follows that v_{k-1} and v_k are mutually adjacent to exactly 2_{k-5} vertices (so all of the vertices x_1, x_2, \dots, x_{k-2} exist) and every other vertex of G is adjacent to exactly one of v_{k-1} and G contains the path $v_1, x_1, v_2, x_2, \dots, v_{k-2}, x_{k-2}, v_k$, producing the desired path. Hence the result [8].

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