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Expected Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits with Inter-Decision Times as Order Statistics

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ABSTRACT: In this paper, the problem of time to recruitment is studied using a univariate policy of recruitment for a single grade manpower system in which attrition takes place due to policy decisions. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the variance of the time to recruitment is obtained when the inter-policy decision times and inter-exit times form an order statistics and an ordinary renewal process respectively.

KEYWORDS: Decision and exit epochs, order statistics, ordinary renewal process, single grade manpower system, univariate policy of recruitment, variance of time to recruitment.

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I. INTRODUCTION

Attrition is a common phenomenon in many marketing organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the breakdown threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. Many researchers have studied several problems in manpower planning using different methods. In [1] and [2] the authors have discussed some manpower planning models for a single and multi-grade manpower system using Markovian and renewal theoretic approach. In [14], the author has analysed the problem of time to recruitment for a single grade manpower system which is subjected to attrition, using a univariate policy of recruitment based on shock model approach for replacement of systems in reliability theory. In [11], the author has considered a single grade manpower system and obtained system characteristics when the loss of manpower process and inter-decision time process form a correlated pair of renewal sequence by employing a univariate cum policy of recruitment. In [10], the author has initiated the study of the problem of time to recruitment for a single grade manpower system by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory thresholds for the cumulative loss of manpower in this manpower system. The author [13] has considered the problem of time to recruitment for a single grade manpower system when inter-decision times form an order statistics. Variance of the time to recruitment for a single grade manpower system with optional and mandatory thresholds is obtained in [16] when inter-decision times form an order statistics.

In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decisions points. This aspect is taken into account for the first time in [3] and the variance of the time to recruitment is obtained when the loss of manpower, inter-decision times and exit times are independent and identically distributed continuous random variables according as the mandatory breakdown threshold is an exponential random variable or extended exponential random variable with shape parameter 2 or a continuous random variable with SCBZ property. In [5] the authors have studied the work in [3] when the inter-decision times form a sequence of exchangeable and constantly correlated exponential random variables. In [4], [6], [12] and [7] the authors have extended the research work in [3]

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when the inter-decision times form (i) an ordinary renewal process (ii) a sequence of exchangeable and constantly correlated exponential random variables (iii) a geometric process and (iv) an order statistics respectively using a different probabilistic analysis. In [8] the authors have studied the results of [4], [6], [12], and [7] using univariate Max policy of recruitment. Recently in [9] the authors have studied the work in [5] when the inter-policy decision times form a geometric process. The present paper extends the research work in [9], when the inter-decision times form an order statistics.

II. MODEL DESCRIPTION

Consider an organization taking policy decisions at random epochs in $(0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For $i=1,2,3,\dots$, let X_i be independent and identically distributed exponential random variables representing the amount of depletion of manpower (loss of man hours) due to i^{th} exit point with probability distribution function $M(\cdot)$, and mean $\frac{1}{\alpha}$ ($\alpha > 0$). Let S_k be the cumulative loss of manpower in the first k exit points. It is assumed that A_i , the time between $(i-1)^{\text{th}}$ and i^{th} exits, form an order statistics of independent random variables with $F(\cdot)$ and $f(\cdot)$ as the exponential distribution and probability density function having mean $\frac{1}{\lambda}$ ($\lambda > 0$) for the population. Let $F_{(1)}(\cdot)$ and $F_{(r)}(\cdot)$ be the distribution of the first and r^{th} order statistics respectively and $f_{(1)}(\cdot)$ and $f_{(r)}(\cdot)$ be the corresponding probability density functions. Let B_i , be the time between the $(i-1)^{\text{th}}$ and i^{th} exit times. It is assumed that B_i 's be independent and identically distributed random variables with probability distribution function $G(\cdot)$, and mean $\frac{1}{\delta}$ ($\delta > 0$). Let $V_k(t)$ be the probability that there are exactly k exit points in $(0, t]$. Let Y be the independent breakdown threshold level for the cumulative depletion of manpower in the organization with probability distribution function $H(\cdot)$. Let q be the probability that every policy decision has exit of personnel. Since $q=0$ means exits due to policy decisions are impossible, it is assumed that $q \neq 0$. Let T be the random variable denoting the time to recruitment with probability distribution function $L(\cdot)$, density function $l(\cdot)$, mean $E(T)$ and variance $V(T)$. Let $\tilde{U}(\cdot)$ and $u^*(\cdot)$ be the Laplace - Stieltjes transform and the Laplace transform of $U(\cdot)$ and $u(\cdot)$ respectively. The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the breakdown threshold.

III. MAIN RESULT

By the recruitment policy and using law of total probability we get

$$P(T > t) = \sum_{K=0}^{\infty} V_K(t) P(S_K < Y) \tag{1}$$

We now obtain explicit analytical expression for $E(T)$ and $V(T)$ by assuming $M(x) = 1 - e^{-\alpha x}$ and $H(x) = 1 - e^{-\theta x}$. In this case from (1) we get

$$P(T > t) = \sum_{k=0}^{\infty} [G_k(t) - G_{k+1}(t)] a^k \tag{2}$$

$$L(t) = \bar{a} \sum_{k=1}^{\infty} G_k(t) a^{k-1} \tag{3}$$

and

$$l^*(s) = \frac{\bar{a} q f^*(s)}{1 + (\bar{a} q - 1) f^*(s)} \tag{4}$$

$$\text{where } a = E[e^{-\theta X}] \text{ and } \bar{a} = 1 - a \tag{5}$$

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From the theory of order statistics [15], it is known that $f_{a(j)}(t) = j[F(t)]^{j-1} f(t), j = 1, \dots, r.$

By hypothesis $F(t) = 1 - e^{-\lambda t}, \lambda > 0.$

Suppose $f(t) = f_{a(1)}(t)$ (6)

In this case from (4) it can be shown that

$$E(T) = \left(\frac{1}{\bar{a} r \lambda q} \right)$$
 (7)

and

$$V(T) = \frac{1}{(\bar{a} \lambda r q)^2}$$
 (8)

Suppose $f(t) = f_{a(r)}(t)$ (9)

In this case from (4) it can be shown that

$$E(T) = \left(\frac{1}{\bar{a} \lambda q} \right) \left(\sum_{j=1}^r \frac{1}{j} \right)$$
 (10)

and

$$V(T) = \left(\frac{1}{\bar{a} \lambda^2 q} \right) \left(\sum_{j=1}^r \frac{1}{j^2} \right) + \left(\frac{1 - \bar{a} q}{(\bar{a} \lambda q)^2} \right) \left(\sum_{j=1}^r \frac{1}{j} \right)^2$$
 (11)

While (7) and (8) give mean and variance of the time to recruitment for sub case (i), (10) and (11) give the same for sub case (ii) where \bar{a} is given by (5).

Note

When $q=1$ our results agree with those in [13]. Our results are also consistent with those in [4] when $r=1$. Computation of $E(T)$ and $V(T)$ for extended exponential and SCBZ Property possessing thresholds is similar as their distribution will have just additional exponential terms.

IV. FINDINGS

From the above results the following observations are presented which agree with reality,

- i. When α increases and keeping all the other parameter fixed, the average loss of manpower increases. Therefore the mean and variance of time to recruitment increase for all the three cases.
- ii. As λ increases, the average time between two consecutive decisions decreases and consequently the mean and variance of time to recruitment decrease for all the three cases when the other parameters are fixed.
- iii. As q , the probability that every policy decision has exit of personnel increases, the mean and variance of time to recruitment decrease for all the three cases when the other parameters are fixed.
- iv. When r increases and keeping all the other parameter fixed, the average loss of manpower decreases. Therefore the mean and variance of time to recruitment decrease for all the three cases.

V. CONCLUSION

The model discussed in this paper is found to be more realistic in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs (ii) associating a probability for any decision to have exit points and (iii) bringing a kind of dependency for inter-decision times using order statistics. From the organization's point of view, our model is more suitable than the

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corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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