

# EDGE DOMINATION IN BOOLEAN FUNCTION GRAPH $B(G, L(G), NINC)$ OF A GRAPH

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**ABSTRACT:** For any graph  $G$ , let  $V(G)$  and  $E(G)$  denote the vertex set and edge set of  $G$  respectively. The Boolean function graph  $B(G, L(G), NINC)$  of  $G$  is a graph with vertex set  $V(G) \cup E(G)$  and two vertices in  $B(G, L(G), NINC)$  are adjacent if and only if they correspond to two adjacent vertices of  $G$ , two adjacent edges of  $G$  or to a vertex and an edge not incident to it in  $G$ . For brevity, this graph is denoted by  $B_1(G)$ . In this paper, edge domination number of Boolean Function Graph  $B(G, L(G), NINC)$  of some standard graphs and its bounds are obtained.

**KEYWORDS:** Boolean Function graph, Edge Domination Number

**2010 MATHEMATICS SUBJECT CLASSIFICATION:**05C69

## I. INTRODUCTION

Graphs discussed in this paper are undirected and simple graphs. For a graph  $G$ , let  $V(G)$  and  $E(G)$  denote its vertex set and edge set respectively. The graph  $G$  is denoted by  $G = (V, E)$ . A subset  $D \subseteq V$  is called a dominating set of  $G$  if every vertex not in  $D$  is adjacent to some vertex in  $D$ . The domination number  $\gamma(G)$  of  $G$  is the minimum cardinality taken over all dominating sets of  $G$ . The open neighborhood  $N(v)$  of  $v$  in  $V$  is the set of vertices adjacent to  $v$  and the set  $N[v] = N(v) \cup \{v\}$  is the closed neighborhood of  $v$ . An edge  $e$  of a graph is said to be incident with the vertex  $v$  if  $v$  is an end vertex of  $e$ . In this case, we also say that  $v$  is incident with  $e$ .

A subset  $F \subseteq E$  is called an edge dominating set of  $G$  if every edge not in  $F$  is adjacent to some edge in  $F$ . The edge domination number  $\gamma'(G)$  of  $G$  is the minimum cardinality taken over all edge dominating sets of  $G$ . The maximum order of a partition of  $E$  into edge dominating sets of  $G$  is called the edge domatic number of  $G$  and is denoted by  $d'(G)$ . The concept of edge domination was introduced by Mitchell and Hedetniemi [8]. Jayaram [6] studied line (edge) dominating sets and obtained bounds for the line (edge) domination number and obtained Nordhaus-Gaddum results for the line domination number. Arumugam and Velammal [1] discussed edge domination number and edge domatic number. The complementary edge domination in graphs was studied by Kulli and Soner [7]. Vaidya and Pandit [9] determined edge domination number of middle graphs, total graphs and shadow graphs of Paths and Cycles. For graph theoretic notations and terminology, Harary [2] is followed. Janakiraman et al., introduced the concept of Boolean function graphs [3 - 5]. For a real  $x$ ,  $\lfloor x \rfloor$  denotes the greatest integer less than or equal to  $x$ .

**Theorem 1.1.** [6] For any  $(p, q)$  graph  $G$ ,  $\gamma' \leq \lfloor p/2 \rfloor$

**Theorem 1.2.** [6] For any  $(p, q)$  graph  $G$ ,  $\gamma' \leq q - \beta_1 + q_0$ , where  $\beta_1$  is the edge independence number and  $q_0$  is the number of isolated edges in  $G$ .

**Theorem 1.3.** [6] For any  $(p, q)$  graph  $G$ ,  $\gamma' \leq q - \Delta'$ , where  $\Delta'$  denotes the maximum degree of an edge in  $G$ .

**Observation [3]**

**1.4.**  $G$  and  $L(G)$  are induced subgraphs of  $B_1(G)$ .

**1.5.** Number of vertices in  $B_1(G)$  is  $p + q$  and if  $d_i = \deg_G(v_i)$ ,  $v_i \in V(G)$ , then the number of edges in  $B_1(G)$  is  $q(p-2) + \frac{1}{2} \sum_{i=1}^p d_i^2$ .

# International Journal of Innovative Research in Science, Engineering and Technology

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**1.6.** The degree of a vertex of  $G$  in  $B_1(G)$  is  $q$  and the degree of a vertex  $e'$  of  $L(G)$  in  $B_1(G)$  is  $\deg_{L(G)}(e') + p - 2$ . Also if  $d^*(e')$  is the degree of a vertex  $e'$  of  $L(G)$  in  $B_1(G)$ , then  $0 \leq d^*(e') \leq p + q - 3$ . The lower bound is attained, if  $G \cong K_2$  and the upper bound is attained, if  $G \cong K_{1,n}$ , for  $n \geq 2$ .

**Theorem 1.7. [3]**  $B_1(G)$  is disconnected if and only if  $G$  is one of the following graphs.  
 $nK_1, K_2, 2K_2$  and  $K_2 \cup nK_1$ , for  $n \geq 1$ .

In this paper, edge domination number of Boolean Function Graph  $B(G, L(G), NINC)$  of some standard graphs and its bounds are obtained.

## II. MAIN RESULTS

In the following edge domination number of  $B_1(P_n), B_1(C_n), B_1(K_p), B_1(K_{1,n})$  are found.

**Theorem 2.1:** For the Path  $P_n$  on  $n$  ( $n \geq 3$ ) vertices,  $\gamma'(B_1(P_n)) = n - 1$

**Proof:** Let  $v_1, v_2, v_3, \dots, v_n$  be the vertices and  $e_{12}, e_{23}, \dots, e_{n-1,n}$  be the edges in  $P_n$ , where  $e_{i,i+1} = (v_i, v_{i+1}), i = 1, 2, \dots, n-1$ . Then  $v_1, v_2, \dots, v_n, e_{12}, e_{23}, \dots, e_{n-1,n} \in V(B_1(P_n))$ .  $B_1(P_n)$  has  $2n - 1$  vertices and  $n(n-1) - 1$  edges.

Let  $f_i = (v_i, e_{i+1,i+2}) \in E(B_1(P_n))$ , where  $i = 1, 2, \dots, n - 2$  and  $f_{n-1} = (v_{n-1}, e_{12})$ . Let  $D' = \{f_1, f_2, \dots, f_{n-1}\} \subseteq E(B_1(P_n))$ . Since each edge in  $E(B_1(P_n))$  is either in  $D'$  or is adjacent to an edge in  $D'$ , it follows that the set  $D'$  is an edge dominating set of  $B_1(P_n)$ . Therefore,  $\gamma'(B_1(P_n)) \leq |D'| = n - 1$ . Also  $D'$  is a minimum edge dominating set of  $B_1(P_n)$ , since for any edge  $f_i \in D'$ , the set  $D' - \{f_i\}$  does not dominate the edge  $f_i$ . Any edge of the form  $(v_i, e_{jk})$  ( $j, k \neq i$ ), where  $e_{jk}$  is not incident with  $v_i$ , dominates  $(n-1)$  edges. But  $B_1(P_n)$  has  $n^2 - n - 1 = n(n-1) - 1$  edges. To dominate all the edges of  $B_1(P_n)$ , an edge dominating set of  $B_1(P_n)$  must contain at least  $(n-1)$  edges. That is, any edge dominating set of  $B_1(P_n)$  has at least  $(n-1)$  edges. Therefore,  $\gamma'(B_1(P_n)) \geq (n-1)$ , Hence,  $\gamma'(B_1(P_n)) = n - 1$ .

**Theorem 2.2:** For the cycle  $C_n$  on  $n$  ( $n \geq 4$ ) vertices,  $\gamma'(B_1(C_n)) = n - 1$ .

**Proof:** Let  $v_1, v_2, v_3, \dots, v_n$  be the vertices and  $e_{12}, e_{23}, \dots, e_{n-1,n}$  be the edges in  $C_n$ , where  $e_{i,i+1} = (v_i, v_{i+1}), i = 1, 2, \dots, n - 1$ . Then  $v_1, v_2, \dots, v_n, e_{12}, e_{23}, \dots, e_{n-1,n} \in V(B_1(C_n))$ .  $B_1(C_n)$  has  $2n$  vertices and  $n^2$  edges. Let  $f_i = (v_i, e_{i+1,i+2}) \in E(B_1(C_n))$  where  $i = 1, 2, \dots, n-2, f_{n-1} = (v_{n-1}, e_{12})$ . Let  $D' = \{f_1, f_2, \dots, f_{n-1}\} \subseteq E(B_1(C_n))$ . Since each edge in  $E(B_1(C_n))$  is either in  $D'$  (or) is adjacent to an edge in  $D'$ , it follows that set  $D'$  is an edge dominating set of  $B_1(C_n)$ . Therefore,  $\gamma'(B_1(C_n)) \leq |D'| = n - 1$ . Also  $D'$  is a minimum edge dominating set of  $B_1(C_n)$ , since for any edge  $f_i \in D'$ , the set  $D' - \{f_i\}$  does not dominate the edge  $f_i$ . Any edge of the form  $(v_i, e_{jk})$  ( $j, k \neq i$ ), where  $e_{jk}$  is not incident with  $v_i$ , dominates  $(n-1)$  edges. But  $B_1(C_n)$  has  $n^2$  edges. To dominate all the edges of  $B_1(C_n)$  an edge dominate set of  $B_1(C_n)$  must contain at least  $(n-1)$  edges. Therefore,  $\gamma'(B_1(C_n)) \geq n - 1$  and hence  $\gamma'(B_1(C_n)) = n - 1$ .

**Theorem 2.3:** For the complete graph  $K_p$  on  $p$  ( $p \geq 4$ ) vertices,  $\gamma'(B_1(K_p)) = (p(p-1))/2$ .

**Proof:** Let  $v_1, v_2, v_3, \dots, v_n$  be the vertices and  $e_{12}, e_{23}, \dots, e_{n-1,n}$  be the edges in  $K_p$ , where  $e_{i,i+1} = (v_i, v_{i+1}), i = 1, 2, \dots, n-1$ . Then  $v_1, v_2, \dots, v_n, e_{12}, e_{23}, \dots, e_{n-1,n} \in V(B_1(K_p))$ .  $B_1(K_p)$  has  $(p(p+1))/2$  and  $(p(p-1)(2p-3))/2$  edges. Let  $f_i = (v_i, e_{i+1,i+2}) \in E(B_1(K_p))$ , where  $i = 1, 2, \dots, n-2$  and  $f_{n-1} = (v_{n-1}, e_{12})$ . Let  $D' = \{f_1, f_2, \dots, f_{n-1}\} \subseteq E(B_1(K_p))$ . Since each edge in  $E(B_1(K_p))$  is either in  $D'$  (or) is adjacent to an edge in  $D'$ , it follows that the set  $D'$  is an edge dominating set of  $B_1(K_p)$ . Therefore,  $\gamma'(B_1(K_p)) \leq |D'| = (p(p-1))/2$ . Also  $D'$  is a minimum edge dominating set of  $B_1(K_p)$ , since for any edge  $f_i \in D'$  the set  $D' - \{f_i\}$  does not dominate the edge  $f_i$ . Any edge of the form  $(v_i, e_{jk})$  ( $j, k \neq i$ ), where  $e_{jk}$  is not incident with  $v_i$ , dominates  $(p(p-1))/2$  edges. But  $B_1(K_p)$  has  $(p(p-1)(2p-3))/2$  edges. To dominate all the edges of  $B_1(K_p)$  an edge dominate set of  $B_1(K_p)$  must contain atleast  $(p(p-1))/2$  edges.

## International Journal of Innovative Research in Science, Engineering and Technology

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Therefore,  $\gamma'(B_1(K_p)) \geq (p(p-1))/2$  and hence  $\gamma'(B_1(K_p)) = (p(p-1))/2$ .

**Theorem 2.4:** For the star  $K_{1,n}$  on  $(n+1)$  vertices,  $\gamma'(B_1(K_{1,n})) = (n+4)/3$ , where  $n \geq 2$ .

**Proof:** Let  $v_1, v_2, v_3, \dots, v_n$  be the vertices with  $v_1$  as the central vertex and  $e_{12}, e_{13}, \dots, e_{1n}$  be the edges in  $K_{1,n}$ , where  $e_{1,i+1} = (v_1, v_{i+1})$ ,  $i = 2, 3, \dots, n$  and. Then  $v_1, v_2, \dots, v_n, e_{12}, e_{13}, \dots, e_{1n} \in V(B_1(K_{1,n}))$ .  $B_1(K_{1,n})$  has  $2n+1$  vertices and  $2n+1$  and  $(n(3n-1))/2$  edges.

Let  $f_i = (v_i, e_{1,i+1}) \in E(B_1(K_{1,n}))$ , where  $i = 1, 2, \dots, n$  and  $f_{n-1} = (v_{n-1}, e_{12})$ .

Let  $D' = \{f_1, f_2, \dots, f_{n-1}\} \subseteq E(B_1(K_{1,n}))$ . Since each edge in  $E(B_1(K_{1,n}))$  is either in  $D'$  (or) is adjacent to an edge in  $D'$ , it follows that the set  $D'$  is an edge dominating set of  $B_1(K_{1,n})$ .

Therefore,  $\gamma'(B_1(K_{1,n})) \leq |D'| = (n+4)/3$ .

Also  $D'$  is a minimum edge dominating set of  $B_1(K_{1,n})$ , since for any edge  $f_i \in D'$  the set  $D' - \{f_i\}$  does not dominate the edge  $f_i$ . Any edge of the form  $(v_i, e_{jk})$  ( $j, k \neq i$ ), where  $e_{jk}$  is not incident with  $v_i$ , dominates  $n+4$  edges. But  $B_1(K_{1,n})$  has  $(n(3n-1))/2$  edges. To dominate all the edges of  $B_1(K_{1,n})$  an edge dominating set of  $B_1(K_{1,n})$  must contain atleast  $(n+4)/3$  edges.

Therefore,  $\gamma'(B_1(K_{1,n})) \geq (n+4)/3$  and hence  $\gamma'(B_1(K_{1,n})) = (n+4)/3$ .

A Wheel  $W_n$  on  $n$  vertices is defined as  $W_n = C_{n-1} + K_1$ , where  $n \geq 4$ .

In the following, edge domination number of  $B_1(W_n)$  is found.

**Theorem 2.5:** For the Wheel  $W_n$  on  $n$  ( $n \geq 7$ ) vertices,  $\gamma'(B_1(W_n)) = n-1$ .

**Proof:** Let  $v_1, v_2, v_3, \dots, v_n$  be the vertices with  $v_1$  as the central vertex and  $e_{12}, e_{13}, \dots, e_{1n}$  be the edges in  $W_n$ , where  $e_{1,i+1} = (v_1, v_{i+1})$ ,  $i = 2, 3, \dots, n$ . Then  $v_1, v_2, \dots, v_n, e_{12}, e_{13}, \dots, e_{1n} \in V(B_1(W_n))$ .

$B_1(W_n)$  has  $2n-1$  vertices and  $((n-1)(3n-4))/2$ .

Let  $f_i = (v_i, e_{1,i+1}) \in E(B_1(W_n))$ , where  $i = 1, 2, \dots, n$  and  $f_{n-1} = (v_{n-1}, e_{12})$ .

Let  $D' = \{f_1, f_2, \dots, f_{n-1}\} \subseteq E(B_1(W_n))$

Since each edge in  $E(B_1(W_n))$  is either in  $D'$  (or) is adjacent to an edge in  $D'$ , it follows that the set  $D'$  is an edge dominating set of  $B_1(W_n)$ . Therefore  $\gamma'(B_1(W_n)) \leq |D'| = n-1$ .

Also  $D'$  is an minimum edge dominating set of  $B_1(W_n)$ , since for any edge  $f_i \in D'$  the set  $D' - \{f_i\}$  does not dominate the edge  $f_i$ . Any edge of the form  $(v_i, e_{jk})$  ( $j, k \neq i$ ), where  $e_{jk}$  is not incident with  $v_i$ , dominates  $n-1$  edges. But  $B_1(W_n)$  has  $((n-1)(3n-4))/2$  edges.

To dominate all the edges of  $B_1(W_n)$ , an edge dominating set of  $B_1(W_n)$  must contain atleast  $(n-1)$  edges.

Therefore,  $\gamma'(B_1(W_n)) \geq n-1$  and hence  $\gamma'(B_1(W_n)) = n-1$ .

**Theorem 2.6:** If  $G \cong nK_2$ ,  $n \geq 2$ , then  $\gamma'(B_1(nK_2)) = n$ .

**Proof:** Let  $D'$  be a minimum edge dominating set of  $nK_2$ . Then  $|D'| = n$ . Since  $L(nK_2)$  is totally disconnected,  $D'$  is also an edge dominating set of  $B_1(G)$ . Also any edge dominating set of  $B_1(G)$  having less than  $n$  edges of  $B_1(G)$  is not an edge dominating set of  $B_1(G)$ . Therefore,  $\gamma'(B_1(nK_2)) = n$ .

**Remark 2.1:** Since  $G$  and  $L(G)$  are induced subgraphs of  $B_1(G)$ ,  $\gamma'(B_1(G)) \geq \gamma'(G)$  and  $\gamma'(B_1(G)) \geq \gamma'(L(G))$ . But any edge dominating set of  $G$  or  $L(G)$  is not an edge dominating set of  $B_1(G)$ ,  $\gamma'(B_1(G)) > \gamma'(G)$  and  $\gamma'(B_1(G)) > \gamma'(L(G))$ .

In the following, upper bounds of  $\gamma'(B_1(G))$  are obtained.

**Theorem 2.7:** For any graph  $G$ ,  $\gamma'(B_1(G)) \leq \gamma'(G) + \gamma'(L(G))$ , where  $L(G)$  is the line graph of  $G$ .

**Proof:** Let  $D$  be a dominating set of  $G$  and let  $D'$  be a dominating set of  $L(G)$ . Then the set  $D \cup D'$  is an edge dominating set of  $B_1(G)$ . Hence  $\gamma'(B_1(G)) \leq \gamma'(G) + \gamma'(L(G))$ .

This bound is attained if  $G \cong P_3, C_3$  or  $C_4$ .

**Theorem 2.8.:** For any graph  $G$  with  $p$  vertices,  $\gamma'(B_1(G)) \leq p + \gamma'(L(G))$

**Proof:** Let  $D'$  be a dominating set of  $L(G)$ . Then  $V(G) \cup D'$  is an edge dominating set of  $B_1(G)$  and hence  $\gamma'(B_1(G)) \leq |V(G) \cup D'| = p + \gamma'(L(G))$ .

**Theorem 2.9:** For any graph  $G$  with  $q$  edges,  $\gamma'(B_1(G)) \leq q + \gamma'(G)$

## International Journal of Innovative Research in Science, Engineering and Technology

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**Proof:** Let  $D$  be a dominating set of  $G$ . Then  $V(L(G)) \cup D$  is an edge dominating set of  $B_1(G)$  and hence  $\gamma'(B_1(G)) \leq |V(L(G)) \cup D| = q + \gamma'(G)$ .

**Theorem 2.10:** For any graph  $G$ ,  $\gamma'(B_1(G)) > \gamma'(G)$ , if  $P_3$  is a subgraph of  $G$ .

**Proof:** Assume  $P_3$  is a subgraph of  $G$ . Then  $L(G)$  has atleast one edge. Let  $D'$  be a minimum edge dominating set of  $G$ . Since  $G$  is an induced subgraph of  $B_1(G)$ ,  $D'$  dominates edges of  $B_1(G)$  corresponding to the edges in  $G$ . To dominate the edges in  $E(L(G)) \cap E(B_1(G))$ , atleast one edge in  $B_1(G)$  is to be included with  $D'$ . Hence  $\gamma'(B_1(G)) > \gamma'(G)$ , if  $P_3$  is a subgraph of  $G$ .

**Theorem 2.11:**  $\gamma'(B_1(G)) = 1$  if and only if  $G \cong K_2$

**Proof:** Let  $D'$  be a dominating set of  $B_1(G)$  such that  $|D'| = 1$ . Let  $D' = (x_1, x_2) \in E(B_1(G))$

**Case i:** Let  $e_1 = (x_1, x_2)$ . Then  $e_1 \in E(G)$  and if there exists an  $e_2 \in E(G)$  such that  $e_2$  is adjacent to  $e_1$ , then the edge  $(e_1, e_2) \in E(B_1(G))$  is adjacent  $e_1$  in  $B_1(G)$ .

**Case ii:**  $x_1 \in V(G)$ ,  $x_2 \in E(G)$ .

Let  $x_2 = (u_2, v_2)$  where  $u_2, v_2 \in V(B_1(G))$  and the edge

$(x_1, x_2) \in E(B_1(G))$  and the edge  $x_2 \in E(G)$  is not incident with  $x_1$  in  $G$ .

That is  $u_2, v_2 \neq x_1$  in  $G$ . Then the edge  $(u_2, v_2) \in E(B_1(G))$  is not adjacent to both  $x_1$  and  $x_2$ .

**Case iii:**  $x_1, x_2 \in E(G)$ .

Let  $x_1 = (u_1, v_1)$ ,  $x_2 = (u_2, v_2)$  where  $u_1, v_1, u_2, v_2 \in V(G)$ . Then  $x_1, x_2$  are adjacent edges in  $G$ . Let  $v_1 = u_2$ . Then the edge  $(u_1, v_1) \in E(B_1(G))$  is not adjacent to the edge  $(x_1, x_2)$  in  $B_1(G)$ . Therefore,  $G$  has exactly one edge  $G \cong K_2$ .

Conversely, if  $G \cong K_2$ , then  $\gamma'(B_1(G)) = 1$ .

**Theorem 2.12:** Let  $e_1$  and  $e_2$  be two edges in  $G$ . Then  $\{e_1, e_2\}$  is a edge dominating set of  $B_1(G)$  if and only if  $G \cong 2K_2$ .

**Proof:** Let  $e_1 = (x_1, x_2)$  and  $e_2 = (x_3, x_4)$  where  $x_1, x_2, x_3, x_4 \in V(G)$ .

**Case i:**  $D' = \{e_1, e_2\} \subseteq E(B_1(G))$ .

Then they are adjacent in  $B_1(G)$ . Let  $e_1 = (x_1, x_2)$  and  $e_2 = (x_2, x_4)$ . Then the edge  $(e_1, e_2) \in E(B_1(G))$  is not adjacent to both  $e_1$  and  $e_2$ .

**Case ii:**  $e_1$  and  $e_2$  are nonadjacent edges in  $G$ .

Then they are nonadjacent in  $B_1(G)$ . Let  $e_1 = (x_1, x_2)$  and  $e_2 = (x_3, x_4)$  then the edge  $(e_1, e_2) \in E(B_1(G))$  and  $D'$  is a edge dominating set of  $B_1(G)$  if  $G \cong 2K_2$ .

Conversely, if  $G \cong 2K_2$ , then the vertices in  $B_1(G)$  corresponding to two edges in  $2K_2$  forms a minimum edge dominating set of  $B_1(G)$ .

**Remark 2.2:** Let  $v_1, v_2 \in V(G)$  and  $e_i \in E(G)$  ( $i = 1, 2$ ) be the edges in  $G$  not incident with  $v_i$  and let  $e_{11} = (v_1, e_1)$ ,  $e_{12} = (v_2, e_2)$ . Then  $\{e_{11}, e_{12}\} \subseteq E(B_1(G))$  is an edge dominating set of  $B_1(G)$  if and only if  $G \cong P_3$ .

**Remark 2.3:** For any connected graph  $G$ , let  $D'$  be an edge dominating set of  $G$  and let  $v$  be a vertex in  $G$  not incident with any of the edges in  $D'$ . Let  $D''$  be the set of edges in  $G$  corresponding to the saturated vertices of maximum matching in  $L(G)$ . If the edges of  $G$  not incident with  $v$  are elements of  $D''$ , then  $D' \cup D''$  is an edge dominating set of  $B_1(G)$  and hence  $\gamma'(B_1(G)) \leq \gamma'(G) + \beta_1'(L(G))$ , where  $\beta_1'$  is the independence number of  $L(G)$ .

**Theorem 2.13:** Let  $G$  be a graph having  $P_3$  as an induced subgraph and let  $H$  be the set of all edges joining vertices of the subgraph  $G - P_3$  of  $G$  and  $P_3$ . Then  $\gamma'(B_1(G)) \leq \gamma'(\langle H \rangle) + 3$ .

**Proof:** Let  $V(P_3) = (v_1, v_2, v_3)$  and  $E(P_3) = \{e_{12}, e_{23}\}$ , where  $e_{12} = (v_1, v_2)$ ,  $e_{23} = (v_2, v_3)$ .

Then  $(v_1, v_2), (v_2, v_3), (e_{12}, e_{23}) \in E(B_1(G))$ . Let  $D$  be a minimum edge dominating set of  $\langle H \rangle$ . Therefore,  $|D| = \gamma'(\langle H \rangle)$

Then the set  $D' = \{(v_1, v_2), (v_2, v_3), (e_{12}, e_{23})\} \cup D \subseteq E(B_1(G))$  is an edge dominating set of  $B_1(G)$  and hence

$\gamma'(B_1(G)) \leq |D'| = 3 + \gamma'(\langle H \rangle)$ .

## International Journal of Innovative Research in Science, Engineering and Technology

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**Remark 2.3:** (1) In general, if  $P_n$  is an induced subgraph of  $G$  and if  $H$  be the set of edges joining vertices of the subgraph  $G - P_n$  of  $G$  to vertices of  $P_n$ , then  $\gamma'(B_1(G)) \leq |E(P_n)| + |E(L(P_{n-1}))| + \gamma'(\langle H \rangle)$

$$= n - 1 = n - 2 + \gamma'(\langle H \rangle) = 2n - 3 + \gamma'(\langle H \rangle)$$

(2) If  $C_n$  ( $n \geq 3$ ) is the smallest cycle in  $G$  and if  $H$  is the set of all edges joining vertices of the subgraph  $G - C_n$  of  $G$  to vertices of  $C_n$ , then  $\gamma'(B_1(G)) \leq 2n + \gamma'(\langle H \rangle)$ .

**Theorem 2.14:** Let  $G$  be a unicyclic graph with  $p$  vertices. Then  $\gamma'(B_1(G)) \leq p$ ,  $p \geq 3$ .

**Proof:** Since  $G$  is unicyclic, the number of edges in  $G$  is  $p$ . Let  $v_1, v_2, \dots, v_p$  be the vertices of  $G$  and let  $\deg_G(v_i) = d_i$ . Then there are  $t_i = p - d_i$  edges in  $G$  not incident with  $v_i$ . For each  $v_i$  in  $G$ , choose an edge  $e_i$  not incident  $v_i$ , such that  $e_i$  are distinct. Let  $D = \{(v_i, e_i) : i = 1, 2, \dots, p\}$ . Then  $D$  is an edge dominating set of  $B_1(G)$ .

Hence  $\gamma'(B_1(G)) \leq |D| = p$ .

### III. CONCLUSION

In this paper, edge domination numbers of Boolean Function Graph  $B(G, L(G), NINC)$  of paths, cycles, complete graphs, stars, wheels and union of  $K_2$  are found and bounds are obtained.

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