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## Effects of Slip and Heat Transfer on the Peristaltic MHD Flow of Carreau Fluid with Different Wave Forms

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**ABSTRACT:** The effects of slip and heat transfer on the peristaltic MHD flow of Carreau fluidare studied under long wavelength and low Reynolds number conditions. The channel asymmetry produced by choosing the different peristaltic wave train likely sinusoidal, triangle, square, trapezoidal and sawtooth. The nonlinear governing equations are solved using a perturbation technique based on small Weissenberg number. Expressions for the stream function, pressuregradient, temperature distribution, axial induced magnetic force function and shear stress on the wall are obtained. The effects of various parameters assessed numerically are presented and discussed. The comparison of the various considered wave forms on the flow is discussed.

KEYWORDS : Peristaltic flow, Carreau fluid, Magnetic field, Partial slip, Heat transfer, different waveforms.

#### I. INTRODUCTION

There has been a recent interest in peristaltic pumping which can produce a flow in a duct completely isolated from the pump mechanism. The term peristalsis is used for the mechanism by which a fluid can be transported through a distensible tube when progressive waves of area contraction and expansion propagate along its length. It plays an indispensable role in transporting many physiological fluids in the body such as the movement of ovum in the fallopian tubes, urine transport from kidney to bladder, the swallowing of food through esophagus, the transport of spermatozoa in the ducts efferentes of the male reproductive tract, the movement of chime in the gastrointestinal tracts, vasomotion of small blood vessels and bile duct and other glandular ducts. Many modern mechanical divices have been modeled on the principle of peristaltic pumping for transporting fluids without internal moving parts, for example, roller pumps, used to pump blood, food or corrosive liquids and transport of noxius fluid in nuclear industry operate under this principle. Considerable analysis of this mechanism has been carried out, primarily for a Newtonian fluid with a periodic train of sinusoidal peristaltic waves. The mechanism of peristaltic transport has attracted the attention of many investigators. Since the first investigation of Latham [1], the inertia free peristaltic flow with a long wavelength at low Reynolds number was studied by Shapiro et al. [2]. The mechanics of peristalsis for different fluids has been examined (analytical, numerical and experimental) by a number of researchers [3-16], all of these papers have been studied the effect of peristaltic transport for different fluids through channels or tubes.

The magneto hydrodynamic (MHD) flow in a channel (tube) having, rhythmically contracting walls (peristaltic transports) is of interest in recent years. Some recent studies have considered the effect of magnetic field on peristaltic flow of Newtonian and non-Newtonian fluids [16–24], all of these investigations have been found that the maximum amplitude of the pressure gradient elevates as the Hartmann number increase. In these studies, the effect of the induced magnetic field has been neglected. The first contribution of the effect of the induced magnetic field on peristaltic flow was studied by Vishnyakov and Pavlov [25]. Eldabe et al. [26] studied the effect of the induced magnetic fieldon peristaltic transport of a non-Newtonian fluid (biviscosity fluid). Recently Abd elmaboud[27] studied the influence of the induced magnetic field on the peristaltic flow in an annulus.



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Very recently, the combined effects of heat transfer and the induced magnetic field on the peristaltic transport of a Carreau fluid havebeen discussed By Hayat et al.[28]. The existing literature indicates that the heat transfer and magnetic field characteristics on the peristalsishave not been studied so far when no-slip condition is no longer valid. In general, the velocity is sensitive to variation in boundary, while the temperature is less sensitive so we consider the slip condition in the boundaries for the velocity field. Turkyilmazoglu [29-30] reported the contribution of the slip over the exact analytical solutions for heat and mass transfer of MHD slip flow in nanofluids and dual and triple solutions for MHD slip flow of non-Newtonian fluid over a shrinking surface. Inall these previous investigations authors have discussed the velocity field, pressure gradient, pressure rise and heat transfer distribution but they did not study the different wave forms and shear stress on the peristaltic flow of a Carreau fluid.

In this paper, we study the MHD flow of Carreau fluid flow in an asymmetric channel under the effects of slip and heat transfer with long wavelength and low Reynolds number assumptions. The problem is examined numerically using a perturbation technique by small Weissenberg number, which provides an effective computational tool for the solution of nonlinear equations. The influence of effected parameters on axial velocity, pressure gradient, pressure rise, shear stress, axial induced magnetic field, temperature distribution, trapping and different wave forms phenomena are presented and discussed. A motivation of the present paper is the hope that such a problem will be applied in many clinical and industrial applications such as transport of bio-fluids in ureters, intestines, arterioles and blood pump.

#### II. PROBLEM DEFINITION



#### **2.1. MATHEMATICAL FORMULATION:**

Fig. 1. Schematic diagram of a two-dimensional asymmetric channel

Let us consider the peristaltic movement of an incompressible Carreau fluid under the effect of the induced magnetic field in a two-dimensional channel of width  $d_1 + d_2$  (Fig.1). The flow is generated by sinusoidal wave trains propagating with constant speed(*C*) along the asymmetric channel. The geometry of the wall surfaces is defined as

$$h_{1}(\bar{X},\bar{t}) = \bar{d}_{1} + \bar{a}_{1} \cos\left[\frac{2\pi}{\lambda}(\bar{X}-c\bar{t})\right] \dots \text{upper wall}$$
(1)  
$$h_{2}(\bar{X},\bar{t}) = -\bar{d}_{2} - \bar{a}_{2} \cos\left[\frac{2\pi}{\lambda}(\bar{X}-c\bar{t}) + \phi\right] \dots \text{lower wall}$$
(2)

In which  $\overline{a}_1$  and  $\overline{a}_2$  are the amplitudes of the waves,  $\lambda$  is the wavelength, C is the wave speed,  $\phi$  ( $0 \le \phi \le \pi$ ) is the phase difference,  $\overline{X}$  and  $\overline{Y}$  are the rectangular coordinates with  $\overline{X}$  measured along the axis of the channel and  $\overline{Y}$  is perpendicular to  $\overline{X}$ . Let ( $\overline{U}, \overline{V}$ ) be the velocity components in a fixed frame of reference ( $\overline{X}, \overline{Y}$ ). It should be noted that



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 $\phi = 0$  corresponds to symmetric channel with waves out of phase and for  $\phi = \pi$  the waves are in phase. Furthermore,  $\overline{a_1}$ ,  $\overline{a_2}$ ,  $\overline{d_1}$ ,  $\overline{d_2}$  and  $\phi$  satisfy the condition.

$$\overline{a}_{1}^{2} + \overline{a}_{2}^{2} + 2\overline{a}_{1}\overline{a}_{2}\cos\phi \leq \left(\overline{d}_{1} + \overline{d}_{2}\right)^{2}$$
(3)

Introducing a wave frame  $(\bar{x}, \bar{y})$  moving with velocity (c) away from the fixed frame  $(\bar{X}, \bar{Y})$  by the transformation  $\bar{x} = \bar{X} - c\bar{t}, \quad \bar{y} = \bar{Y}, \ \bar{u}(\bar{x}, \bar{y}) = \bar{U} - c, \ \bar{v}(\bar{x}, \bar{y}) = \bar{V}.$ (4)

A constant magnetic field of strength  $\overline{H}_0$  is applied in transverse direction. This gives rise to an induced magnetic field  $\overline{H}^+(\overline{h}_{\overline{x}}(\overline{X},\overline{Y},\overline{t}), \overline{h}_{\overline{y}}(\overline{X},\overline{Y},\overline{t}), 0)$  and hence the total magnetic field becomes  $\overline{H}^+(\overline{h}_{\overline{x}}(\overline{X},\overline{Y},\overline{t}), \overline{H}_0 + \overline{h}_{\overline{y}}(\overline{X},\overline{Y},\overline{t}), 0)$ . The basic equations of magneto hydrodynamics, neglecting the displacement currents and free charges, are

(5)

 $\nabla \cdot \overline{H} = 0, \ \widetilde{N} \cdot \overline{\varepsilon} = \theta,$ 

$$\nabla \times \overline{H} = \overline{J}, \overline{J} = \sigma \left\{ \overline{\varepsilon} + \mu_e \left( \overline{V} \times \overline{H}^+ \right) \right\}, \tag{6}$$
$$\nabla \times \overline{\varepsilon} = -\mu_e \frac{\partial \overline{H}}{\partial \overline{t}}, \tag{7}$$

$$\tilde{\mathbf{N}}.\vec{\boldsymbol{V}}=\boldsymbol{0}, \tag{8}$$

$$\rho \underbrace{\stackrel{\circ}{\underbrace{\$}}_{\underline{\$}}}_{\underline{\$}} + (\overline{V}.\,\widetilde{N}) \underbrace{\stackrel{\circ}{\underbrace{\$}}_{\underline{t}}}_{\underline{\$}} = -\widetilde{N}\overline{p} + div\overline{\tau} - \mu_{e} \underbrace{\stackrel{\circ}{\underbrace{1}}}_{\underline{t}} (\overline{H}^{+}.\,\widetilde{N}) - \frac{1}{2} (\overline{H}^{+})^{2} \, \underbrace{\stackrel{\circ}{N}\stackrel{i}{\underbrace{\$}}_{\underline{t}}}_{\underline{b}}, \tag{9}$$

$$\rho C_p \frac{dT}{dt} = \kappa \tilde{N}^2 T + \tau_1 L.$$
<sup>(10)</sup>

In the above equations  $\tau_1$  is the Cauchy stress tensor. L = gradV,  $\overline{p}$  is the pressure,  $\overline{H}$  is the induced electrical field,  $\overline{J}$  is the current density,  $m_e$  is magnetic permeability, s is the electrical conductivity,  $\overline{\epsilon}$  is the electrical field,  $C_p$  is the specific heat at a constant volume,  $\kappa$  is the thermal conductivity and T is temperature. The velocity  $\overline{V}$  is defined by

$$\overline{V} = (\overline{u}, \overline{v}, 0). \tag{11}$$

The constitutive equation for a Carreau fluid is

$$\bar{\tau} = -\left[\eta_{\infty} + \left(\eta_0 - \eta_{\infty}\right) \left(1 + \left(\Gamma \bar{\dot{\gamma}}\right)^2\right)^{\frac{n-1}{2}} \right] \bar{\dot{\gamma}}, \tag{12}$$

Where  $\tau$  is the extra stress tensor,  $\eta_{\infty}$  is the infinite shear-rate viscosity,  $\eta_0$  is the zero shear-rate viscosity,  $\Gamma$  is the time constant, *n* is the dimensionless power law index. The above model reduces to the Newtonian Model for n=1 or  $\Gamma=0$  and  $\dot{\gamma}$  is defined as

$$\overline{\dot{\gamma}} = \sqrt{\frac{1}{2} \sum_{i} \sum_{j} \overline{\dot{\gamma}_{ij}} \overline{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2} \Pi}$$
(13)



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Combining equations (5) - (7), we obtain theinduction equations

$$\frac{\|\bar{H}^{+}}{\|\bar{t}} = \tilde{N} \times \bar{V} \times \bar{H}^{+} + \zeta \tilde{N}^{2} \bar{H}^{+}.$$
(14)

In which  $\zeta = \frac{1}{\sigma \mu_e}$  is the magnetic diffusivity.

The following non-dimensional quantities are also defined:

$$x = \frac{\overline{x}}{\lambda}, \quad y = \frac{\overline{y}}{\overline{d_1}}, \quad u = \frac{\overline{u}}{c}, \quad v = \frac{\overline{v}}{c}, \quad t = \frac{c}{\lambda}\overline{t}, \quad h_1 = \frac{\overline{h_1}}{\overline{d_1}}, \quad h_2 = \frac{\overline{h_2}}{\overline{d_1}}, \quad \tau = \frac{d_1}{\mu c}\overline{\tau}, \quad \Phi = \frac{\overline{\Phi}}{H_0\overline{d_1}}, \quad S_t = \frac{H_0}{C}\sqrt{\frac{\mu_e}{\rho}}, \quad \dot{\gamma} = \frac{\overline{\gamma}\overline{d_1}}{c}, \quad \delta = \frac{\overline{d_1}}{\lambda}, \quad W_e = \frac{\Gamma c}{d_1}, \quad p = \frac{\overline{d_1}^2}{c\lambda\mu}\overline{p}, \quad R_e = \frac{\rho c\overline{d_1}}{\mu}, \quad R_m = \sigma\mu_e ac, \quad P_r = \frac{\rho vC_p}{\kappa}, \quad \theta = \frac{T-T_0}{T_1-T_0}, \quad Ec = \frac{c^2}{C_p(T_1-T_0)}, \quad \psi = \frac{\overline{\psi}}{c\overline{d_1}}, \quad a = \frac{\overline{a_1}}{\overline{d_1}}, \quad b = \frac{\overline{a_2}}{\overline{d_1}}, \quad d = \frac{\overline{d_2}}{\overline{d_1}}, \quad M^2 = \operatorname{Re} \mathbf{S}_r^2 \mathbf{R}_m.$$
(15)

To proceed, we non-dimensional Eqs. (9) and (14), the resulting equations in terms of the stream function  $\psi$  and the magnetic force function  $\Phi$  can be written as:

$$\frac{\partial \psi}{\partial y} - \delta \left( \frac{\partial \psi}{\partial y} \frac{\partial \Phi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \Phi}{\partial y} \right) + \frac{1}{R_m} \nabla^2 \Phi = E, \qquad (16)$$

$$\operatorname{Re}\delta\left\{\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial y}\right\}=-\frac{\partial p_{m}}{\partial x}+\delta\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\operatorname{Re}\delta S_{t}^{2}\left(\frac{\partial\Phi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\Phi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\Phi}{\partial y}+\operatorname{Re}S_{t}^{2}\frac{\partial^{2}\Phi}{\partial y^{2}},$$
(17)

$$-\operatorname{Re}\delta^{3}\left\{\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial x}\right\}=-\frac{\partial p_{m}}{\partial y}+\delta^{2}\frac{\partial\tau_{xy}}{\partial x}+\delta\frac{\partial\tau_{yy}}{\partial x}-\operatorname{Re}\delta^{3}S_{t}^{2}\left(\frac{\partial\Phi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\Phi}{\partial y}\frac{\partial}{\partial y}\right)\frac{\partial\Phi}{\partial x}-\operatorname{Re}\delta^{2}S_{t}^{2}\frac{\partial^{2}\Phi}{\partial x\partial y},\qquad(18)$$

$$\frac{\partial^2 \theta}{\partial y^2} + Br \left\{ 1 + \frac{(n-1)}{2} W e^2 \dot{\gamma}^2 \right\} \left( \frac{\partial^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right) \frac{\partial^2 \psi}{\partial y^2} = 0 .$$
<sup>(19)</sup>

Where

$$u = \frac{\partial \psi}{\partial y}, \quad \upsilon = -\delta \frac{\partial \psi}{\partial x}, \quad \mathbf{h}_{x} = \frac{\partial \Phi}{\partial y}, \quad \mathbf{h}_{y} = -\delta \frac{\partial \Phi}{\partial x}, \quad \mathbf{Br} = \mathbf{E}\mathbf{c}\mathbf{P}\mathbf{r},$$
 (20)

$$p_m = p + \frac{1}{2} \operatorname{Re} \delta \frac{\mu_e \left(H^+\right)^2}{\rho c^2},\tag{21}$$

$$\nabla^2 = \delta^2 \frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial y^2},\tag{22}$$

$$\tau_{xx} = -2 \left[ 1 + \frac{(n-1)}{2} W e^2 \dot{\gamma}^2 \right] \frac{\partial^2 \psi}{\partial x \partial y},$$
(23)



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$$\tau_{xy} = -\left[1 + \frac{(n-1)}{2}We^2\dot{\gamma}^2\right] \left(\frac{\partial^2\psi}{\partial y^2} - \delta^2\frac{\partial^2\psi}{\partial x^2}\right),\tag{24}$$

$$\tau_{yy} = 2\delta \left[ 1 + \frac{(n-1)}{2} W e^2 \dot{\gamma}^2 \right] \frac{\partial^2 \psi}{\partial x \partial y},$$
(25)

$$\dot{\gamma} = \left[ 2\delta^2 \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right) + 2\delta^2 \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)^2 \right]^{\frac{1}{2}}.$$
(26)

Here  $\delta$  is a wave number,  $R_e$  the Reynolds number,  $R_m$  the magnetic Reynolds number,  $S_t$  the Strommer's number,  $p_m$  the magnetic pressure, We the Weissenberg number, M the Hartmann number, Pr the Prandtl number, E the Eckert number and Br the Brickman number.

Eqs. (16) - (19) after using the long wavelength and low Reynolds number conditions reduce to:

$$\frac{\partial \psi}{\partial y} + \frac{1}{R_m} \frac{\partial^2 \Phi}{\partial y^2} = E,$$
(27)

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left[ \left\{ 1 + \frac{(n-1)}{2} W e^2 \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \right\} \frac{\partial^2 \psi}{\partial y^2} \right] + M^2 \left( E - \frac{\partial \psi}{\partial y} \right), \tag{28}$$

$$\frac{\partial p}{\partial y} = 0,\tag{29}$$

$$\frac{\partial^2 \theta}{\partial y^2} + Br \left[ \left\{ 1 + \frac{(n-1)}{2} W e^2 \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \right\} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \right] = 0,$$
(30)

$$\frac{\partial^2}{\partial y^2} \left[ \left\{ 1 + \frac{(n-1)}{2} W e^2 \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \right\} \frac{\partial^2 \psi}{\partial y^2} \right] - M^2 \frac{\partial^2 \psi}{\partial y^2} = 0.$$
(31)

#### 2.2. BOUNDARY CONDITIONS :

The fluid adheres to the surface partially and the motion of the fluid exhibits partial slip conditions. In the wave frame, the boundary conditions are

$$\psi = \frac{q}{2}, \quad \frac{\partial \psi}{\partial y} + \beta \frac{\partial^2 \psi}{\partial y^2} = -1, \quad \theta = 0, \quad \Phi = 0 \quad at \quad y = h_1 = 1 + a \cos 2\pi x, \tag{32}$$

$$\psi = -\frac{q}{2}, \quad \frac{\partial \psi}{\partial y} - \beta \frac{\partial^2 \psi}{\partial y^2} = -1, \quad \theta = 1, \quad \Phi = 0 \quad at \quad y = h_2 = -d - b\cos\left(2\pi x + \phi\right). \tag{33}$$

Where  $\beta$  is the non-dimensional slip parameter.

In laboratory frame, the dimensional volume flow rate is

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$$Q = \int_{\overline{h}_2(\overline{X},\overline{t})}^{\overline{h}_1(\overline{X},t)} \overline{U}(\overline{X},\overline{Y},\overline{t}) d\overline{y}.$$
(34)

In which  $\overline{h_1}$  and  $\overline{h_2}$  are functions of  $\overline{X}$  and  $\overline{t}$ , the above expression in wave frame becomes  $q = \int_{\overline{h_2}(\overline{x})}^{\overline{h_1}(\overline{x})} \overline{u}(\overline{x}, \overline{y}) d\overline{y}$ . (35)

Where  $\overline{h_1}$  and  $\overline{h_2}$  are functions of  $\overline{x}$  alone. Form eqs. (4), (34) and (35) we can write

$$Q = q + c\overline{h_1}(\overline{x}) - c\overline{h_2}(\overline{x}). \tag{36}$$

the Time- averaged flow over a period T at a fixed position  $\overline{X}$  is

$$\theta = \frac{1}{T} \int_{0}^{T} \mathcal{Q}dt.$$
(37)

Upon making use of eq.(36)into eq.(37) and then integrating one can write

$$\theta = F + c\overline{d}_1 + c\overline{d}_2 \tag{38}$$

I. If we define the dimensionless mean flows  $\overline{q}$  in the laboratory frame and q in the wave frame, according to

$$\overline{Q} = \frac{\theta}{c\overline{d}_1}, \quad q = \frac{F}{c\overline{d}_1}.$$
(39)

in which

$$F = \int_{h_2(x)}^{h_1(x)} \frac{\partial \psi}{\partial y} dy = \psi(h_1(x)) - \psi(h_2(x)).$$
(40)

One finds that eq. (38) becomes

$$\overline{Q} = q + 1 + d \,. \tag{41}$$

We note that  $h_1(x)$  and  $h_2(x)$  represent the dimensionless form of the surfaces of the peristaltic walls

$$h_1(x) = 1 + a\cos 2\pi x, \tag{42}$$

$$h_2(x) = -d - b\cos(2\pi x + \phi).$$
 (43)



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#### **III SOLUTION METHOD**

#### **3.1. PERTURBATION SOLUTION**

For perturbation solution, we expand  $\psi$ , q, pand  $\Phi$  as

$$\begin{split} \psi &= \psi_0 + W e^2 \psi_1 + O(W e^4), \quad (44) \\ q &= q_0 + W e^2 q_1 + O(W e^4), \quad (45) \\ p &= p_0 + W e^2 p_1 + O(W e^4), \quad (46) \\ \Phi &= \Phi_0 + W e^2 \Phi_1 + O(W e^4). \quad (47) \end{split}$$

The perturbation series solution up to second order for stream function  $\psi$ , pressure gradient  $\frac{dp}{dx}$ , pressurerise  $\Delta P_{\lambda}$ , magnetic force function  $\Phi$  and shear stress  $\tau_{xy}$  can be summarized as

$$\begin{split} \psi &= C_1 + C_2 y + C_3 \cosh My + C_4 \sinh My \\ &+ We^2 \Big[ C_5 + C_6 y + C_7 \cosh My + C_8 \sinh My \\ &- \frac{M^4}{64} \big( n - 1 \big) \Big( C_3 \Big( C_3^2 + 3C_4^2 \Big) \cosh 3My + C_4 \Big( C_4^2 + 3C_3^2 \Big) \sinh 3My \Big) \\ &- \frac{3}{16} M^5 \big( n - 1 \big) \Big( C_3^2 - C_4^3 \big) \Big( (2C_3 + MC_4 y) \cosh My + \big( MC_3 y + 2C_4 \big) \sinh My \big) \Big], \end{split}$$
(48)  
$$\begin{aligned} \frac{dp}{dx} &= M^2 E + \frac{M^2 \Big[ \Big( 2 + M^2 q \beta \Big) \sinh M \Big( \frac{h_1 - h_2}{2} \Big) \Big] + Mq \cosh M \Big( \frac{h_1 - h_2}{2} \Big) \Big] \\ &- M^2 E + \frac{M^2 \Big[ \Big( 2 - M^2 \beta \big( h_1 - h_2 \big) \Big) \sinh M \Big( \frac{h_1 - h_2}{2} \Big) \Big] - M \big( h_1 - h_2 \big) \cosh M \Big( \frac{h_1 - h_2}{2} \Big) \Big] \\ &- We^2 \Big[ \frac{M^2}{(h_1 - h_2)} \Big( q - \frac{qL_5}{(h_1 - h_2) L_7} - \frac{L_8 L_5}{L_7} - L_6 \Big) \Big], \end{aligned}$$
(49)  
$$\begin{aligned} \Delta P_{\lambda} &= \int_{0}^{1} \frac{dp}{dx} dx, \end{aligned}$$
(50)



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$$\begin{aligned} \theta &= -\frac{M^2 Br}{8} \Big( C_3^2 + C_4^2 \Big) \cosh 2My - \frac{M^2 Br}{4} C_3 C_4 \sinh 2My - \frac{M^4 Br}{4} \Big( C_4^2 - C_3^2 \Big) y^2 + C_9 y + C_{10} \\ &- We^2 \bigg[ Br \bigg( \bigg( 2 \bigg( L_9 + L_{10} y - \frac{L_{12}}{M} \bigg) + \frac{(n-1)}{4} \bigg( C_3^4 - C_4^4 \bigg) \bigg) \frac{\cosh 2My}{4M^2} \\ &+ \bigg( 2 \bigg( L_{11} + L_{12} y - \frac{L_{10}}{M} \bigg) + \frac{(n-1)}{2} C_3 C_4 \bigg( C_3^2 - C_4^2 \bigg) \bigg) \frac{\sinh 2My}{4M^2} \\ &+ \bigg( 2 L_{13} + \frac{(n-1)}{16} \bigg( C_3^4 + C_4^4 + 6 C_3^2 C_4^2 \bigg) \bigg) \frac{\cosh 4My}{16M^2} + \bigg( 2 L_{14} + \frac{(n-1)}{4} C_3 C_4 \bigg( C_3^2 + C_4^2 \bigg) \bigg) \frac{\sinh 4My}{16M^2} \\ &+ \bigg( 2 L_{15} + \frac{3}{8} \bigg( C_3^2 - C_4^2 \bigg)^2 \bigg) \frac{y^2}{2} \bigg) - C_{11} y - C_{12} \bigg], \end{aligned}$$
(51)

$$\Phi = R_m \left( \left( E - C_2 \right) \frac{y^2}{2} - \frac{C_3}{M} \sinh My - \frac{C_4}{M} \cosh My \right) + C_{13}y + C_{14}$$

$$-We^2 \left[ R_m \left( C_6 \frac{y^2}{2} + \frac{C_7}{M} \sinh My + \frac{C_8}{M} \cosh My - \frac{(n-1)}{192} M^3 \left( C_3 \left( C_3^2 + 3C_4^2 \right) \sinh 3My + C_4 \left( C_4^2 + 3C_3^2 \right) \cosh 3My \right) - \frac{3(n-1)}{16} M^3 \left( C_3^2 - C_4^2 \right) \left( \frac{M \left( C_3 y \cosh My + C_4 y \sinh My \right)}{-(C_3 \sinh My + C_4 \cosh My)} - C_{15}y - C_{16} \right],$$
(52)

$$\tau_{xy} = -M^{2}C_{3}\cosh My - M^{2}C_{4}\sinh My - We^{2} \left[ \frac{(n-1)}{2} \left( M^{2}C_{3}\cosh My + M^{2}C_{4}\sinh My \right)^{3} - \frac{9}{64} M^{6} (n-1) \left( C_{3} \left( C_{3}^{2} + 3C_{4}^{2} \right) \cosh 3My + C_{4} \left( C_{4}^{2} + 3C_{3}^{2} \right) \sinh 3My \right) + M^{2}C_{7}\cosh My + M^{2}C_{8}\sinh My - \frac{3}{16} M^{6} (n-1) \left( C_{3}^{2} - C_{4}^{2} \right) \left( (2C_{3} + MC_{4}y) \cosh My + (MC_{3}y + 2C_{4}) \sinh My \right) \right]$$
(53)

Where  $L_i$  and  $C_i$  are set in the Appendix.

In order to check the results obtained above, it can be observed from Eqs. (47) - (51) that as  $\beta \rightarrow 0$ , the results reduced to that obtained by Hayat et al. [28].

#### **IV. RESULTS AND DISCUSSION**

The Carreu fluid provides a good understanding of peristaltic motion involved in the intestines and the small blood vessels. The chyme in the small intestine is an example of the Carreau fluid. Blood also behaves like a non-Newtonian fluid at low shear rates in tubes with small diameters. This fluid model exhibits the macroscopic wall slip. This



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situation corresponds to the slip flow, in which a fluid presents the loss of adhesive forces at the wall, making the fluid slide over the wall. The effects of the wall slip parameter  $\beta$ , the Weissenberg number We, the magnetic Reynolds number  $R_m$ , Brinkman number Br, and the Hartman number M onstream function  $\psi$ , pressure gradient  $\frac{dp}{dx}$ , pressure

rise  $\Delta P_{\lambda}$ , magnetic force function  $\Phi$  and shear stress  $\tau_{xy}$  are presented and discussed.

The axial velocity component 
$$u = \frac{\partial \psi}{\partial y}$$
, is obtained from Eq. (47) as:

 $u = C_2 + C_3 M \sinh My + C_4 M \cosh My$ 

 $+We^{2}[C_{6}+C_{7}M\sinh My+C_{8}M\cosh My$ 

$$-\frac{3M^{5}}{64}(n-1)\left(C_{3}\left(C_{3}^{2}+3C_{4}^{2}\right)\sinh 3My+C_{4}\left(C_{4}^{2}+3C_{3}^{2}\right)\cosh 3My\right)$$
(54)  
$$-\frac{3}{16}M^{6}(n-1)\left(C_{3}^{2}-C_{4}^{3}\right)\left(\binom{C_{4}\cosh My+(2C_{3}+MC_{4}y)\sinh My}{C_{3}\sinh My+(MC_{3}y+2C_{4})\cosh My}\right)^{2}.$$



Fig. 2: The axial velocity  $\mathcal{U}$  with  $\mathcal{Y}$  for a=0.43, b=0.5, d=1,  $\overline{\mathcal{Q}} = 2$ , x=1,  $\phi = \pi/6$ ; (a) We=0.001, n=0.398,  $\beta=0.01$ ; (b) We=0.001, n=0.398, M=1; (c) M=1, n=0.398, \beta=0.01; (d) We=0.001, M=1,  $\beta=0.01$ .

Fig. 2 illustrates the behavior of emerging flow parameters on the longitudinal velocity u. It is noticed that the variations of several parameters at the center and near the walls of the channel are different. However, at the central line of the channel y = 0, the largest disparity occurs. It is found that the Weissenberg number We and Hartmann number M have opposite effect on the velocity at the center of the channel as shown Fig. 2(a) and 2(c).

The effect of the slip parameter on the axial velocity is discussed through Fig. 2(b). The figure shows that the slip velocity effect appears near the channel walls but, as we move to the inner layers of the fluid this effect is disappearing under the effect of the fluid flow. It can be also seen that the velocity increases by decreasing  $\beta$  at the center line of the channel. According to the Navier conditions, the existence of slip at the surface is linearly proportional to the shear stress at the surface. Therefore, the blood flow in microcirculation under the slip condition can also be the expected



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physiological benefit of this study. Fig. (2d) demonstrates that an increase of the index power law of the fluid increases the velocity that means the velocity of non-Newtonian fluid (carreau n=0.398) is small compared to Newtonian fluid (n=1).

All figures represent the temperature are plotted at the cross section of the channel (namely x=0.3). To study the behavior of the emerging parameters on the temperature some figures have been carried out.



Fig. 3: Temperature distribution  $\theta$  versus y for a = 0.5, b = 0.5, d = 1, x = 0.3,  $\phi = \pi/6$ , M=1, n = 0.398, We=0.001 (a) Br=0.1;

(b)  $\beta = 0.01$ .

In Fig (3.a, 3.b, 3.c, 3.d), we observe the effects of the mean volume flow rate  $\overline{Q}$ , partial slip  $\beta$ , Weissenberg number  $W_e$  and brinkman number Br on the temperature profile with fixing the other parameters. This figure indicates that an increase in  $\overline{Q}$ ,  $W_e$  and Br lead to an increase in the temperature field while, an increase in  $\beta$  leads to decrease in the temperature field.



Fig. 4: Axial induced magnetic field  $h_x$  versus y for a = 0.5, b = 0.5, d = 1,  $\phi = \pi/6$ ,  $w_e = 0.001$ , x = 0.2, n = 0.398, M=1; (a)  $\overline{Q} = 1$ , E = 0.1,  $\beta = 0.01$ ; (b)  $\overline{Q} = 1$ ,  $\beta = 0.01$ ,  $R_m = 10$ ; (c)  $\beta = 0.01$ ,  $R_m = 10$ , E = 0.1; (d)  $R_m = 10$ , E = 0.1,  $\overline{Q} = 1$ .



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Fig. 4 display the effect of the magnetic Reynolds number  $R_m$ , the Eckert number E, volume flow rate  $\overline{\rho}$  and partial slip  $\beta$  on the axial induced magnetic field. We observed that the magnitude of the axial magnetic field  $h_x$  decreases by increasing the magnetic Reynolds number  $R_m$ , the Eckert number E and the slip parameter  $\beta$ . Moreover, in the half region of the channel, the induced magnetic field is one direction and in the other half it is in the opposite direction as in Figs. 4(a), 4(c) and 4(d). And also we have seen in Fig. 4(b) the opposite behavior of volume flow rate  $\overline{\rho}$ .



Fig. 5: The pressure distribution dp/dx with x for a = 0.5, b = 0.5,  $d = 1, \overline{Q} = -2$ , E = 0.1,  $We = 0.001, \phi = \pi/6, n = 0.398$ ; (a)  $\beta = 0.01$ ; (b) M=1.

Fig.5 is plotted to see the effect of the parameters M and  $\beta$  on the pressure gradient dp/dx. It can be noticed that in the wider part of the channel  $x \in [0, 0.3]$  and [0.7, 1], the pressure gradient is relatively small, that is, the flow can easily pass without imposition of a large pressure gradient. On the other hand, in a narrow part of the channel  $x \in [0.3, 0.7]$  a much pressure gradient is required to maintain the same flux to pass it especially near x = 0.5. Fig. 5(a) indicates that the pressure gradient dp/dx increases with increasing Hartmann number M.

Fig. 5(b) shows that the pressure gradient dp/dx decreases with increasing the partial slip  $\beta$ .



Fig. 6: The pressure rise  $\Delta P_{\lambda}$  with  $\overline{Q}$  for a = 0.5, b = 0.5, d = 1,  $\phi = \pi/6$ , We = 0.001, E = 0.1, n = 0.398; (a)  $\beta = 0.01$ ; (b) M=1.

Fig. 6 depicts the variation of the dimensionless pressure rise  $\Delta P_{\lambda}$  versus the variation of time-average flux  $\overline{Q}$ . The graph is sectored so that the quadrant (I) designated as region the peristaltic pumping ( $\overline{Q} > 0$  and  $\Delta P_{\lambda} > 0$ ). Quadrant (II) is denoted the augmented flow when  $\overline{Q} > 0$  wet  $\Delta P_{\lambda} < 0$ . Quadrant (IV) such that  $\overline{Q} < 0$  and  $\Delta P_{\lambda} > 0$  is called



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retrograde (or) backward pumping. The figure also shows that there is a nonlinear relation between  $\Delta P_{\lambda}$  and  $\overline{Q}$ . Fig. 6(a) is a graph of the pressure rise  $\Delta P_{\lambda}$  per wavelength versus the mean flow rate  $\overline{Q}$  of an asymmetric channel for fixed values of other parameters. We observed that an increase in the Hartmann number M results to decrease in the peristaltic pumping rate. Fig 6(b) shows the variation of the pressure rise  $\Delta P_{\lambda}$  with the flow rate  $\overline{Q}$  for different values of the slip parameter. It is clear that the peristaltic pumping rate decreases with increasing the values of the slip parameter  $\beta$ . This is because of the velocity of the fluid on the walls hinder the peristaltic movement (i.e the peristaltic pumping region becomes wider by decreasing  $\beta$ ).



Fig. 7: The shear stress  $\mathcal{T}_{xy}$  with y for a = 0.5, b = 0.5,  $d = 1, \overline{Q} = 2$ ,  $x = 1, \phi = \pi/6$ , we = 0.001, n = 0.398; (a)  $\beta = 0.01$ ; (b) M=1.

The influences of various emerging parameters of our analysis on the shear stress  $\tau_{xy}$  are illustrated in Fig 7. The variation of shear stress  $\tau_{xy}$  versus y is calculated from Eq. (43). In Fig. 7(a) we observed that the curves intersect at the origin and the shear stress  $\tau_{xy}$  decreases with increasing the Hartmann number M in the upper wall and an opposite behavior is observed in the lower wall of the channel. An important observation here is that the value of shear stress completely vanishes at the center of the channel. The relation between the shear stress  $\tau_{xy}$  and y with different values of the partial slip parameter  $\beta$  is depicted in Fig. 7 (b). We observe that the curves intersect at the origin and the shear stress  $\tau_{xy}$  increases with increasing the partial slip parameter  $\beta$  at the upper half of the channel while an opposite behavior is observed in the lower half of the channel.



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Fig. 8: The different wave forms for a = 0.5, b = 0.5, d = 1, x = 0.2,  $\phi = 0$ , w = 0.001, M = 1,  $\beta = 0.01$ , n = 0.398, Br = 0.1,  $\overline{Q} = 2$ ; a) velocity profiles; (b) pressure gradient; (c) pressure rise; (d) temperatue distribution.

In Figs. 8(a)-8(d), we noted that tremendous results is obtained for the velocity, pressure gradient, pressure rise and temperature in the square wave and very worst in the triangular wave compared among the triangular, sawtooth, sinusoidal, trapezoidal and square wave respectively.

Another interesting phenomenon in peristaltic motion is the trapping. It is basically the formation of an internally circulating bolus of fluid by closed stream lines. The trapped bolus will be pushed ahead along the peristaltic waves. It has been shown in [31] that under certain conditions, the streamlines  $\psi = 0$  at the central line split to enclose a bolus of fluid particles circulating along closed streamline. This bolus moves at the wave velocity and therefore appears to be trapped by the wave. A criterion for the presence of trapping is the existence of stagnation points in the wave frame which are located at the intersection of the curve  $\psi = 0$  and the central line .(need not be a whole streamline, but can be composed of streamlines separated by zeros of u, that is, by stagnation points).

The stream lines are calculated from Eq. (38) and plotted in Figs. 9-12.



Fig. 9: The stream lines for a = 0.5, b = 0.5,  $d = 1, \overline{Q} = 1.5$ ,  $\phi = 0$ , We = 0.001, n = 0.398,  $\beta = 0.01$ ; (a) M=1; (b) M=3; (c) M=3.5.



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It is shown in Figs. 9 that the size of bolus decreases with increasing the Hartmann number M while the bolus disappears for M = 3.5



Fig. 10: The stream lines for  $a = 0.5, b = 0.5, d = 1, \overline{Q} = 1.5$ ,  $\phi = 0, n = 0.398, M = 1, We = 0.001$ ; (a)  $\beta = 0.00$ ; (b)  $\beta = 0.25$ ; (c)  $\beta = 0.5$ 

Fig.10 is depicted for various values of the partial slip parameter  $\beta$ . It is found that the volume of the trapping bolus decreases as the partial slip parameter  $\beta$  increases, moreover, the bolus disappears at  $\beta$ =0.5. In some medical cases there exist a loss of elasticity in the walls of the channels (tubes) hence the muscles give the shape an abnormal situation as in arteriosclerosis and irritable bowel syndrome. Moreover, the esophageal wall matches the shape of the bolus during the bolus transport. This study attempts more rational assumptions to enable an accurate analysis of the physiological and the pathological phenomenon occurring during peristalsis. This study focuses on Carreau fluidmodeling of peristaltic transport in an asymmetric channel.



Fig. 11: The stream lines for a = 0.5, b = 0.5,  $d = 1, \overline{Q} = 1.5$ ,  $\phi = 0$ , n = 0.398, M=1,We=0.001,  $\beta = 0.00$ ; (a) Sinusoidal; (b) Trapeziodal; (c) Square; (d) Triagulr; (e) Sawtooth waves.



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Fig. 12: The stream lines for a = 0.5, b = 0.5,  $d = 1, \overline{Q} = 1.5$ ,  $\phi = 0$ , n = 0.398, M=1,We=0.001;  $\beta = 0.01$ ; (a) Sinusoidal; (b) Trapeziodal; (c) Square; (d) Triagular; (e) Sawtooth waves.

Figs.11-12 has been allocated for a comparison between the different wave forms like sinusoidal, triangular, trapezoidal, square and sawtooth wave. It is finally observed that the volume of the trapping bolus of the Carreau fluid decreases by increasing the partial slip  $\beta$  in a different wave forms.

#### V. CONCLUSIONS

In the present note, we have discussed theperistaltic transport of a Carreau fluid under the effect of magnetic field with partial slip. The two-dimensional governing equations have been modeled and then simplified using the long wavelength approximation and then solved by using the perturbation technique.

The main findings can be summarized as follows:

- The maximum velocity in the non-Newtonian fluid is small compared with the Newtonian fluid.
- The effect of slip appears near the wall of the channel, but disappears in the middle of the channel under the influence of the fluid movement.
- In the wider part of the channel the pressure gradient is relatively small, that is, the flow can easily pass without imposition of a large pressure gradient.
- The peristaltic pumping region becomes wider by decreasing the slip parameter. This result shows that the slip parameter has a direct effect on the peristaltic pumping.
- The square wave has given great yield compare with triangular, sawtooth, sinusoidal and trapezoidal wave in the velocity, pressure gradient, pressure rise and temperature distribution.
- In the peristaltic pumping region ( $\overline{Q} > 0$  and  $\Delta P_{\lambda} > 0$ ) the pressure rise decreases with elevating the slip parameter.
- The shear stress distribution decreases in the upper half of the channel and increases in the lower half of the channel with increasing Hartmann number. But opposite behavior with slip parameter is noticed.
- The size of tapping bolus decreases with increase slip parameter and Hartmann number.

#### NOMENCLATURE

#### **ROMAN SYMBOLS:**

Br = Brickman number



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- $C_p$  =Specific heat at constant pressure
- E = Eckert number
- $\overline{H}$  = Induced electrical field
- $\overline{J}$  = Current density
- M= Hartmann number
- n = Power-law index
- $\overline{P}$  = Pressure
- $p_m =$  Magnetic pressure
- Pr = Prandtl number
- Q = Dimensionless mean flow rate
- $R_e$  = Reynolds number
- $R_m$  = Magnetic Reynolds number
- $S_t =$  Strommer's number
- T = Temperature
- $(\overline{U}, \overline{V})$  = Velocity components of fixed frame of reference
- $(\overline{u}, \overline{v})$  = Velocity components of wave frame of reference
- $(\overline{X}, \overline{Y})$  = Cartesian co-ordinates in a fixed frame
- $(\overline{x}, \overline{y})$  = Cartesian coordinates in a wave frame
- *We*= Weissenberg number

#### GREEK SYMBOLS:

 $\beta$  = Non-dimensional slip parameter

- $\delta$  = Wave number
- $\Delta P_{\lambda} =$  Pressure rise
- $\overline{\epsilon}$  = Electrical field

 $\zeta = \frac{1}{\sigma \mu_e} = \text{Magnetic diffusivity}$ 

- $\eta_0$  =Zero shear-rate viscosity
- $\eta_{\infty}$  =Infinite shear- rate viscosity
- $\kappa \equiv$  Thermal conductivity
- $m_{e}$  =Magnetic permeability
- s = Electrical conductivity
- $\tau = \text{Extra stress tensor}$
- $\tau_{xy}$  = Shear stress
- $\Phi$  = Magnetic force function

 $\psi =$  Stream function



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#### APPENDIX

$$\begin{split} & C_{1} = \frac{q}{2} + \frac{1}{L_{0}} \bigg( Mqh_{1} \cosh M \bigg( \frac{h_{1} - h_{2}}{2} \bigg) + \Big[ (2 + M^{2}q\beta)h_{1} - (q + h_{1} - h_{2}) \Big] \sinh M \bigg( \frac{h_{1} - h_{2}}{2} \bigg) \Big], \\ & C_{2} = -\frac{1}{L_{0}} \bigg( (2 + M^{2}q\beta) \sinh M \bigg( \frac{h_{1} - h_{2}}{2} \bigg) + Mq \cosh M \bigg( \frac{h_{1} - h_{2}}{2} \bigg) \bigg), \\ & C_{3} = -\frac{1}{L_{0}} \bigg( q + h_{1} - h_{2} \bigg) \sinh M \bigg( \frac{h_{1} + h_{2}}{2} \bigg), \\ & C_{4} = \frac{1}{L_{0}} \bigg( q + h_{1} - h_{2} \bigg) \cosh M \bigg( \frac{h_{1} + h_{2}}{2} \bigg), \\ & C_{5} = \frac{q}{2} + \frac{h_{1}}{(h_{1} - h_{2})} \bigg( \frac{qL_{5}}{L_{7}} + \frac{L_{8}L_{5}}{(h_{1} - h_{2})L_{7}} + L_{6} - q \bigg) - \frac{\sinh Mh_{1}}{L_{7}} \bigg( \frac{q}{h_{1} - h_{2}} + L_{8} \bigg) \\ & + \frac{\cosh Mh_{1}}{M (\sinh Mh_{1} - \sinh Mh_{2})} \bigg( \frac{M}{L_{7}} (\cosh Mh_{1} - \cosh Mh_{2}) \bigg( \frac{q}{h_{1} - h_{2}} + L_{8} \bigg) + L_{3} - L_{4} \bigg) - L_{1}, \\ & C_{6} = \frac{1}{(h_{1} - h_{2})} \bigg( q - \frac{qL_{5}}{(h_{1} - h_{2})L_{7}} - \frac{L_{8}L_{5}}{L_{7}} - L_{6} \bigg), \\ & C_{7} = \frac{-1}{(\sinh Mh_{1} - \sin M h_{2})} \bigg( \bigg( \frac{\cosh Mh_{1} - \cosh Mh_{2}}{L_{7}} \bigg) \bigg( \frac{q}{(h_{1} - h_{2})} + L_{8} \bigg) + \frac{(L_{3} - L_{4})}{M} \bigg), \\ & C_{8} = \frac{1}{L_{7}} \bigg( \frac{q}{(h_{1} - h_{2})} + L_{8} \bigg), \\ & C_{9} = \frac{1}{(h_{2} - h_{1})} \bigg[ 1 - \frac{BrM^{2}}{8} \bigg( \bigg( \frac{C_{3}^{2} + C_{4}^{2}}{(2} \bigg) (\cosh 2Mh_{1} - \sinh 2Mh_{2}) \bigg) \\ & C_{10} = \frac{1}{(h_{1} - h_{2})} \bigg[ h_{1} - \frac{BrM^{2}}{8} \bigg( \bigg( \frac{C_{3}^{2} + C_{4}^{2}}{(h_{2} \cosh 2Mh_{1} - \sinh 2Mh_{2})} + 2 \bigg( C_{4}^{2} - C_{3}^{2} \bigg) h_{1}h_{2} \bigg( h_{1}^{2} - h_{2}^{2} \bigg) \bigg) \bigg], \end{split}$$



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$$\begin{split} C_{11} &= -Br \bigg[ \bigg( 2 \bigg( L_9 - \frac{L_{12}}{M} \bigg) + \frac{(n-1)}{2} L_{16} \bigg) \frac{1}{4M^2} (\cosh 2Mh_2 - \cosh 2Mh_1) + \frac{L_{10}}{2M^2} (h_2 \cosh 2Mh_2 - h_1 \cos 2Mh_1) \\ &+ \bigg( 2 \bigg( L_{11} - \frac{L_{10}}{M} \bigg) + \frac{(n-1)}{2} L_{17} \bigg) \frac{1}{4M^2} (\sinh 2Mh_2 - \sinh 2Mh_1) \\ &+ \bigg( 2 L_{13} + \frac{(n-1)}{2} L_{18} \bigg) \frac{1}{16M^2} (\cosh 4Mh_2 - \cosh 4Mh_1) \bigg) \\ &+ \bigg( 2 L_{14} + \frac{(n-1)}{2} L_{19} \bigg) \frac{1}{16M^2} (\sinh 4Mh_2 - \sinh 4Mh_1) + \frac{L_{15}}{3} (h_2^3 - h_1^3) + (2L_{15} + L_{20}) \frac{(h_2^2 - h_1^2)}{2} \bigg], \\ C_{12} &= -Br \bigg[ \bigg( 2 \bigg( L_9 - \frac{L_{12}}{M} \bigg) + \frac{(n-1)}{2} L_{16} \bigg) \frac{1}{4M^2} (h_2 \cosh 2Mh_1 - h_1 \cosh 2Mh_2) + \frac{L_{10}}{2M^2} h_1 h_2 (\cosh 2Mh_1 - \cos 2Mh_2) \\ &+ \bigg( 2 L_{14} - \frac{L_{10}}{M} \bigg) + \frac{(n-1)}{2} L_{19} \bigg) \frac{1}{4M^2} (h_2 \cosh 2Mh_1 - h_1 \cosh 2Mh_2) + \frac{L_{10}}{2M^2} h_1 h_2 (\cosh 2Mh_1 - \cos 2Mh_2) \\ &+ \bigg( 2 L_{13} - \frac{L_{10}}{M} \bigg) + \frac{(n-1)}{2} L_{19} \bigg) \frac{1}{16M^2} (h_2 \sinh 4Mh_1 - h_1 \cosh 4Mh_2) \\ &+ \bigg( 2 L_{13} + \frac{(n-1)}{2} L_{19} \bigg) \frac{1}{16M^2} (h_2 \sinh 4Mh_1 - h_1 \cosh 4Mh_2) \\ &+ \bigg( 2 L_{14} + \frac{(n-1)}{2} L_{19} \bigg) \frac{1}{16M^2} (h_2 \sinh 4Mh_1 - h_1 \sinh 4Mh_2) + \frac{L_{15}}{3} h_1 h_2 (h_1^2 - h_2^2) + (2L_{15} + L_{20}) \frac{h_1 h_2 (h_1 - h_2)}{2} \bigg], \\ C_{13} &= \frac{R_m}{(h_2 - h_1)} \bigg[ \bigg( E - C_2 \bigg) \frac{(h_1^2 - h_2^2)}{2} - \frac{C_3}{M} (\sinh 4Mh_1 - h_1 \sinh Mh_2) - \frac{C_4}{M} (\cosh Mh_1 - \cosh Mh_2) \bigg], \\ C_{14} &= \frac{R_m}{(h_1 - h_2)} \bigg[ \bigg( E - C_2 \bigg) \frac{h_1 h_2 (h_1 - h_2)}{2} - \frac{C_3}{M} (h_2 \sinh Mh_1 - h_1 \sinh Mh_2) - \frac{C_4}{M} (h_2 \cosh Mh_1 - h_1 \cosh Mh_2) \bigg], \\ C_{15} &= \frac{R_m}{(h_2 - h_1)} \bigg[ C_8 \bigg( \frac{h_2^2 - h_1^2}{2} \bigg) + \frac{C_7}{M} (\sinh Mh_2 - \sinh Mh_1) + C_8 \bigg( \cosh hMh_2 - \cosh Mh_2) \bigg], \\ C_{15} &= \frac{R_m}{(h_2 - h_1)} \bigg[ C_8 \bigg( \frac{h_2^2 - h_1^2}{2} \bigg) + \frac{C_7}{M} (\sinh Mh_2 - \sinh Mh_1) + C_4 \bigg( (C_4^2 + 3C_3^2) (\cosh 3Mh_2 - \cosh 3Mh_1) \bigg) \\ &- \frac{3(n-1)}{16} M^3 \bigg( C_3^2 - C_4^2 \bigg) \bigg( M (h_2 \sinh Mh_2 - \sinh Mh_1) - (\cosh Mh_2 - \sinh Mh_1) \bigg) \bigg], \end{aligned}$$



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$$\begin{split} & C_{16} = \frac{R_m}{(h_1 - h_2)} \bigg[ C_6 \frac{h_1 h_2 \left(h_2 - h_1\right)}{2} + \frac{C_7}{M} \left(h_1 \sinh M h_2 - h_2 \sinh M h_1\right) + \frac{C_8}{M} \left(h_1 \cosh h M h_2 - h_1 \cosh M h_2\right) \\ & - \frac{(n-1)}{192} M^3 \left(C_3 \left(C_3^2 + 3C_4^2\right) \left(h_1 \sinh M h_2 - h_2 \sinh M h_1\right) + C_4 \left(C_4^2 + 3C_3^2\right) \left(h_1 \cosh M h_2 - h_2 \cosh M h_1\right) \right) \\ & - \frac{3(n-1)}{16} M^4 C_3 \left(C_3^2 - C_4^2\right) \left(M h_1 h_2 \left(\cosh M h_2 - \cosh M h_1\right) - \left(h_1 \sinh M h_2 - h_2 \sinh M h_1\right)\right) \\ & - \frac{3(n-1)}{16} M^4 C_3 \left(C_3^2 - C_4^2\right) \left(M h_1 h_2 \left(\cosh M h_2 - \cosh M h_1\right) - \left(h_1 \sinh M h_2 - h_2 \sinh M h_1\right)\right) \\ & - \frac{3(n-1)}{16} M^4 C_3 \left(C_3^2 - C_4^2\right) \left(M h_1 h_2 \left(\sinh M h_2 - \sinh M h_1\right) - \left(h_1 \cosh M h_2 - h_2 \sinh M h_1\right)\right) \\ & - \frac{3(n-1)}{16} M^4 C_3 \left(C_3^2 - C_4^2\right) \left(M h_1 h_2 \left(\sinh M h_2 - \sinh M h_1\right) - \left(h_1 \cosh M h_2 - h_2 \cosh M h_1\right)\right) \\ & - \frac{3(n-1)}{16} M^4 \left(n-1\right) \left(C_3 \left(C_3^2 + 3C_4^2\right) \cosh M h_1 + C_4 \left(C_4^2 + 3C_3^2\right) \sinh M h_1\right) \\ & - \frac{3}{16} M^4 \left(n-1\right) h_1 \left(C_3^2 - C_4^2\right) \left(C_3 \sinh M h_1 + C_4 \cosh M h_1\right), \\ & L_2 = -\frac{M^4}{64} \left(n-1\right) h_2 \left(C_3^2 - C_4^2\right) \left(C_3 \sinh M h_2 + C_4 \left(C_4^2 + 3C_3^2\right) \sinh M h_2\right) \\ & - \frac{3M^3 \left(n-1\right)}{64} \left(C_3 \left(C_3^2 + 3C_4^2\right) \cosh M h_2 + C_4 \left(C_4^2 + 3C_3^2\right) \sinh M h_2\right) \\ & - \frac{3M^3 \left(n-1\right)}{64} \left(C_3 \left(-C_3^2 + 3C_4^2\right) + 3\beta M C_4 \left(C_4^2 + 3C_3^2\right) \right) \sinh M h_1 + \left(M h_6 C_3 + C_4\right) + 3\beta M \left(2C_3 + M h_6 C_4\right) \right) \cosh M h_2 \\ & - \frac{3M^3 \left(n-1\right)}{64} \left(C_3 \left(C_3^2 + 3C_4^2\right) + 3\beta M C_4 \left(C_4^2 + 3C_3^2\right) \right) \sinh M h_2 + \left(C_4 \left(C_4^2 + 3C_3^2\right) + 3\beta M C_5 \left(C_3^2 + 3C_4^2\right) \right) \cosh M h_2 \\ & - \frac{3M^3 \left(n-1\right)}{64} \left(C_3 \left(-C_3^2 + 3C_4^2\right) + 3\beta M C_4 \left(C_4^2 + 2C_4\right) \right) \sinh M h_2 + \left(M h_6 C_3 + C_4\right) + 3\beta M \left(2C_3 + M h_6 C_4\right) \right) \cosh M h_2 \\ & - \frac{3M^3 \left(n-1\right)}{64} \left(C_3 \left(C_3^2 + 3C_4^2\right) + 3\beta M C_4 \left(C_4^2 + 2C_4\right) \right) \sinh M h_2 + \left(M h_6 C_3 + C_4\right) + 3\beta M \left(2C_3 + M h_6 C_4\right) \right) \cosh M h_2 \\ & - \frac{3M^3 \left(n-1\right)}{64} \left(C_3 \left(C_3^2 + 3C_4^2\right) + 3\beta M C_4 \left(C_4^2 + 2C_4\right) \right) \sinh M h_2 + \left(M h_6 C_3 + C_4\right) + 3\beta M \left(2C_3 + M h_6 C_4\right) \right) \cosh M h_2 \\ & - \frac{3M^3 \left(n-1\right)}{64} \left(C_3 \left(C_3^2 + 3C_4^2\right) + 3\beta M \left(M h_6 C_3 + 2C_4\right) \right) \sinh M h_2 + \left(M h_6 C_3 + C_4\right) + 3\beta M \left(2C_3 + M h_6 C_4\right) \right) \cosh M h_2 \\ &$$



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$$\begin{split} L_{9} &= \frac{M^{3}}{2} \bigg[ C_{3} \bigg( MC_{7} - \frac{3}{8} M^{5} (n-1) C_{3} \big( C_{3}^{2} - C_{4}^{2} \big) - \frac{9}{64} M^{5} (n-1) C_{3} \big( C_{3}^{2} + 3C_{4}^{2} \big) \bigg) \\ &+ C_{4} \bigg( MC_{8} - \frac{3}{8} M^{5} (n-1) C_{4} \big( C_{3}^{2} - C_{4}^{2} \big) + \frac{9}{64} M^{5} (n-1) C_{4} \big( C_{4}^{2} + 3C_{3}^{2} \big) \bigg) \bigg] , \\ L_{10} &= -\frac{3}{16} M^{9} (n-1) C_{3} C_{4} \big( C_{3}^{2} - C_{4}^{2} \big) , \\ L_{11} &= \frac{M^{4}}{2} \bigg( C_{3} C_{8} - C_{4} C_{7} - \frac{11}{64} M^{4} (n-1) C_{3} C_{4} \big( C_{3}^{2} - C_{4}^{2} \big) \bigg) , \\ L_{12} &= -\frac{3}{16} M^{5} \big( n-1 \big) \big( C_{3}^{4} - C_{4}^{4} \big) , \\ L_{13} &= -\frac{9}{128} M^{8} \big( n-1 \big) \big( C_{3}^{4} + C_{4}^{4} + 6C_{3}^{2} C_{4}^{2} \big) , \\ L_{14} &= -\frac{9}{32} M^{8} \big( n-1 \big) C_{3} C_{4} \big( C_{3}^{2} + C_{4}^{2} \big) , \\ L_{15} &= \frac{M^{4}}{2} \bigg( C_{3} C_{7} - C_{4} C_{8} - \frac{3}{8} M^{5} \big( n-1 \big) \big( C_{3}^{2} - C_{4}^{2} \big)^{2} \bigg) , \\ L_{18} &= \frac{1}{8} \big( C_{3}^{4} + C_{4}^{4} + 6C_{3}^{2} C_{4}^{2} \big) , \\ L_{18} &= \frac{1}{8} \big( C_{3}^{4} + C_{4}^{4} + 6C_{3}^{2} C_{4}^{2} \big) , \\ L_{19} &= \frac{1}{2} C_{3} C_{4} \big( C_{3}^{2} + C_{4}^{2} \big) , \\ L_{10} &= \frac{3}{8} \big( C_{3}^{4} - C_{4}^{4} \big) , \\ L_{10} &= \frac{1}{2} \big( C_{3}^{4} - C_{4}^{4} \big) \bigg) . \end{split}$$

#### **DEFINITION OF WAVE SHAPES**

The non-dimensional expressions of the considered wave forms are given by the following equations:

#### I. SINUSOIDAL WAVE:

$$h_{1}(x) = 1 + a\sin(2\pi x)$$

$$h_{2}(x) = -d - b\sin(2\pi x + \phi)$$
(55)
(56)

II. TRIANGULAR WAVE:

$$h_1(x) = 1 + a \left\{ \frac{8}{\pi^3} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)^2} \sin[(2m-1)x] \right\}$$
(57)

$$h_{2}(x) = -d - b \left\{ \frac{8}{\pi^{3}} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)^{2}} \sin[(2m-1)x + \phi] \right\}$$
(58)

#### III. SQUARE WAVE:

$$h_{1}(x) = 1 + a \left\{ \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)} \cos\left[(2m-1)x\right] \right\}$$
(59)

$$h_2(x) = -d - b \left\{ \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)} \cos\left[(2m-1)x + \phi\right] \right\}$$
(60)



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#### **IV. TRAPEZOIDAL WAVE:**

$$h_{1}(x) = 1 + a \left\{ \frac{32}{\pi^{2}} \sum_{m=1}^{\infty} \frac{\sin \frac{\pi}{8} (2m-1)}{(2m-1)^{2}} \sin[(2m-1)x] \right\}$$

$$\left\{ 32 \sum_{m=1}^{\infty} \frac{\sin \frac{\pi}{8} (2m-1)}{(2m-1)} \right\}$$
(61)
(62)

$$h_{2}(x) = -d - b \left\{ \frac{32}{\pi^{2}} \sum_{m=1}^{\infty} \frac{\sin \frac{\pi}{8} (2m-1)}{(2m-1)^{2}} \sin[(2m-1)x + \phi] \right\}$$
(62)

#### V. SAWTOOTH WAVE:

$$h_{1}(x) = 1 + a \left\{ \frac{8}{\pi^{3}} \sum_{m=1}^{\infty} \frac{\sin[2\pi mx]}{m} \right\}$$
(63)  
$$\left\{ 8 \sum_{m=1}^{\infty} \frac{\sin[2\pi mx + \phi]}{m} \right\}$$
(64)

$$h_2(x) = -d - b \left\{ \frac{8}{\pi^3} \sum_{m=1}^{\infty} \frac{\sin[2\pi mx + \phi]}{m} \right\}$$
(64)

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