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Pre Generalised Closed Maps and Homeomorphisms in Extended Topological Spaces

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ABSTRACT: This paper focuses on coining a novel concept of generalized closed maps and homeomorphisms in the context of pre-open sets in simple extension topology. We also intend to characterize the properties of the newly formed maps and homeomorphisms.

KEYWORDS: $\tau^+_{p,g}$ closed, $\tau^{+*}_{p,g}$ closed maps, $\tau^+_{p,g}$ - homeomorphisms and $\tau^{+*}_{p,g}$ homeomorphism

I. INTRODUCTION

Many topologists like Mashour et al [9], Levine [6], [8], Njastad [17], Maki et al [9,10] Noiri [16], have respectively introduced and contributed much to general topology via the category of the weaker form of sets. In 1963, Levine [7], defined simple expansion of τ by B as $\tau(B) = \{O \cup [(O' \cap B)] / O, O' \in \tau\}$. Nirmala and Arockiarani [13] have initiated the new idea of $\tau^+_{p,g}$ closed sets in the light of simple extension topology. The notion of continuity and irresoluteness earns great importance because of its broad application in topological spaces. Balachandran et al [3] and Arockiarani et al [1] have defined and characterized g -continuous maps and rg continuous maps.

Crossley and Hildebrand [5] introduced and studied irresolute maps which are stronger than semi-continuous maps and independent of continuous maps. Nirmala and Arockiarani [14] have devised the concept of continuity and irresoluteness under simple extension topological spaces.

The concept of pre-closed maps was introduced and studied by Sen and Bhattacharyya [18]. Mashour et al [11,12], Arockiarani [2] have put forth the notion of pre-open maps and weak pre-open, and rg -closed maps respectively.

Biswas [4] and Noiri [15] brought out the idea of semi-homeomorphisms. Maki and Sundram et al [19] introduced and studied g -homeomorphism and gc -homeomorphisms.

Several researchers have generalized the notion of homeomorphisms in topological spaces [2,5]

This paper aims at defining and deducing the properties of $\tau^+_{p,g}$ closed and $\tau^{+*}_{p,g}$ closed maps. Another form of homeomorphism namely $\tau^+_{p,g}$ homeomorphism and $\tau^{+*}_{p,g}$ homeomorphism are proposed. In this article we prove that $\tau^{+*}_{p,g}gh(X, \tau^+)$ forms a group under composition of maps and also prove another characterizing theorem on $\tau^{+*}_{p,g}$ homeomorphism as the one which induces an isomorphism from the group $\tau^{+*}_{p,g}gh(X, \tau^+)$ onto $\tau^{+*}_{p,g}gh(Y, \sigma)$. Also, we prove that $\tau^{+*}_{p,g}$ homeomorphism is an equivalence relation under compositions of maps.

II. PRELIMINARIES

DEFINITION 2.1

A subset A of a space (X, τ) is called

- (i) a semi-open set [6] if $A \subset cl(int(A))$ and a semi-closed set if $int(cl(A)) \subset A$,
- (ii) a pre-open set [11] is $A \subset int(cl(A))$ and a preclosed set if $cl(int(A)) \subset A$,

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- (iii) a generalized closed set (briefly g-closed) [8] if $cl(A) \subset U$ whenever $A \subset U$ and U is open.
- (iv) a generalized pre-closed set (briefly gp-closed) [16] if $pcl(A) \subset U$ whenever $A \subset U$ and U is open
- (v) τ_p^+g closed [13] set if $\tau^+cl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre-open in (X, τ) , where $\tau^+cl(A) = \bigcap \{ S \subseteq X / A \subseteq S \text{ and } S \text{ is closed in } \tau^+(B) \}$

DEFINITION 2.2

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then f is said to be

- (i) pre-continuous [11] if $f^{-1}(V)$ is pre-closed in X for every closed set V of Y ;
- (ii) gp-continuous [2] if $f^{-1}(V)$ is gp-closed in X for every closed set V of Y ;
- (iii) τ_p^+g continuous [14] if $f^{-1}(V)$ is τ_p^+g closed in (X, τ) for every closed set
- (iv) V of (Y, σ) irresolute [5] if $f^{-1}(V)$ is semi-open in X for every semi-open set V of Y .
- (v) τ_p^+g irresolute [14] (resp. τ^+ irresolute) if the inverse image of every τ_p^+g closed (resp. τ^+ open) set in Y is a τ_p^+g closed (resp. τ^+ open) set in X .

DEFINITION 2.3

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then f is said to be

- (i) pre-closed [18] if $f(V)$ is pre-closed in Y for every closed set V of X ,
- (ii) α -open [12] if $f(V)$ is α -open in Y for every open set V of X .

DEFINITION 2.4

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then f is said to be

- (i) a generalized homeomorphism [19] if f is g -continuous, g -open and bijective.
- (ii) g homeomorphism [19] if f is g -continuous, g -irresolute and its inverse is also g -irresolute.
- (iii) semihomomorphism (B) [4] if f is continuous, semi-open and bijective.
- (iv) semihomomorphism (C.H) [5] if f is irresolute, pre-semi-open and bijective.

Lemma 2.5:

Let $A \subset Y \subset X$. Then

- (i) If Y is open in X , then $pcl_Y(A) = pcl_X(A) \cap Y$.
- (ii) If Y is pre-closed in X , then $pcl_Y(A) = pcl_X(A)$

Proposition 2.6:

Let $A \subset Y \subset X$ and Y be open pre-closed in X . Then $A \in \tau_p^+GC(Y)$ implies $A \in \tau_p^+GC(X)$.

3.1 τ_p^+g closed and $\tau^{+*}g$ closed maps

Two different types of closed sets are defined in this section and we further intend to discuss their properties.

Definition 3.1.1 :

A map $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is τ_p^+g closed if the image $f(A)$ is τ_p^+g closed in Y , for every closed set A in X .

Similar to τ_p^+g closed map we can also define τ_p^+g open map.

Remark 3.1.2 :

Every pre-closed map is a τ_p^+g closed map.

Proposition 3.1.3:

If a mapping $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is τ_p^+g closed, then for every subset A of X , $\tau_p^+g cl(f(A)) \subset f(cl(A))$.

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Proof:

Let $A \subset X$. Since f is τ_p^+ closed, $f(\text{cl}(A))$ is τ_p^+ closed in Y . Now $f(A) \subset f(\text{cl}(A))$. As closure respects inclusion, $\tau_p^+ \text{cl}(f(A)) \subset \tau_p^+ \text{cl}(f(\text{cl}(A))) = f(\text{cl}(A))$.

Proposition 3.1.4 :

If $\text{cl}(\text{int}(f(A)) \subset f(\text{cl}(A)))$ for every subset A of X , then $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is τ_p^+ closed .

Proof:

If $\text{cl}(\text{int}(f(A)) \subset f(\text{cl}(A)))$ for every subset A of X , then we know that f is pre-closed. Thus f is τ_p^+ closed .

Proposition 3.1.5 :

A map $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is τ_p^+ closed if and only if for every subset S of Y and for each open set U containing $f^{-1}(S)$, there is a τ_p^+ open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof:

(Necessary condition) : Let f be τ_p^+ closed. Let S be a subset of Y and U be an open set of Y such that $f^{-1}(S) \subset U$. Then $V = Y - f(X-U)$ is a τ_p^+ open set containing S such that $f^{-1}(V) \subset U$.

(Sufficiency condition): Assume that A is closed in X . Then $f^{-1}(Y - f(A)) \subset X - A$ and $X - A$ is open . By hypothesis, there is a τ_p^+ open set V of Y such that $Y - f(A) \subset V$ and $f^{-1}(V) \subset X - A$. Thus $A \subset X - f^{-1}(V)$. Hence $Y - V \subset f(A) \subset f(X - f^{-1}(V)) \subset Y - V$ which implies $f(A) = Y - V$. As $Y - V$ is τ_p^+ closed, $f(A)$ is τ_p^+ closed. Hence f is a τ_p^+ closed map.

Proposition 3.1.6 :

Let $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ be a pre-irresolute and τ_p^+ closed. Then for every τ_p^+ closed set $A \subset X$, $f(A)$ is τ_p^+ closed in Y .

Proof:

Suppose A is τ_p^+ closed in X and $f(A) \subset R$, where R is pre-open in Y . Then $A \subset f^{-1}(R)$. As A is τ_p^+ closed, $\tau^+ \text{cl}(A) \subset f^{-1}(R)$, ie) $f(\tau^+ \text{cl}(A)) \subset R$. Since f is τ_p^+ closed, $\tau^+ \text{cl}(f(\tau^+ \text{cl}(A))) \subset R$ and thus we have $\tau^+ \text{cl}(f(A)) \subset R$. Hence $f(A)$ is τ_p^+ closed.

Proposition 3.1.7 :

If $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is τ_p^+ closed, and A is a closed subset of X , then $f|_A: A \rightarrow Y$ is τ_p^+ closed.

Proof:

Assume that $B \subset A$ is closed in A . Then B is closed in X , as A is closed in X . Therefore $f(B)$ is τ_p^+ closed in Y . But $f(B) = (f|_A)(B)$. Thus $(f|_A)(B)$ is τ_p^+ closed in Y . Hence $(f|_A)$ is τ_p^+ closed.

Theorem 3.1.8:

Let $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ be a τ_p^+ closed map and $g: (Y, \sigma^+) \rightarrow (Z, \mu^+)$ be τ_p^+ closed and pre-irresolute, then $g \circ f: (X, \tau^+) \rightarrow (Z, \mu^+)$ is τ_p^+ closed.

Proof :

Suppose that A is closed in X . As f is τ_p^+ closed, $f(A)$ is τ_p^+ closed in Y . Since g is τ_p^+ closed and pre-continuous by proposition 3.1.6, $g(f(A)) = (g \circ f)(A)$ is τ_p^+ closed in Z . Thus $(g \circ f)$ is τ_p^+ closed .

Proposition 3.1.9:

If $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is closed and $g: (Y, \sigma^+) \rightarrow (Z, \mu^+)$ is τ_p^+ closed, then $(g \circ f): (X, \tau^+) \rightarrow (Z, \mu^+)$ is τ_p^+ closed.

Proof:

Let A be closed in X . As f is closed, $f(A)$ is closed in Y . Since g is τ_p^+ closed, $g(f(A))$ is τ_p^+ closed in Z . Thus $(g \circ f)$ is τ_p^+ closed.

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Theorem 3.1.10:

Let $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ and $g : (Y, \sigma^+) \rightarrow (Z, \mu^+)$ be two mappings and let $g \circ f : (X, \tau^+) \rightarrow (Z, \mu^+)$ be a τ_p^+g closed mappings. Then

- (i) If f is continuous and surjective, then g is τ_p^+g closed
- (ii) If g is τ_p^+g irresolute and injective, then f is τ_p^+g closed

Proof:

- (i) Let us assume that A is closed in Y . Since f is continuous, we have $f^{-1}(A)$ is closed in X . As $(g \circ f)$ is τ_p^+g closed and f is surjective, $(g \circ f) f^{-1}(A) = g(f f^{-1}(A)) = g(A)$ is τ_p^+g closed in Z , which implies that g is τ_p^+g closed.
- (ii) Assume that B is closed in X . τ_p^+g closedness of $(g \circ f)$ shows that $(g \circ f)(B)$ is τ_p^+g closed in Z . g is injective $g^{-1}((g \circ f)(B)) = f(B)$. Since g is τ_p^+g irresolute, $f(B)$ is τ_p^+g closed in Y , that is f is τ_p^+g closed.

Definition 3.1.11:

A map $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ is said to be a τ_p^+g closed map if the image $f(A)$ of every τ_p^+g closed set A in X is τ_p^+g closed in Y .

Remark 3.1.12:

If $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ is pre-irresolute and τ_p^+g closed, then it is τ_p^+g closed

Lemma 3.1.13:

A map $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ is pre-irresolute if and only if $f(\text{pcl}(A)) \subset \text{pcl}(f(A))$ for every subset A of X .

Proof:

Necessity: Assume $A \subset X$. Then $\text{pcl}(f(A))$ is pre-closed in Y . As f is pre-irresolute, $f^{-1}(\text{pcl}(f(A)))$ is pre-closed in X . As $A \subset f^{-1}(f(A)) \subset f^{-1}(\text{pcl}(f(A)))$, $\text{pcl}(A) \subset f^{-1}(\text{pcl}(f(A)))$. Then $f(\text{pcl}(A)) \subset \text{pcl}(f(A))$.

Sufficiency: Let B be pre-closed in Y . By hypothesis, $f(\text{pcl}(f^{-1}(B))) \subset \text{pcl}(f(f^{-1}(B))) \subset \text{pcl}(B) = B$. Hence $\text{pcl}(f^{-1}(B)) \subset f^{-1}(B)$. Thus $f^{-1}(B)$ is pre-closed. i.e f is pre-irresolute.

3.2 τ_p^+g - homeomorphisms

In this section we introduce two types of homeomorphisms namely $(\tau_p^+g$ & $\tau_{p^*}^+g$ homeomorphisms) and investigate its properties.

Definition 3.2.1:

A bijection $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ is called a τ_p^+g homeomorphisms if f is both τ_p^+g continuous and τ_p^+g open. The family of all τ_p^+g homeomorphisms from (X, τ^+) onto itself is denoted by $\tau_p^+g(X, \tau^+)$.

Remark 3.2.2:

- (i) Every homeomorphism is a τ_p^+g homeomorphisms
- (ii) Every g homeomorphism is a τ_p^+g homeomorphisms

Proposition 3.2.3:

For any bijection $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$, the following statements are equivalent.

- (i) f is a τ_p^+g open map
- (ii) f is a τ_p^+g closed map
- (iii) $f^{-1} : (Y, \sigma^+) \rightarrow (X, \tau^+)$ is τ_p^+g continuous.

Proof :

- (i) \Rightarrow (ii). Assume that A is closed in (X, τ^+) . Then $(X-A)$ is open in X . By (i), $f(X-A)$ is τ_p^+g open in Y . Therefore, $Y - f(X-A) = f(A)$ is

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$\tau^+_p g$ closed in Y .

(ii) \Rightarrow (iii) Assume that A is closed in (X, τ^+) . As f is $\tau^+_p g$ closed map, $f(A)$ is $\tau^+_p g$ closed in (Y, σ^+) . Since $f(A) = (f^{-1})^{-1}(A)$, f^{-1} is $\tau^+_p g$ continuous

(iii) \Rightarrow (i) Assume that A is open in (X, τ^+) . By hypothesis, $(f^{-1})^{-1}(A)$ is $\tau^+_p g$ open in Y . Therefore $f(A)$ is $\tau^+_p g$ open in Y . Thus (i) holds.

Proposition 3.2.4:

Let $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ be a bijective, $\tau^+_p g$ continuous map, Then the following are equivalent.

- (i) f is a $\tau^+_p g$ open map
- (ii) f is a $\tau^+_p g$ homeomorphism
- (iii) f is a $\tau^+_p g$ closed map

Proof:

The proof is immediate.

Remark:3.2.5

The composition of two $\tau^+_p g$ homeomorphism is not always a $\tau^+_p g$ homeomorphism

Definition 3.2.6

A map $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is called $\tau^{+*}_p g$ open if $f(V)$ is a $\tau^+_p g$ open in (Y, σ^+) for every $\tau^+_p g$ open set V of (X, τ^+) .

Proposition 3.2.7:

For any bijection $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$, the following statements are equivalent.

- (i) the inverse map $f^{-1}: (Y, \sigma^+) \rightarrow (X, \tau^+)$ is $\tau^+_p g$ irresolute
- (ii) f is a $\tau^{+*}_p g$ open map
- (iii) f is a $\tau^{+*}_p g$ closed map

Proof:

(i) \Rightarrow (ii) Assume that A is $\tau^+_p g$ open in (X, τ^+) . By (i),

$(f^{-1})^{-1}(A) = f(A)$ is $\tau^+_p g$ open in (Y, σ^+) . Thus (ii) holds.

(iii) \Rightarrow (ii) Assume that A is $\tau^+_p g$ closed in (X, τ^+) . Then $(X-A)$ is $\tau^+_p g$ open and by (iii), $f(X-A) = Y - f(A)$ is $\tau^+_p g$ open in (Y, σ^+) .

Therefore $f(A)$ is $\tau^+_p g$ closed in (Y, σ^+) . Thus f is $\tau^{+*}_p g$ closed map

(iii) \Rightarrow (i), Assume that A is $\tau^+_p g$ closed in (X, τ^+) . Now by (iii), $f(A)$ is $\tau^+_p g$ closed in (Y, σ^+) but $f(A) = (f^{-1})^{-1}V$. Thus (i) holds.

Definition 3.2.8:

A bijection $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is called a $\tau^{+*}_p g$ homeomorphism if f is $\tau^+_p g$ irresolute and f its inverse f^{-1} is also $\tau^+_p g$ irresolute.

Remark 3.2.9:

The spaces $(X, \tau^+) \rightarrow (Y, \sigma^+)$ are $\tau^{+*}_p g$ homeomorphism if there exists a $\tau^{+*}_p g$ homeomorphism from (X, τ^+) onto (Y, σ^+)

The family of all $\tau^{+*}_p g$ homeomorphism from (X, τ^+) onto itself is denoted by $\tau^{+*}_p g h(X, \tau^+)$.

Proposition 3.2.10:

For any space (X, τ^+) , $\tau^{+*}_p g h(X, \tau^+) \subset \tau^+_p g h(X, \tau^+)$.

Proof:

The proof follows from the fact that every $\tau^+_p g$ irresolute function is $\tau^+_p g$ continuous and every $\tau^+_p g$ open map is $\tau^{+*}_p g$ open.

Proposition 3.2.11:

The set $\tau^{+*}_p g h(X, \tau^+)$ forms a group.

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Proof:

Now let us define $\mu : \tau^{+*}_p g h (X, \tau^+) \times \tau^{+*}_p g h (X, \tau^+) \rightarrow \tau^{+*}_p g h (X, \tau^+)$

By $\mu (f, g) = g \circ f$ for every $f, g \in \tau^{+*}_p g h (X, \tau^+)$.

The composition of functions is associative. The identity mapping

$I : (X, \tau^+) \rightarrow (X, \tau^+)$ which belongs to $\tau^{+*}_p g h (X, \tau^+)$ is the identity element

$f \in \tau^{+*}_p g h (X, \tau^+)$, $f \circ f^{-1} = f^{-1} \circ f = I$. Thus inverse exists for each element of

$\tau^{+*}_p g h (X, \tau^+)$. Hence $\tau^{+*}_p g h (X, \tau^+)$ is a group under the operation of composition of map.

Definition 3.2.12:

A topological space (X, τ^+) is called as $\tau^+_p g$ space if every subset of X is $\tau^+_p g$ closed.

Proposition 3.2.13:

If (X, τ^+) is a $\tau^+_p g$ space, then $h (X, \tau^+)$ is a subgroup of $\tau^{+*}_p g h (X, \tau^+)$.

Proof :

Since (X, τ^+) is a $\tau^+_p g$ space, any function $f : (X, \tau^+) \rightarrow (X, \tau^+)$ is $\tau^+_p g$ irresolute and $\tau^{+*}_p g$ open.

Thus then $h (X, \tau^+)$ is a subset of $\tau^{+*}_p g h (X, \tau^+)$. Hence $h (X, \tau^+)$ is a subgroup of $\tau^{+*}_p g h (X, \tau^+)$.

. Since $h (X, \tau^+)$ is a group with the binary operation μ of $\tau^{+*}_p g h (X, \tau^+)$.

Theorem 3.2.14:

If $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ is a $\tau^{+*}_p g$ homeomorphism, then it induces an isomorphism from the group $\tau^{+*}_p g h (X, \tau^+)$ onto $\tau^{+*}_p g h (Y, \sigma^+)$ where $\tau^{+*}_p g h (X, \tau^+)$ denotes the group of all $\tau^{+*}_p g$ homeomorphism from (X, τ^+) onto itself.

Proof :

Assume that $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ is a $\tau^{+*}_p g$ homeomorphism Now define

$f^* : \tau^{+*}_p g h (X, \tau^+) \rightarrow \tau^{+*}_p g h (Y, \sigma^+)$ by $f^*(h) = f \circ h \circ f^{-1}$ for every $h \in \tau^{+*}_p g h (X, \tau^+)$. f^* is a bijection, since composition of bijections is a bijection.

Now $f^*(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = f^*(h_1) \circ f^*(h_2)$.

Thus f^* is an isomorphism induced by f .

Proposition 3.2.15:

If a $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ is $\tau^{+*}_p g$ irresolute, then for every subset A of X ,

$f(\tau^+_p g \text{ cl}(A)) \subset p^+ \text{ cl}(f(A))$.

Proof :

For each $A \subset X$, $p^+ \text{ cl}(f(A))$ is $\tau^+_p g$ closed in Y . By hypothesis, $f^{-1}(p^+ \text{ cl}(f(A)))$ is $\tau^+_p g$ closed in X . Since $A \subset f^{-1}(f(A)) \subset f^{-1}(p^+ \text{ cl}(f(A)))$ and by the definition of $\tau^+_p g$ closure, $\tau^+_p g \text{ cl}(A) \subset f^{-1}(p^+ \text{ cl}(f(A)))$. Thus $f(\tau^+_p g \text{ cl}(A)) \subset p^+ \text{ cl}(f(A))$.

Proposition 3.2.16:

If a function $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ is $\tau^{+*}_p g$ irresolute, then for all $B \in Y$,

$\tau^+_p g \text{ cl}(f^{-1}(B)) \subset f^{-1}(p^+ \text{ cl}(B))$.

Proof:

$p^+ \text{ cl}(B)$ is $\tau^+_p g$ closed in Y and by hypothesis, $f^{-1}(p^+ \text{ cl}(B))$ is $\tau^+_p g$ closed in X . As $f^{-1}(B) \subset f^{-1}(p^+ \text{ cl}(B))$, $\tau^+_p g \text{ cl}(f^{-1}(B)) \subset f^{-1}(p^+ \text{ cl}(B))$.

Theorem 3.2.17:

$\tau^{+*}_p g$ homeomorphism is an equivalence relation

Proof:

Reflexivity & symmetry are obvious. The composition of $\tau^+_p g$ irresolute maps is $\tau^+_p g$ irresolute proves transitivity.

IV. CONCLUSION

Thus a new set is defined which can be added to the array of existing sets. The properties and applications of these new sets are deduced which forms a platform for further research.

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