

Heat Conduction Problem in a Thick Annular Disc Due to Arbitrary Axisymmetric Heat Flux

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ABSTRACT: In this paper attempt has been made to discuss steady state quasi static thermal stresses in a thick annular disc $a \leq r \leq b$, $-h \leq z \leq h$ subjected to arbitrary heat flux at upper and lower surface of annular disc while inner and outer circular surface of thick annular disc are maintained at zero degree temperature. The governing heat conduction have been solved by using integral transform technique. The results are obtained in series form in term of Bessel's functions, The result of displacement and stresses have been computed numerically and illustrated graphically.

KEYWORDS: Quasi-static, Thermoelastic problem, Thermal Stresses, Axisymmetric Thermal Stresses.

I. INTRODUCTION

Nowacki (1957) [6] has determined steady-state thermal stresses in circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge. Roy Choudhary (1972) (1973) and Wankhede (1982) [13] determined Quasi static thermal stresses in thin circular plate. Gogulwar and Deshmukh (2005) [1] determined thermal stresses in thin circular plate with heat sources. Whereas Qian & Batra (2004) [7] studied transient thermoelastic deformation of thick functionally graded plate under lateral loads and obtained the results for radial and axial displacements and temperature change moreover Sharma et al (2004) [10] studied the behavior of thermoelastic thick plate under lateral loads and obtained the results for radial and axial displacements and temperature change have been computed numerically and illustrated graphically for different theories of generalized thermoelasticity. Recently Ruhi et al (2005) [9] did thermoelastic analysis of thick walled finite length cylinders of functionally graded materials and obtained the results for stress, strain and displacement components through the thickness and along the length are presented due to uniform internal pressure and thermal loading. V. S. Kulkarni, K. C. Deshmukh [12] considered a thick annular disc which is subjected to a transient axisymmetric temperature field on the radial and axial directions of the cylindrical coordinate system and determined the expression for temperature, displacement and stress functions due to arbitrary heat flux on the upper and lower surface. Deshmukh et al [3] studied two dimensional nonhomogeneous boundary value problem of heat conduction and it's thermal deflection of a semi-infinite circular plate on the outer curved surface for an infinite length. Deshmukh et al [4] determined the thermal stresses induced by a point heat source in a circular plate by quasi static approach.

II. STATEMENT OF THE PROBLEM

Consider a thick annular disc defined by $a \leq r \leq b$, $-h \leq z \leq h$. Let the disc be subjected to a axisymmetric temperature field on the radial direction in the cylindrical co-ordinate system. Initially the plate kept at zero temperature the arbitrary the heat flux $Q f(r)/\lambda$ is prescribed over the upper surface ($z = h$) and the lower surface ($z = -h$) the fixed circular edges ($r = a$, $r = b$) are at zero temperature. Assume the lower and upper surface of thick annular disc are traction free. Under this more realistic prescribed condition, the quasi state thermal stresses are required to be determined.

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Displacement Potential:

The differential equation and governing the displacement potential function $\phi(r, z, t)$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = k\tau \quad (1)$$

where K is restraint coefficient and temperature change $\tau = T - T_i$, T_i is the initial temperature. The displacement potential function ϕ is known as Goodier's thermoelastic displacement potential.

Heat Condition Equation:

The temperature of the plate at time t satisfies the Heat Condition equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \quad (2)$$

with the boundary conditions

$$\frac{\partial T}{\partial z} = \pm Q f(r) / \lambda \quad \text{at} \quad z = \pm h \quad \text{for} \quad a \leq r \leq b \quad (3)$$

$$T = 0 \quad \text{at} \quad r = a \quad \text{for} \quad -h \leq z \leq h \quad (4)$$

$$T = 0 \quad \text{at} \quad r = b \quad \text{for} \quad -h \leq z \leq h \quad (5)$$

Thermal Stresses:

The displacement function in a cylindrical co-ordinate system is represented by the Michell's function defined as in [5].

$$U_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 M}{\partial r \partial z} \quad (6)$$

$$U_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \quad (7)$$

the Michell's function M must satisfy

$$\nabla^2 \nabla^2 M = 0 \quad (8)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \quad (9)$$

the Component stresses are represented by thermoelastic displacement potential ϕ and Michell's function M as

$$\sigma_{rr} = 2G \left[\frac{\partial^2 \phi}{\partial r^2} - k\tau + \frac{\partial}{\partial z} \left(\nu \nabla^2 M - \frac{\partial^2 M}{\partial r^2} \right) \right] \quad (10)$$

$$\sigma_{\theta\theta} = 2G \left[\frac{1}{r} \frac{\partial \phi}{\partial r} - k\tau + \frac{\partial}{\partial z} \left(\nu \nabla^2 M - \frac{1}{r} \frac{\partial M}{\partial r} \right) \right] \quad (11)$$

$$\sigma_{zz} = 2G \left[\frac{\partial^2 \phi}{\partial r^2} - k\tau + \frac{\partial}{\partial z} \left((2-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \right) \right] \quad (12)$$

and

$$\sigma_{rz} = 2G \left[\frac{\partial^2 \phi}{\partial r \partial z} - k\tau + \frac{\partial}{\partial z} \left((1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \right) \right] \quad (13)$$

where G and ν are shear modulus and Poisson's ratio respectively.

For traction free surface stress function.

$$\sigma_{rr} = \sigma_{rz} = 0 \quad \text{at} \quad z = \pm h \quad (14)$$

the equation (1) to (14) constitute mathematical formulation of the problem.

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III. SOLUTION OF THE PROBLEM

The temperature change Introducing the finite the Hankel transform over the variable r and its inverse its transform defined as in.[11]

$$\bar{T}(\alpha_m, z) = \int_a^b r K_0(\alpha_m r) T(r, z) dr \tag{15}$$

$$T(r, z) = \sum_{m=1}^{\infty} \bar{T}(\alpha_m, z) K_0(\alpha_m r) \tag{16}$$

where

$$K_0(\alpha_m r) = \frac{R_0(\alpha_m r)}{\sqrt{N}} \tag{17}$$

$$R_0(\alpha_m r) = \left[\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right] \tag{18}$$

and the normality constant as

$$N = \frac{b^2}{2} R_0^2(\alpha_m b) - \frac{a^2}{2} R_0^2(\alpha_m a) \tag{19}$$

where $\alpha_1, \alpha_2, \dots$ are roots of the transcendental equation

$$\left[\frac{J_0(\alpha a)}{J_0(\alpha b)} - \frac{Y_0(\alpha a)}{Y_0(\alpha b)} \right] = 0 \tag{20}$$

with $J_n(x)$ is the Bessel function of the first kind and $Y_n(x)$ is the Bessel function of the second kind of order n .

The transform satisfies the relation

$$H \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] = -\alpha_m^2 \bar{T}(\alpha_m, z) \tag{21}$$

and

$$H \left[\frac{\partial^2 T}{\partial z^2} \right] = \frac{d^2 \bar{T}}{dz^2} \tag{22}$$

On applying the finite Hankel transform defined in the equation (15) to equation (2) one obtains

$$\frac{\partial^2 \bar{T}}{\partial z^2} - \alpha_m^2 \bar{T} = 0 \tag{23}$$

where \bar{T} is the Hankel transform of T on solving the equation (23) one obtains

$$\bar{T} = A e^{\alpha_m z} + B e^{-\alpha_m z} \tag{24}$$

On solving the equation (24) under the condition given in the equation (3) one obtains

$$A = B = \frac{Q \bar{f}(\alpha_m)}{\alpha_m \lambda} \left[\frac{\cosh(\alpha_m h)}{\sinh(2\alpha_m h)} \right]$$

Using the equation (24) one obtains

$$\bar{T} = \frac{2Q \bar{f}(\alpha_m)}{\alpha_m \lambda} \left[\frac{\cosh(\alpha_m h) \cosh(\alpha_m z)}{\sinh(2\alpha_m h)} \right] \tag{25}$$

here $\bar{f}(\alpha_m)$ is the Hankel transform of $f(r)$.

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On Applying the inverse Hankel transform defined is the equation (16) one obtains

$$T = \frac{Q}{\lambda} \sum_{n=1}^{\infty} \left(\frac{\bar{f}(\alpha_m)}{\sqrt{N}} \right) \left[\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right] \times \left[\frac{\cosh(\alpha_m z)}{\alpha_m \sinh(\alpha_m h)} \right] \quad (26)$$

Since the initial temp $T_i=0$,

$$\tau = T - T_i \Rightarrow T = \tau \quad (27)$$

Michell's function M

Now assume Michell's function M which satisfy the condition (8)

$$M = \frac{QK}{\lambda} \sum_{m=1}^{\infty} \left[\frac{\bar{f}(\alpha_m)}{\sqrt{N}} \left[\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right] \times (H_n \cosh(\alpha_m z) + R_n \alpha_m z \sinh(\alpha_m z)) \right] \quad (28)$$

where H_n and R_n are arbitrary function which can be determined finally using the condition (14)

Goodiers Thermoelastic Displacement Function ϕ

To obtains the displacement potential ϕ using the equation (26) and (27) in equation (1) one have

$$\phi = \frac{QK}{\lambda} \sum_{n=1}^{\infty} \frac{\bar{f}(\alpha_m)}{\sqrt{N}} \left[\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right] \times \left[\frac{(z \sinh(\alpha_m z))}{z \alpha_m^2 \sinh(\alpha_m h)} \right] \quad (29)$$

Displacement and Thermal Stresses

Now using the equation (26), (27), (28) and (29) in the equation (6), (7), (10), (11), (12) and (13) one obtains the expression for displacement and stresses respectively as

$$U_r = \frac{QK}{\lambda} \sum_{m=1}^{\infty} \left(\frac{\bar{f}(\alpha_m)}{\sqrt{N}} \right) \left[\frac{J_1(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_1(\alpha_m r)}{Y_0(\alpha_m b)} \right] \left\{ \left[\frac{-z \sinh(\alpha_m z)}{2\alpha_m \sinh(\alpha_m h)} \right] + \left[\alpha_m^2 H_n \sinh(\alpha_m z) + \alpha_m^2 R_n (\sinh(\alpha_m z) + \alpha_m z \cosh(\alpha_m z)) \right] \right\} \quad (30)$$

$$U_z = \frac{QK}{\lambda} \sum_{m=1}^{\infty} \frac{\bar{f}(\alpha_m)}{\sqrt{N}} \left[\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right] \left\{ \frac{\sinh(\alpha_m z) + \alpha_m z \cosh(\alpha_m z)}{2\alpha_m^2 \sinh(\alpha_m h)} - \alpha_m^2 H_n \cosh(\alpha_m z) + \alpha_m^2 R_n \langle 2(1-2\nu) \cosh(\alpha_m z) - \alpha_m z \sinh(\alpha_m z) \rangle \right\} \quad (31)$$

$$\sigma_{rr} = \frac{2GQK}{\lambda} \sum_{m=1}^{\infty} \frac{\bar{f}(\alpha_m)}{\sqrt{N}} \left\{ \left[\frac{\dot{J}_1(\alpha_m r)}{J_0(\alpha_m b)} - \frac{\dot{Y}_1(\alpha_m r)}{Y_0(\alpha_m b)} \right] \left[\frac{-z \sinh(\alpha_m z)}{2\alpha_m \sinh(\alpha_m h)} \right] - \left[\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right] \left[\frac{\cosh(\alpha_m z)}{\alpha_m \sinh(\alpha_m h)} \right] \right\}$$

$$+ H_n \alpha_m^2 m \left[\frac{\dot{J}_1(\alpha_m r)}{J_0(\alpha_m b)} - \frac{\dot{Y}_1(\alpha_m r)}{Y_0(\alpha_m b)} \right] \sinh(\alpha_m z) + \alpha_m^2 R_n$$

$$\left\langle 2\nu \alpha_m \left(\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right) \sinh(\alpha_m z) \right\rangle$$

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$$\left. \left[\frac{J_1(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_1(\alpha_m r)}{Y_0(\alpha_m b)} \right] \left(\sinh(\alpha_m z) + (\alpha_m z) \cosh(\alpha_m z) \right) \right\} \quad (32)$$

$$\begin{aligned} \sigma_{\theta\theta} = & \frac{2GQK}{\lambda} \sum_{m=1}^{\infty} \left(\frac{\bar{f}(\alpha_m)}{\sqrt{N}} \right) \left\{ \left[\frac{J_1(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_1(\alpha_m r)}{Y_0(\alpha_m b)} \right] \left[\frac{-z \sinh(\alpha_m z)}{2\alpha_m \sinh(\alpha_m h)} \right] \right. \\ & - \left. \left[\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right] \times \left[\frac{\cosh(\alpha_m z)}{\alpha_m \sinh(\alpha_m h)} \right] \right. \\ & + H_n \alpha_m^2 \left(\frac{1}{r} \right) \left[\frac{J_1(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_1(\alpha_m r)}{Y_0(\alpha_m b)} \right] \sinh(\alpha_m z) + R_n \alpha_m^2 \langle 2\nu \alpha_m \\ & \left. \left[\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right] \right. \\ & \left. \left. \sinh(\alpha_m z) + \frac{1}{r} \left[\frac{J_1(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_1(\alpha_m r)}{Y_0(\alpha_m b)} \right] \left[\sinh(\alpha_m z) + (\alpha_m z) \cosh(\alpha_m z) \right] \right\} \quad (33) \end{aligned}$$

$$\begin{aligned} \sigma_{zz} = & \frac{2GQK}{\lambda} \sum_{m=1}^{\infty} \left(\frac{\bar{f}(\alpha_m)}{\sqrt{N}} \right) \left\{ \left[\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right] \left[\frac{z \sinh(\alpha_m z)}{2 \sinh(\alpha_m h)} \right] \right. \\ & \left. - \alpha_m^3 H_n \sinh(\alpha_m z) + \alpha_m^3 R_n \left[(1 - 2\nu) \sinh(\alpha_m z) - (\alpha_m z) \cosh(\alpha_m z) \right] \right\} \quad (34) \end{aligned}$$

$$\begin{aligned} \sigma_{rz} = & \left(\frac{2GQK}{\lambda} \right) \sum_{m=1}^{\infty} \frac{\bar{f}(\alpha_m)}{\sqrt{N}} \left[\frac{J_1(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_1(\alpha_m r)}{Y_0(\alpha_m b)} \right] \times \\ & \left\{ \left[\frac{-\sinh(\alpha_m z) - (\alpha_m z) \cosh(\alpha_m z)}{2\alpha_m \sinh(\alpha_m h)} \right] \right. \\ & \left. + \alpha_m^3 H_n \cosh(\alpha_m z) + \alpha_m^3 R_n \langle 2\nu \cosh(\alpha_m z) + (\alpha_m z) \sinh(\alpha_m z) \rangle \right\} \quad (35) \end{aligned}$$

Determination of Unknown Arbitrary function H_{mn} and R_{mn}

In order to satisfy the condition (14) solving the equation (21) and (22) for H_n and R_n , one obtains.

$$H_n = \frac{[(1 - 2\nu) \sinh(\alpha_m h) \cosh(\alpha_m h)] [\sinh(\alpha_m h) + (\alpha_m h) \cosh(\alpha_m h)]}{2\alpha_m^4 \sinh(\alpha_m h) [\sinh(\alpha_m h) \cosh(\alpha_m h) - (\alpha_m h)]} \quad (36)$$

$$R_n = \frac{\sinh(\alpha_m h)}{2\alpha_m^4 [\sinh(\alpha_m h) \cosh(\alpha_m h) - (\alpha_m h)]} \quad (37)$$

Using the value of H_n and R_n in the equation (15) to (22) one obtains the expression for displacement and stresses

IV. SPECIAL CASE

Setting

$$f(r) = (r^2 - \alpha^2)^2 (r^2 - b^2)^2 \quad (38)$$

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Applying finite Hankel transform to (38), one obtains

$$\bar{f}(\alpha_m) = \int_a^b \frac{1}{\sqrt{N}} \left[\frac{J_0(\alpha_m r)}{J_0(\alpha_m b)} - \frac{Y_0(\alpha_m r)}{Y_0(\alpha_m b)} \right] r(r^2 - \alpha^2)(r^2 - b^2) dr,$$

$$\bar{f}(\alpha_m) = \frac{8 \left\{ (a^2 \alpha_m^2 - 3b^2 \alpha_m^2 + 16) J_0(\alpha_m a) - (b^2 \alpha_m^2 - 3a^2 \alpha_m^2 + 16) J_0(\alpha_m b) \right\}}{\pi \sqrt{N} a_m^6 J_0(\alpha_m a) J_0(\alpha_m b) Y_0(\alpha_m b)} \quad (39)$$

V. NUMERICAL RESULTS, DISCUSSION AND REMARKS

The numerical calculation have been carried out for steel (SN 50C) plate with parameters $a = 1m$, $b = 2 m$, $h=0.3 m$, Thermal diffusivity $k= 15.9 \times 10^{-6} (m^2 s^{-1})$ and Poisson ratio $\nu = 0.281$ with $\alpha_1 = 3.120$, $\alpha_2 = 6.2734$, $\alpha_3 = 9.4182$, $\alpha_4 = 12.5614$, $\alpha_5 = 15.7040$.. are the roots of transcendental equation $Y_0(\alpha b) J_0(\alpha a) - Y_0(\alpha a) J_0(\alpha b) = 0$. In order to examine the influence of heat flux on the upper and lower surface of thick plate, one performed the numerical calculations $r = 1, 1.2, 1.4, 1.6, 1.8, 2 m$ and $z = -0.3, -0.15, 0, 0.15, 0.3 m$ Numerical variations in radial and axial directions are shown in the figures with the help of computer program.

VI. CONCLUSION

In this problem, a thick annular disc is considered which is free from traction and determined the expressions for temperature, displacement and stress function due to arbitrary heat flux under steady state. As a special case mathematical model is constructed for

$$f(r) = (r^2 - a^2)^2 (r^2 - b^2)^2$$

and performed numerical calculations. The thermoelastic behavior is examined such as temperature, displacement and stresses with the help of arbitrary heat applied.

Figure 1: The temperature is maximum at the middle of the thick annular disc and symmetrical to words outer and inner circular surfaces.

Figure 2: The stress function σ_{rr} is maximum in 1/4 the 3/4 middle region of the thick annular disc and reduces to zero towards inner and outer surfaces of the thick annular disc.

Figure 3 : The stress function σ_{zz} is maximum at $r = 1.4$ and varies in the thickness.

Figure 4: The stress function $\sigma_{\theta\theta}$ is maximum near to 1.5 and varies with the thickness of the plate.

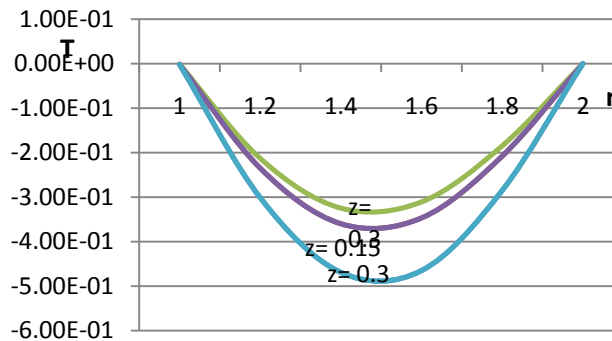


Fig. 1: Temperature T versus r for different values of z at $z = -0.3, -0.15, 0, 0.15, 0.3$

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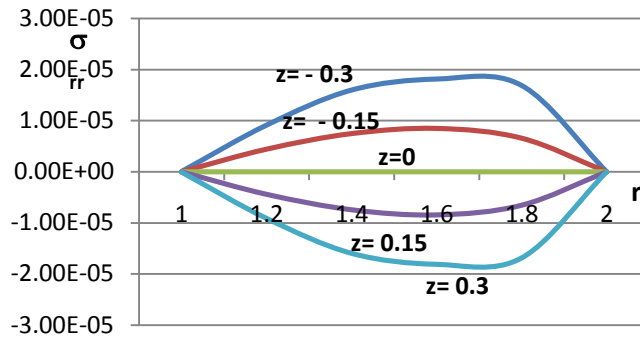


Fig. 2: Stress functions σ_{rr} versus r for different values of z at $z = -0.3, -0.15, 0, 0.15, 0.3$

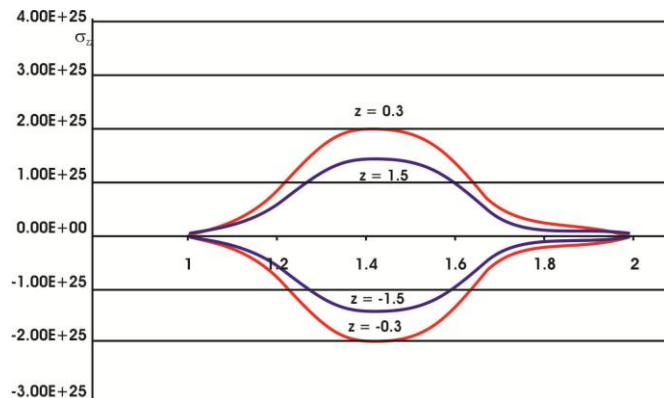


Fig. 3: Stress function σ_{zz} versus r for different values of z at $z = -0.3, -0.15, 0, 0.15, 0.3$

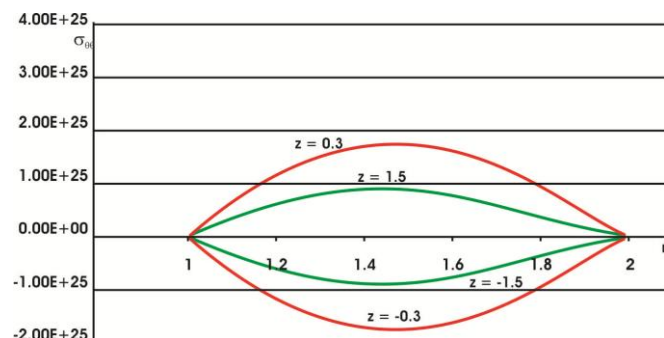


Fig. 4: Stress function $\sigma_{\theta\theta}$ versus r for different values of z at $z = -0.3, -0.15, 0, 0.15, 0.3$

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